

MODULATED MEDIUM FOR GENERATION OF TRANSITION RADIATION

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Abstract

It is shown on an example of amorphous quartz, under the influence of a standing microwave field, at its certain parameters, superlattice is created in the medium where difference in values of dielectric constants of neighboring layers can be up to third order. This superlattice exists during the nanosecond, however it is sufficient for using it as a radiator for generation of transition radiation by relativistic electrons.

INTRODUCTION

The formation and governing of periodically modulated refractive index in media is a most important problem of solid state physics and material science. First of all it is related to the possibility of developing compact UV or X-ray Free-Electron Lasers (FEL) based on emission of transition radiation (TR) (see for example [1]). Currently the following two problems are discussed intensely:

1. A gas-plasma medium with periodically varied ionization density [2–9],
2. A special periodical solid-state superlattice-like (SSL) structures composed of layers with different refraction indexes [10–20].

TR is generated due to the difference in frequency-dependent dielectric constants (permittivity functions) of adjacent layers (remember that the radiation power is proportional to $[\epsilon_1^R(w) - \epsilon_2^R(w)]^2$, where $\epsilon_{1;2}^R(w) = Re[\epsilon_{1;2}(w)]$) [21]. Therefore, the possibility of controlling this difference by means of an external field is highly important. In other words, the problem here is to construct a superlattice with difference in dielectric constants of neighboring domains having the form $[\epsilon_1^R(w, \mathbf{g}) - \epsilon_2^R(w, \mathbf{g})]^2$, where \mathbf{g} describes the controlling parameters, $\epsilon_1(w, \mathbf{g})$ and $\epsilon_2(w, \mathbf{g})$ are dielectric permittivity functions in neighboring regions. According to theoretical and experimental studies, the periodical structures may be formed in condensed matter by means of external electromagnetic or acoustic fields [22–25].

This idea was recently realized in TR generation experiments [26]. In particular, it was shown that at the passage of a beam of 20 Mev electrons through amorphous silicon dioxide $a - SiO_2$ with a standing electromagnetic wave (of 10 GHz frequency) inside, anomalous high short-wave radiation was produced. Preliminary studies explain this

high intensity radiation as a result of multiple passage of the electron beam through interfaces between regions with different permittivity functions. Theoretically the appearance of 1D superlattice order in random media is explained by the polarization of media due to the orientational relaxation of elastic dipoles in the direction of external electromagnetic field propagation [27].

So, the main objective of this work is a systematic investigation of relaxation processes and critical effects in $a - SiO_2$ compound type disordered 3D systems under the action of external electromagnetic field that forms a standing wave in the medium, and in particular, to prove the possibility of formation of 1D periodic superlattice of permittivity function in the scales of space-time periods of standing wave.

FORMULATION OF THE PROBLEM

The starting point in our discussion will be the Clausius-Mossotti relation for dielectric constant. It is known that in isotropic media (as well as in crystals of cubic symmetry) the dielectric constant is well described by the Clausius-Mossotti equation [28–30]:

$$\frac{\epsilon_s - 1}{\epsilon_s + 2} = \frac{4\pi}{3} \sum_m N_m^0 \alpha_m^0, \quad (1)$$

where N_m^0 is the concentration of particles (electrons, atoms, ions, molecules) with given m types of polarizability and α_m^0 correspondingly are polarizability coefficients. It follows from this formula that the static dielectric constant ϵ_s depends on the polarizability properties of particles as well as on their topological order. In the external field the homogeneity and isotropy of the medium is often lost. Then, it is expected that the formula (1) will be applicable after slight generalization.

The object of our investigation are solid state dielectrics of the amorphous silicon dioxide $a - SiO_2$ type. According to numerical *ab initio* simulations [31], the structure of this type compound may be well described by the model of 3D disordered spin system.

In particular the 3D spin system we can represent as a 3D lattice with the lattice's constant $d_0(T) = \{m_0/\rho_0(T)\}^{1/3}$, where m_0 is the molecule mass, ρ_0 is the density and T is the temperature. We will assume also, that in each cell of this lattice there are only one randomly distributed spin (roughly polarized molecule).

We will suppose that the media under the influence of external standing electromagnetic field the electrical part

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of which has a kind:

$$E(x; E_0, \Omega, \lambda_s, \varphi_0) = E(x; \mathbf{g}) = 2E_0 \sin(\varphi_0) \cos(kx), \quad (2)$$

where $\varphi_0 = \Omega t_0$ and t_0 are respectively the initial phase and time, Ω is a wave frequency, $k = 2\pi/\lambda_s$ and λ_s is the wavelength, the symbol \mathbf{g} shows the parameters of standing wave (controlling parameters) ($E_0, \Omega, \lambda_s, \varphi_0$).

Here follows a natural question, how does the dielectric constant change on the scale of wavelength period and in time interval $\Delta t \ll \Omega^{-1} \sim 10^{-9} \text{sec}$, when the relaxation time of molecular dipoles is $\tau \sim 10^{-11} \div 10^{-12} \text{sec} \ll \Omega^{-1}$. This question is important since the processes associated with the Cerenkov and transition radiation produced in media are much faster than the above time scale. Note, that the time, during which a relativistic electron passes the wavelength ($\lambda_s \sim 10^{-4} \text{cm}$) of standing wave and the formation times of transition or Cerenkov photons in this layer is less than 10^{-15}sec . This time interval is essentially less than the time, during which the standing wave is steady-state. Since the wavelength is supposed to be much larger than the inter-dipole distance $\lambda_s \gg d_0$, the Clausius-Mossotti relation is still true. In this case the main problem is to calculate the polarizability coefficient related to orientation effects.

Taking into account the external field, one can express the polarization of matter at an arbitrary point as the macroscopic self-consistent relation:

$$\mathbf{P}(\mathbf{r}) = \sum_{\mathbf{l}} \mathbf{p}(\mathbf{l} - \mathbf{r}) = \sum_m n_m \alpha_m (\mathbf{l} - \mathbf{r}) \mathbf{E}_{loc}(\mathbf{l} - \mathbf{r}), \quad (3)$$

where $\mathbf{l} \equiv \mathbf{l}(l_x, l_y, l_z)$ is 3D lattice vector, \mathbf{p} is respectively the dipole moment of molecule. The second equation in (3) contributes to the value of dipole moment (spin). Note, that the number of the carriers of given polarization type in an elementary cell is $n_m \sim (d_0(T))^{-3}$, α_m being coefficients of the polarizability of corresponding types with due regard for external field and \mathbf{E}_{loc} is the local field, i.e. the effective field that induces the polarization at the site of an individual molecule. The contribution of each effect to the net dipole moment per molecule is linear, that is actually verified by experiments. Under the action of external field the polarization of different types arise in media. However, simple analyzes shows that the values of polarizability coefficients due to orientation effects essentially exceed the others.

Note that the coefficient of elastic orientational polarizability in amorphous media $\alpha_{dip}(\mathbf{l} - \mathbf{r})$ is a random function of cell location. This fact is due to random orientation of local field strengths $\mathbf{E}_{loc}(\mathbf{l} - \mathbf{r})$ with respect to the external field $\mathbf{E}(x; \mathbf{g})$. Therefore, all terms in the right side of (1) are basically known and well studied in literature (see, e.g., [28–30]) except from those connected with the orientation effects.

The orientation effects have a collective nature and are characterized by average value of random sum $\sum_{\mathbf{l}} \alpha_{dip}(\mathbf{l} -$

$\mathbf{r})$ (sum of random coefficients of orientational polarizability).

Multiplying both sides of equation (3) on the field (2), we find:

$$\mathbf{P}(\mathbf{r}, \mathbf{g}) \mathbf{E}(x, \mathbf{g}) = -\delta U(\mathbf{r}, \mathbf{g}) = \sum_m n_m \left[\sum_{\mathbf{l}} \alpha_m (\mathbf{l} - \mathbf{r}) \mathbf{E}_{loc}(\mathbf{l} - \mathbf{r}) \right] \mathbf{E}(x; \mathbf{g}), \quad (4)$$

where $-\delta U(\mathbf{r}, \mathbf{g})$ describes the potential energy of amorphous matter in the external field. The statistical properties of media in the direction of wave propagation will be considered later.

Taking into account (4), one can obtain the following expression for the part of potential energy of 3D spin system which is related with the orientational effects of spins in the external field:

$$-\delta U_{dip}(\mathbf{r}, \mathbf{g}) = \sum_{\mathbf{l}} \alpha_{dip}(\mathbf{l} - \mathbf{r}) \mathbf{E}_{loc}(\mathbf{l} - \mathbf{r}) \mathbf{E}(x; \mathbf{g}). \quad (5)$$

Let us separate a layer with volume $V = L_x \times L_y \times L_z$ in the infinite crystal lattice, where $L_x \sim (\lambda_s) \gg d_0(T)$ and $(L_y, L_z) \rightarrow (\infty, \infty)$. It is easy to see that this volume is filled with the infinite number of steric spin-chains with length L_x .

An important problem is now to calculate the mean value of the interaction potential between the spin layer and the external field.

Formally the following expression may be written for that:

$$\begin{aligned} -\delta U_V(\mathbf{r}, \mathbf{g}) &= -\sum_{\mathbf{l}_{\perp}} \delta U_{L_x}(\mathbf{l}_{\perp} | \mathbf{r}, \mathbf{g}), \quad \mathbf{l}_{\perp} \equiv \mathbf{l}_{\perp}(l_x, l_y), \\ -\delta U_{L_x}(\mathbf{l}_{\perp} | \mathbf{r}, \mathbf{g}) &= \sum_{l_x} \alpha_{dip}(\mathbf{l} - \mathbf{r}) \mathbf{E}_{loc}(\mathbf{l} - \mathbf{r}) \mathbf{E}(x; \mathbf{g}), \quad (6) \end{aligned}$$

where $-\delta U_{L_x}(\mathbf{l}_{\perp} | \mathbf{r}, \mathbf{g})$ is the interaction potential between the 1D steric spin-chain and external field. We will drop out calculating part and go to conclusion.

CONCLUDING REMARKS

In the present article a new microscopic approach has been developed for studying the properties of stationary dielectric constant and permittivity function in dielectric media under the influence of external standing electromagnetic field. The approach consists of the following two general steps:

1. Generalization of the Clausius-Mossotti equation for dielectric constant in the external standing electromagnetic wave;
2. Generalization of the equation for dielectric permittivity function taking into account the previous results.

Mathematically the problem is solved as follows. The dielectric medium in the external electromagnetic field is

modelled as a 3D spin glass system under the influence of external field. Note that all general changes of properties of media take place in the wavelength scale of external field space-time period. We have investigated in detail the layer of medium that consisted of disordered 1D spin-chains with the length of the order of external field's wavelength. Taking into account the fact that on infinite (x, y) plane the distribution of spin-chains is isotropic we can use the Birgoff ergodic hypothesis (see 6) and to reduce the initial 3D spin-glass problem on a two conditionally separated 1D problems. It means that we can investigate each 1D problem separately. However we must remember, that at the solution of the second 1D problem the parameters of a first 1D problem ought to be taken into account.

In the work we have constructed all formulas which are necessary to allow for the contribution of orientational effects at calculation of stationary and frequency-depending dielectric constants.

As was shown in result of catastrophe in $C-M$ equation, in the region of short wave-length, the difference between permittivities of neighboring layers may be essentially big.

Last circumstance allows us in homogenous and the isotropic dielectrics of spin-glasses type, artificially to create a superlattice from different permittivities the parameters of which it is possible to control by external fields and use this structure for generation of extremely intensive transition radiation.

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