STUDY FOR EVALUATION OF UNDULATOR MAGNETIC FIELD USING VIBRATING WIRE METHOD

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Abstract

We have constructed a planer undulator that is Halbach type composed of permanent magnet blocks. The period length of the undulator and the number of periods are 100 mm and 25, respectively. The vibrating wire method is studied to measure the periodic magnetic field of the undulator. A thin copper-beryllium wire is placed on the beam axis in the undulator. An AC current flow is applied in the wire, then by measuring amplitudes and phases of standing waves excited on the wire, we can reconstruct the magnetic field distribution along the wire numerically. In this paper, we discuss relations between reproducibility of the undulator field and the mode harmonics number incorporated into the reconstruction of undulator field.

INTRODUCTION

A test accelerator for a terahertz source project (t-ACTS) has been progressed at the Electron Light Science Centre, Tohoku University, in which a generation of intense coherent terahertz radiation from the very short electron bunch will be demonstrated [1,2]. A narrow-band coherent terahertz radiation using an undulator has been considered to be implemented. We have constructed a planer undulator for the THz radiation.

It is important to know a magnetic characteristic of an undulator before installation to estimate a beam orbit in an undulator and a radiation power from electron beam. Magnetic field of the THz undulator was measured by Hall probes in the manufacturing company. But this measuring with Hall probe requires high precision mechanical positioning systems and a long-linear stage to move the Hall probes along the undulator. The measurement system is very expensive and may not be easily available in our laboratory. Therefore, we started to study and develop a suitable method for the THz undulator.

Several methods are commonly used for magnetic field measurement of an undulator magnetic field. These are mapping with Hall probes, scanning with rotating coils [3], the pulsed wire technique [4], and so on. Though mapping with Hall probes can provide enough accuracy, require high precision mechanical positioning systems. Using the rotating coil, an integrated magnetic field on the rotating axis can be measured precisely. In case of undulator, an amount of beam deflection due to magnetic field error can be derived from this measurement. However, the rotating wire can not measure a distribution of a periodic undulator field. The pulsed wire method does not require special equipment, so that the arrangement can be made simple. However, a serious problem is caused by the distortion of the pulse signal during its propagation along the wire. For this reason, this method is unbefitting for longer undulators.

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The vibrating wire method [5-7], also does not require special equipment, because it uses as a probe wire stretched through the measured magnetic field, the apparatus can be made simple. This method can measure the periodic undulator field and we can find a local magnetic error by comparing with ideal magnetic field distribution. In the case of the undulator having large number of periods or short period length, the number of modes which we have to measure increases and the vibration frequency will also become high. Therefore, this vibration wire method is unsuitable for the undulator has a large number of periods or a short period length. Conversely, the vibrating wire method suits to measure the undulator having a small number of periods and a long period length.

TERAHERTZ UNDULATOR

The THz undulator is a planer undulator of Halbach type made only of permanent magnet blocks [2]. The longitudinal magnetized blocks were installed at both ends of the undulator in order to correspond an axis of injection of electron beam with the magnetic center of undulator. Table 1 shows parameters of our undulator for coherent terahertz radiation. Figure 1 shows the magnetic field distribution of the undulator Bu(z) at the gap of 54 mm measured by Hall probes. Undulator gap moves horizontally and the electron beam oscillates in vertical plane. Height of the beam axis is 750 mm from the floor. The gap changes 44~110 mm, while actual experiment has been considered to be implemented at gap = 54~68 mm, because the undulator has been developed for the terahertz FEL by the free space mode using the optical cavity [1]. This undulator produces the terahertz radiation and its wavelength is 360~170 μm (0.8~1.7 THz) with 17 MeV electron beam.

Table 1: THz Undulator Parameters

Undulator type	Halbach planer type
Size of magnets	110×65×25 mm^3
Material and coating	Nd-Fe-B・TiN
Period length and number	100mm・25 periods
Undulator length	2.532 m
Peak magnetic strength	0.41 T (g = 54mm)
K value	3.82 (max)

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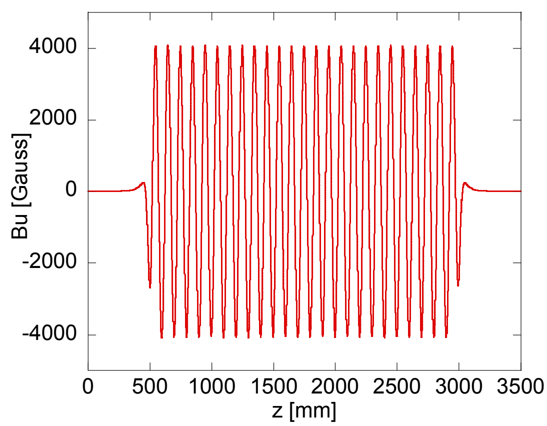

Figure 1: Measured magnetic field of the THz undulator with gap = 54 mm.

THEORY OF VIBRATING WIRE METHOD

The vibrating wire method employs a wire stretched through the measured magnetic field. The Lorentz forces between AC current driving through the wire and surrounding magnetic field cause the string vibration. If the frequency of a string vibration matches to eigenmode frequency, the resonant standing wave is excited on the wire. Measuring the amplitude and phase of the resonance modes, we can derive the harmonic mode of the magnetic field and reconstruct the undulator field by means of the inverse Fourier transformation.

Figure 2 shows the experimental setup. A wire with the tension T and the length L has the fixed ends at z = 0 and z = L. The AC current, I(t), in the wire, depends on time as $I(t) = I_0\exp(i\omega t)$. There are two forces that affect the wire. They are gravity μg, where μ is mass of wire per unit of length, and the Lorentz force $I(t)\times B_u(z)$.

The equation for vertical wire position, U(z,t), will be

$$\mu\frac{\partial^2 U}{\partial t^2}=T\frac{\partial^2 U}{\partial z^2}-\gamma\frac{\partial U}{\partial t}-\mu g+Bu(z)I(t) \qquad (1)$$

Here, the γ is damping constant. A general solution may be written in the form

$$U(z,t)=U_g(z)+U_d(z,t) \qquad (2)$$

$$U_d(z,t)=U_b(z)\exp(i\omega t) \qquad (3)$$

The vertical wire position can be described by the

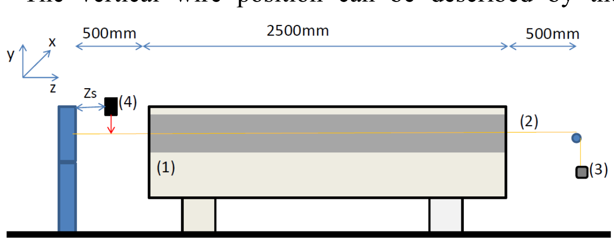

Figure 2: Experimental setup of vibration wire method. (1) Undulator, (2) Copper-beryllium wire (3.5 m, 100μm diameter), (3) Weight, (4) Laser position sensor

gravity term U_g and the dynamical term U_d. The dynamical term U_d is can be represented by Fourier sine series U_b:

$$U_b=\sum_{n=1}^{\infty}U_n\sin(n\pi z/L) \qquad (4)$$

Where U_n is the n-th order Fourier coefficient. The magnetic field $B_u(z)$ may be represented in the same way:

$$Bu(z)=\sum_{n=1}^{\infty}B_n\sin(n\pi z/L) \qquad (5)$$

With these equations, one can find a relation between U_d and B_n:

$$U_d(z,t)=\sum_{n=1}^{\infty}\frac{B_n\sin(n\pi z/L)}{\mu(\omega^2-\omega_n^{\ 2}+i\gamma\omega)}I_0\cdot e^{i\omega t} \qquad (6)$$

where ω_n is the resonance frequency of the n-th mode. We will reconstruct the magnetic field $B_u(z)$ using Eqs.(5) and (6).

UNDULATOR FIELD MEASUREMENT USING VIBRATING WIRE METHOD

Estimation of wire sag

In the experiment, we are planning to use the copper-beryllium wire with 100μm diameter and a length of the wire will be 3.5 m as shown in Fig. 2. An expression for the gravity term $U_g(z)$ is

$$U_g(z)=-\frac{\mu g}{2T}z(z-L). \qquad (7)$$

The $U_g(z)$ corresponds to a wire sag. In the vibrating wire method, the magnetic field at the position in which only the amount of the sag shifts will be measured. Note that at the position z = L/2, the function $U_g(z)$, that is, the wire sag becomes maximum. With the 1 kg weight, the maximum wire sag (S) will be

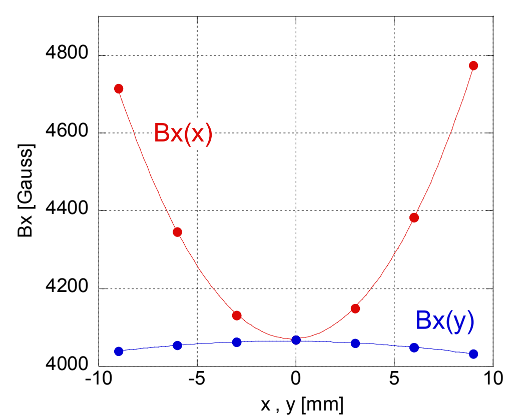

Figure 3: Measured magnetic field distribution in transverse plane at the longitudinal centre of the THz undulator with gap = 54 mm. Since the electron beam oscillates in the x-z plane in the THz undulator, the Bx is the main field component of the undulator.

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$$S = U_g(z=\frac{L}{2}) = \frac{\mu g}{8T}L^2 \approx 99.8[\mu m] \quad (8)$$

From the Eq. (8), the sag is about 100 µm in vertical direction (y-direction). Figure 3 shows the spatial distribution of magnetic field at the longitudinal centre of the undulator obtained with Hall probes measurement. It shows that the difference of the magnetic field strength between at y = 0 and at y = -100µm is 1.62×10^{-5} T which corresponds to 0.003% of peak field. Therefore the measurement error due to the wire sag is negligible in the experimental setup.

Resonance frequency of wire

Angular frequency ω_n and resonant frequency fn of wire vibration at the n-th order resonance are given by

$$\omega_n = 2\pi\frac{n}{2L}\sqrt{\frac{T}{\mu}} \approx 348.043\times n \quad (9)$$

and

$$f_n = \frac{\omega_n}{2\pi} = 55.392\times n, \quad (10)$$

respectively.

In the previously described setup, the fundamental resonant frequency is 55.4 Hz

Reconstruction error

We use the Fourier sine-series to reconstruct the magnetic field $B_u(z)$ of the undulator. The resolution of reconstruction will depend on how many harmonics are included. The reconstructed field distribution is compared with the design field distribution of the undulator. The design field is expanded into Fourier series, the coefficients B_n is the harmonics of a sine Fourier transformation as

$$B_n = \frac{2}{L}\int_0^L Bu(z)\sin(n\pi z/L)dz. \quad (11)$$

The magnetic field can be reconstructed using the inverse Fourier transformation as expressed by Eq. (12).

$$BS_m(z) = \sum_{n=1}^{m} B_n \sin(n\pi z/L). \quad (12)$$

Figure 4 shows the Fourier coefficient Bn up to the 200th harmonics. It shows the peak harmonics of the undulator field at around n = 70. The vibration wavelength of 70th harmonics corresponds to a period length of the undulator. From Eq. (12), we can get the reconstructed field BS_m and Fig. 5 shows the reconstructed field distributions using up to 75, 100, and 200 modes, respectively. Comparing these plots in Fig. 5, the design field and the reconstructed field incorporating higher modes are in good agreement. We define the integral error ERR_m as difference between $B_u(z)$ and $BS_m(z)$ integrating over the wire length in Eq. (13).

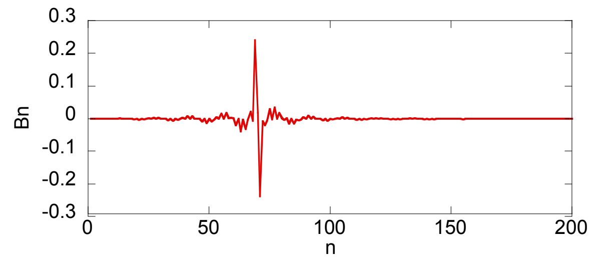

Figure 4: Fourier coefficient of the magnetic field, Bn., as function of mode number *n*.

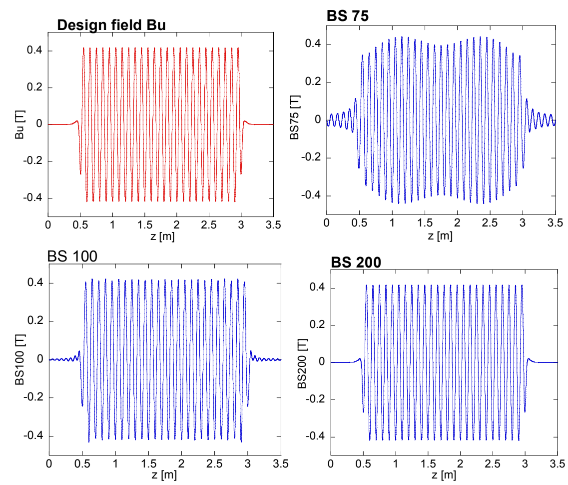

Figure 5: (Upper-left) Design magnetic field Bu(z). The reconstructed magnetic field $BS_m(z)$ using up until 75 (upper-right), 100 (bottom-left) and 200 (bottom-right) harmonics.

$$ERR_m = \int_{z=0}^{L}\sqrt{(Bu(z)-BS_m(z))^2}dz. \quad (13)$$

Figure 6 shows the relation between the mode number m and ERR_m. It is obvious that the error significantly decrease when using more than the 71 harmonics. The value ERR_m/L, in which ERR_m is divided by the length of the wire, can be considered make an average field error. The values for m = 100 and 150 are 1.2×10^{-4} and 2.17×10^{-5} T, respectively. To find a local magnetic field error that is larger than geomagnetism (~0.4 Gauss; 0.01 % of peak field), we should measure more than 135th harmonics. The 135th resonance frequency of wire vibration f_{135} is 7478 Hz.

Vibration amplitude of the wire

Here we consider actual experiments. The B_n is derived from measuring the amplitude and phase of wire vibration. The position of a wire position sensor is assumed to be z = z_s. From Eq. (6), the vibration amplitude of n-th harmonics $U_{d,n}(z_s, t)$ can be written as

$$U_{d,n}(z,t) = \frac{B_n\cdot\sin(\pi n z_s/L)\cdot I_0}{\mu\sqrt{(\omega^2-\omega_n{}^2)^2+(\omega\gamma)^2}}e^{i(\omega t-\theta)}, \quad (14)$$

where

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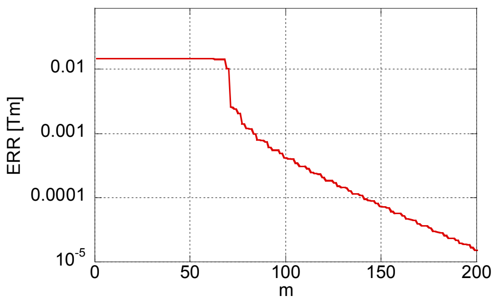

Figure 6: The integral error ERR_m as difference between Bu(z) and $BS_m(z)$ integrating over the wire length.

$$\theta = \tan^{-1}\left(\frac{\omega\gamma}{\omega^2 - \omega_n^2}\right) \tag{15}$$

and the driving current $I(t) = I_0 \exp(i\omega t)$. If the frequency of driving current is close to the resonant frequency ω_n, i.e., $\omega \sim \omega_n$, it causes a strong resonant vibration. The resonance term $U_{d,n}(z_s,t)$ will dominate over the rest harmonics mode. We scan the frequency of driving current through one of the resonant frequency ω_n. The wire vibration is directly measured using the wire position sensor and we record the amplitude and the relative phase between the sensor output signal and the driving AC current. Then we fit this amplitude data as a function of frequency using the formula

$$A_n(\omega) = \frac{a_n}{\sqrt{(\omega^2 - b_n^2)^2 + (\omega c_n)^2}}, \tag{16}$$

where a_n, b_n, and c_n are fitting parameters. The component B_n is obtained from a_n as

$$B_n = a_n \frac{1}{\sin(\pi n z_s / L)} \frac{\mu}{I_0}. \tag{17}$$

We numerically estimate the amplitude of wire at $z = z_s$. Figure 7 shows the vibration amplitude for the 69th harmonics. Here, the parameters are $\mu = 6.518\times10^{-5}$ kg/m, $\omega_n = 348.043\times n$ Hz, $I_0 = 10$ mA, $z_s = 0.025$ m, $\gamma = 1$. At the resonant frequency ω_n, the amplitude becomes the maximum and the width of peak is proportional to the damping constant γ. The phase θ depends on the frequencies ω, ω_n, and the damping constant γ and it flips by π at the ω_n. In an actual experiment, the damping constant will be derived from the fitting using Eq. (16). From Fig. 7, the peak amplitude of 69th harmonics is about 1.5 mm and this is the maximum for the all vibration modes. For comparison, the amplitude of the 135th mode is 0.5 μm in the same experimental condition. From the estimations of the vibration amplitude, the modes up to the 135th harmonic are able to be measured using the laser at least. Because, we plan to use the laser displacement sensor which has position resolution about 20 nm.

On the other hand, we calculate the vibration energies of the wire for the 69th and 135th harmonics mode. The

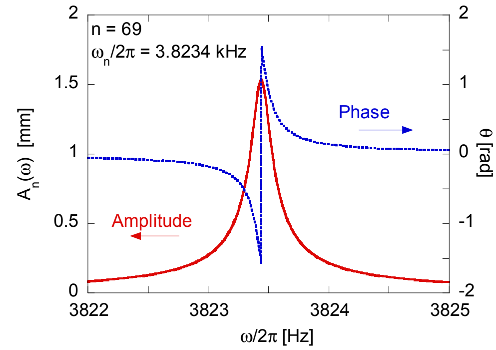

Figure 7: Vibration amplitude ($A_n(\omega)$) and phase (θ) of wire as function of the frequency of the driving current around the 69th harmonics.

vibration energy can be derived from the wire weight, amplitude and frequency of vibration in each mode. In the case of the 69th and 135th harmonic mode, the vibration energies are about 565 W and 4.63×10^{-3} W, respectively. It is understood that high power AC current source is not necessary from this numerical result in the actual experiment.

SUMMARY

We have numerically examined the magnetic field measurement of the THz undulator by using the vibrating wire method. The relation between reproducibility of the undulator field and the mode harmonics number used for the reconstruction of undulator field was derived by comparing the design field and the reconstructed field. To suppress the measurement error comparable with geomagnetism, it is necessary for the reconstruction up to the 135th harmonics. The vibration amplitude of wire was estimated by assuming an experimental setup. From the numerical result, the various vibration modes can be measured using a commercial laser displacement sensor. Moreover it is understood that the energy consumed in vibration of wire is small from the calculation. As a result of numerical examination, the vibrating wire method can be used for the magnetic field measurement of the THz undulator in the t-ACTS project.

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