CHANNELED POSITRONS AS A SOURCE OF GAMMA RADIATION

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Abstract

A possibility of channeling of low-energy ($5 \div 20\,Mev$) relativistic positrons with coaxial symmetry around separate crystal axes of negative ions in some types of crystals, is shown. The annihilation processes of positrons with medium electrons are investigated in details. The lifetime of a positron in the regime of channeling is estimated $10^{-6}\,sec$ which on a $10^9 \div 10^8$ is bigger than at usual cases.

INTRODUCTION

Studying of ways of generation of short-wave coherent radiation always was an essential problem of a science and was stimulated by its wide applied application. In work [1] one of the first the mechanism of radiation of relativistic electrons at their movement in periodic structures (see also [2]) was offered (later such types of structures have been named undulators). It at first sight imperceptible work later has essentially accelerated a process of creation of of modern devices the synchrophasotrons and the lasers on free electrons (see for example review [3]). In spite of the fact that the technology of generation of undulator radiation steadily is being developed and successes are obvious [4], however the problems also are obvious and they remain unresolved up to now. Let's note that the frequency of undulator radiation is being defined by length of its periodic element which in FELs and the undulators devices are macroscopic. Other defects of undulators their big sizes and used big energies of electrons. After opening of channeling of electrons (positrons) in crystals [5–7] and accompanying by its short-wave radiation [8] was appeared a hope to solve all the aforementioned problems. However these hopes up to now is justified only partially. In particular the problems of generation of short-wave radiation (x-ray) from less energetic electrons (of order a several Mev) on very small distances (of order a several micron) have been solved. Now a new problem arise. The point is that in the regime of channeling the particle (electrons and positrons) usually live very short time $\sim 10^{-14} \div 10^{-15}\,sec$. However this time is very short for conversion of a appreciable part of particle energy to energy of radiation. The short lifetime as well doesn't conduce to use of external factors for control by beam of channeling particles and to improving of spectral characteristics of radiation.

The quantum theory of channeling for electrons and positrons has been elaborated by many authors [8–10]. It is important to note that an electron in a crystal can commit both planar and axial channeling. At the same time only one type of real channeling for the positrons is known, the regime where a particle is localized between two adjacent planes. The possibility of axial channeling of positive particles has not been investigated, seriously up to now, because the crystallographic axes, irrespective of grade of crystal are been charged positively. However investigation of possibilities of axial channeling of positrons and, hence, the formation of metastable relativistic positron systems (PS) is a problem of utmost importance for a radiation physics.

In earlier studies [11] we investigated the possibilities of ionic crystals of type $CsCl$ and have shown that at channeling of positrons around the axises of negatively charged ions Cl^- the main factor of de-channeling the scattering of particles on phonons subsystem is absent. However such channels for positrons, as shows an analysis there are and in other more realistic crystals, for example in crystal SiO_2.

In this work a role of different processes (scattering of positron on media electrons, the annihilation processes etc) in the expansion of energy levels of relativistic PS is analyzed and shown that PSs are metastable.

FORMATION OF RELATIVISTIC POSITRON SYSTEM (PS)

In previous work [12] we have shown, that if a low-energy relativistic positrons ($5 \div 20\,Mev$) are scattering under a small corners $\vartheta \leq \vartheta_L$ (where ϑ_L is a Lindhard angle) on the axis $\langle 100 \rangle$ of chlorine ions Cl^- that they fall into regime of axial channeling. Moreover the motions of positrons concentrate between two cylinders that is very important from the point of view of movement stability. In particular, as was shown the effective $2D$ potential of channeling don't depend from temperature of media in a broad range of temperatures and has an order $-10\,eV$ depths of potential which is sufficient for formation a several quantum states of transverse motion. Recall that this type of effective potential can be in other crystals too. For example the effective potential of negatively charged ions O_2 axes often used in experiments of crystal SiO_2 is such.

In other words, the relativistic positrons in described regime of channeling don't interact with phonons subsystem. That means a main factor of de-channeling of particles in considered case is absent.

Taking into account the symmetry of effective potential for positrons around of the negative ions axis we can write the following analytical formula:

$$U_0(\rho) \quad = \quad D_0(e^{-2\alpha\overline{\rho}} - 2e^{-\alpha\overline{\rho}}), \overline{\rho} = (\rho - \rho_0)/\rho_0,$$

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ISBN 978-3-95450-126-7

$$\rho = \sqrt{x^2+y^2}, \tag{1}$$

where the parameters for usual crystals are being found into intervals $D_0 = 5 \div 10eV$, $\alpha = 0.5 \div 0.8$ and $\rho_0 \sim 0.5d$, where d is a lattice constant.

The full wavefunction of positron in the potential (1) in atomic units $\hbar = c = 1$ is being solved exactly and represented by the form [12]:

$$\Psi(\mathbf{r}) = \frac{1}{\sqrt{2\pi d}}\, e^{ip_z z}\Phi(\rho,\varphi), \qquad \mathbf{r} = \mathbf{r}(z,\rho,\varphi), \tag{2}$$

where $\Phi(\rho,\varphi)$ describes the wavefunction of localized state of positron system (PS) with the relativistic mass μ and is being characterised by quantum numbers of vibration n and rotation m correspondingly [13]:

$$\begin{aligned}\Phi(\rho,\varphi) &= \frac{1}{\sqrt{2\pi\rho}}\, e^{im\varphi} y^s \exp\Big(-\frac{y}{2}\Big)\, {}_1F_1(a,b,y),\\ m &= 0, \pm 1, \pm 2, ...,\end{aligned} \tag{3}$$

where the notations are made: $y = 2\gamma_2\alpha^{-1}\exp(-\alpha\overline{\rho})$ and $s = \beta\alpha^{-1}$, in addition:

$$a = \frac{1}{2}\Big(1+\frac{2\beta}{\alpha}\Big) - \frac{1}{\alpha}\Big(\frac{\gamma_1}{\gamma_2}\Big)^2, \quad b = \frac{2\beta}{\alpha}+1,$$

$$\beta^2 = -2\mu\varepsilon\rho_0^2 + Mc_0, \gamma_1^2 = 2\mu D_0\rho_0^2 - \frac{M}{\alpha}\Big(2-\frac{3}{\alpha}\Big),$$

$$\gamma_2^2 = 2\mu D_0\rho_0^2 - \frac{M}{\alpha}\Big(1-\frac{3}{\alpha}\Big), \quad M = m^2 - 1/4.$$

The eigenvalues of localized state is presented by the following formula:

$$\begin{aligned}\epsilon_{nm} &= \frac{1}{2\mu\rho_0^2}\Big\{-\gamma_0^2 + 2\alpha\gamma_0\Big(n+\frac{1}{2}\Big) - \alpha^2\Big(n+\frac{1}{2}\Big)^2\Big\}\\ &+ \frac{1}{2\mu\rho_0^2}\Big\{M - \frac{9}{4}\Big(\frac{\alpha-1}{\gamma_0}\Big)^2\frac{M^2}{\alpha^4}\Big\},\end{aligned} \tag{4}$$

where $\gamma_0 = \sqrt{2\mu D_0}\rho_0$, in addition: n is the vibrational quantum number and m correspondingly a quantum number characterizing the rotational motion.

PROCESSES LEADING TO DECAY OF POSITRON-SYSTEMS

After exception of major factor of dechanneling the three different processes still remain which lead to decay of PS:

a) The annihilation of positron with electron of media on one γ photon;

b) The annihilation of positron with electron of media on two γ photons;

c) The scattering of positron on electrons of media.

The main problem now is investigation of processes investment in lifetime of PS.

Decay PA on One γ Photon

It is obvious that PS may be illustrated as a system, with total zero spin (like of parapositronium), because the skeleton of negatively charged ions axis does not possess spin and accordingly there are not spin-spin interaction between it and the positron. In other words the interaction between positron and the axis only is an electromagnetic. Another distinction between positronium and PS is a possibility of decay of the last on one γ photon. Recall that in this case the conservation laws of energy and momentum takes place in view of presence of media.

This process is being defined by matrix element of first order [14]:

$$\langle f|S^{(1)}|i\rangle = \frac{2\pi ie}{\sqrt{2\omega}}\int \Psi^*(\mathbf{r})\hat{e}e^{-i\mathbf{qr}}\psi(\mathbf{r})d^3r\cdot\delta(\varepsilon_p+\varepsilon_e-\omega), \tag{5}$$

where Ψ and ε_p are the wavefunction and the total energy of positron, $\psi(\mathbf{r})$ and ε_e designed the wavefunction and the energy of media's electron, $\mathbf{q}$-is the momentum of γ photon, ω-its frequency. Taking into account that the distribution of electrons in positive as well in negative ions is given by spherical model JMGR, we can write the wavefunction of electrons system in factorized form:

$$\psi(\mathbf{r}) \equiv \psi(\rho,z,\varphi) = \frac{1}{\sqrt{d}}\, e^{i\eta(z)z}\chi(\rho), \tag{6}$$

where $\eta(z)$ is a momentum of media electron in the point z, it is supposed that the random function $\eta(z)$ has a zero average value. This means that the statistical averaging of random term in (6) is equal to unit $\langle e^{i\eta z/\hbar}\rangle = 1$. In addition $\chi(\rho)$ Gaussian function which is normalized to unit:

$$\chi(\rho) = \frac{1}{(\pi)^{3/4}\rho_0}\, e^{-\bar{\rho}^2/2}. \tag{7}$$

For the effective differential cross-section of annihilation we can write the following expression:

$$d\sigma = \frac{e^2}{2(2\pi)^2\omega}\sum_{\nu_e\,\nu_p}|Q|^2\delta(\varepsilon_p+\varepsilon_e-\omega)\omega^2\, do_\gamma, \tag{8}$$

where $do_\gamma = \sin\vartheta d\vartheta d\varphi$ the solid angle element in which is situated a photon momentum, ν_e and ν_e are a summation indices of electron and positron spins in the initial state. However for the considered problem the orientation of electrons spin does not play important role and correspondingly later we will ignore the summation. In (8) the term Q in weakly relativistic case is being written in the kind:

$$Q = \int \Psi^*(\mathbf{r})\mathbf{e}\hat{\mathbf{v}}\, e^{-i\mathbf{qr}}\psi(\mathbf{r})d^3r, \quad \hat{\mathbf{v}} = \frac{1}{i\mu}\nabla_{\mathbf{r}}, \tag{9}$$

where $\hat{\mathbf{v}}$ is the operator of positron velocity, correspondingly $\mathbf{e}$ - describes the unit vector of photon polarization. The expression (9) may be transformed to the form:

$$Q = \frac{(2\pi)^3}{\mu}\int(\mathbf{e}\mathbf{p}')\widehat{\Psi}^*(\mathbf{p}')\widehat{\psi}(\mathbf{q}-\mathbf{p}')\, d^3p',$$

ISBN 978-3-95450-126-7

where $\widehat{\Psi}^*(\mathbf{p}')$ and $\widehat{\psi}(\mathbf{q}-\mathbf{p}')$ are Fourier transforms of corresponding wavefunctions:

$$\widehat{\Psi}(\mathbf{p}') = \frac{1}{(2\pi)^3}\int \exp\left(-i\mathbf{p}'\mathbf{r}\right)\Psi(\mathbf{r})d^3r,$$
$$\widehat{\psi}(\mathbf{q}-\mathbf{p}') = \frac{1}{(2\pi)^3}\int \exp\left[-i(\mathbf{q}-\mathbf{p}')\mathbf{r}\right]\psi(\mathbf{r})d^3r. \quad (10)$$

In (10) $\mathbf{p}'$-designates the momentum of positron which in considered case in toto coincides with its p_z projection. Analyzing of expression for $\widehat{\psi}(\mathbf{q}-\mathbf{p}')$ shows that the transition amplitude is more probably if positron and photon momentums coincides i.e. $\mathbf{q}=\mathbf{p}'$. Using the equality $(\mathbf{e}\mathbf{p}') = p'\sin\vartheta'$ and above mentioned conclusion the expression (10) we can write in the following kind (see Fig 2):

$$Q = \frac{(2\pi)^3}{\mu}\widehat{\psi}(0)\int p'\,\widehat{\Psi}^*(\mathbf{p}')\,\sin\vartheta' d^3p', \quad (11)$$

where $d^3p' = p'_\perp d\theta dp'_\perp dp'_z$.
The function $\widehat{\Psi}(\mathbf{p}')$ is being calculated simply:

$$\widehat{\Psi}(\mathbf{p}') = \frac{(-1)^m}{(2\pi)^2}\frac{e^{im\theta}}{\sqrt{d}}\delta(p_z-p'_z)\Theta_m(p'_\perp),$$
$$\Theta_m(p'_\perp) = \int_0^\infty \Phi(\rho,0)J_m(p'_\perp\rho)\sqrt{\rho}\,d\rho. \quad (12)$$

The calculation of cross-section we will begin with averaging of expression (11) by the photon polarization. It is obviously when we rotate the polarization vector $\mathbf{e}$ around photon's momentum $\mathbf{q}$ on the angle π at this time as may be shown, the angle ϑ' respectively is being changed in the limits $\pi/2-(\beta_q-\beta_{p'}) \le \vartheta' \le \pi/2+(\beta_q-\beta_{p'})$, where $\beta_q = \arccos(q_z/q)$ and $\beta_{p'} = \arccos(p'_z/p')$. After integration by angle ϑ' in Eq. (11) it is easy to find:

$$Q = -\frac{2(2\pi)^3}{\mu}\widehat{\psi}(0)\int p'\,\widehat{\Psi}^*(\mathbf{p}')\,\sin(\beta_q-\beta_{p'})d^3p'. \quad (13)$$

Now substituting (12) into (13) and making the elementary calculations we find:

$$Q = -\frac{2(2\pi)^2}{\mu d}\,\delta(p_z-q_z)\widehat{\psi}(0)$$
$$\times \int_0^\infty \bar{p}\,\Theta_0(p'_\perp)\,\sin(\beta_q-\beta_{\bar{p}})p'_\perp dp'_\perp, \quad (14)$$

where $\bar{p} = \sqrt{{p_z}^2+{p'_\perp}^2}$, the δ- function shows the momentums conservation law on z- axis.

Finally let's analyze the amplitude Q forward which is more probable direction of PS decay on one γ photon, when in the PS doesn't rotate. In this case with high accuracy take place the following equalities $\beta_q = 0$, $\beta_{\bar{p}} = \arccos(p_z/\bar{p})$ and $m=0$.

Using these equalities we can substantially simplify expression (14):

$$Q = \frac{2(2\pi)^{5/2}}{\mu d}\,\delta(p_z-q_z)\widehat{\psi}(0)p_z^{1/2}\int_0^\infty \Theta_0(p'_\perp){p'_\perp}^{3/2}dp'_\perp. \quad (15)$$

Now using the expression (15) and standard connection between amplitude and transition probability [14] for the one-photon decay of PS we can write the following assessment:

$$P_\gamma \sim 10^6 sec^{-1}. \quad (16)$$

Recall that for finding assessment (16) we used the parameters of crystal $CsCl$.

Decay PA on Two γ Photons

It is easy to understand that the PA similar to positronium and, correspondingly the processes of their decaying should be similar too. This mean that the probability of annihilation of PA is a similar to positronium annihilation and we can connect of PA decay with probability of annihilation of free pair of positron and electron. It is obvious that annihilation process in a considered case don't depend from orientations of electron and positron spins.

The analysis of Fourier image of PA wavefunction $\Theta_0(p_\perp)$ in a basic state shows, that the probability amplitude find positron and electron with moments $p_\perp$ and $-p_\perp$ is a substantial for a moments $p_\perp \sim 1/\rho_0$. Taking into account last, the cross-section of process for a low-energies positron (namely such the localization energies of positron) may be represented by the formula (see for example [14]):

$$\sigma = \pi r_0^2/v_\perp, \quad (17)$$

where r_0 the electron's classical radius, $v_\perp = p_\perp/\mu$ the positron speed on plane (x,y), where motion of positron is localized between two circles (see Fig 1).

Finally using (2) and (17) we can write the expression for probability of PA decay on two γ photons [14]:

$$P_{2\gamma} = \frac{1}{(2\pi)^2\rho_0 d}\left|\Phi_{0,0}(\rho_0)\right|^2\left(v_\perp\sigma\right)_{v_\perp\to 0}$$
$$= \frac{r_0^2}{4\pi\rho_0 d}\left|\Phi_{0,0}(\rho_0)\right|^2 \sim 10^6 sec^{-1}, \quad (18)$$

where $\Phi_{n,m}(\rho)$ describes the radial part of wavefunction PS which in "ground state" is written at kind $\Phi_{0,0}(\rho)$.

At scattering of positron on a media's electrons the amplitude of transition is being described by S-matrix element of second order, similar to process of two photons' decay.

CONCLUSION

The problem of creation of sources for the generation of short waves radiation is a problem of utmost importance. The experimental discovery of the phenomenon of channeling of relativistic charged particles in crystals has given a good hope for creation of compact devices for generation of powerful radiation in X-rays region. Despite the made big efforts these hopes for a long time were not justified. The point is that at the channeling arises difficultly solvable problem connected with the short length of de-channeling. The characteristic times of channeling in axial or planar regimes accordingly of order $10^{-14}sec$ and $10^{-15}sec$, that

ISBN 978-3-95450-126-7

does not allow to increase the share of radiated energy and a fortiori to control process. As have shown our recent investigations this problem is being solved if we consider the channeling of positrons of energy $5 \div 20\, Mev$ in particular, in ionic crystals of type $CsCl$ along the chlorine ions axis $< 100 >$. In this case in crystal forms $2D$ relativistic positron-systems which with phonons subsystem of lattice practically are not interacting. All other types of influence on PS, collisions with electrons of media, the scattering on lattice discreteness etc, are perturbations of essentially small or similar to PS annihilation processes order. In other words in mentioned way with high probability it is possible to create of $2D$ relativistic PSs in media with the lifetimes bigger than $t \sim 10^{-6} \div 10^{-7}\, sec$. This means that we solved the main problem on way of creation of nano-undulators which have very large lifetimes and by which we can control.

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ISBN 978-3-95450-126-7