OPTIMIZATION OF DIELECTRIC LOADED METAL WAVEGUIDES FOR ACCELERATION OF ELECTRON BUNCHES USING SHORT THZ PULSES *

A. Fallahi †, F. X. Kärtner, CFEL-DESY, Hamburg, Germany
L. J. Wong, A. Sell, RLE-MIT, Cambridge, USA

Abstract

The last decade has witnessed extensive research efforts to reduce the size of charged particle accelerators to achieve compact devices for providing relativistic particles. To this end, various methods such as laser plasma and dielectric wakefield acceleration are investigated and their pros and cons are studied. With the advent of efficient THz generation techniques based on optical rectification, THz waveguides are also considered to be proper candidates for compact accelerators. Sofar, the proposed schemes toward high power THz generation are capable of producing short pulses, which dictates the study of particle acceleration in the pulsed regime rather than continuous-wave regime. Therefore, THz waveguides are more suitable than cavities for the considered purpose. Consequently, various effects such as group velocity mismatch and group velocity dispersion start to influence the acceleration scenario and impose limits on the maximum energy gain from the pulse. In this contribution, we investigate electron bunch acceleration and compression in dielectrically loaded metal waveguides for the THz wavelength range and present design methodologies to optimize their performance.

INTRODUCTION

Linear accelerators are the major tools for providing relativistic particles [1]. Linear colliders for high-energy physics, high-power proton linacs for advanced neutron sources, small linacs for medical applications, free electron lasers for small wavelength sources, and high frequency power generators are examples of devices in which accelerators play significant roles. The current research on linacs focuses on new performance levels with better beam quality and lower power requirements. Acceleration of a particle using a high frequency beam is one of the potential ways to enhance the performance of the particle accelerators. The main concept of this technique is exposing an electron bunch to a traveling beam with a properly selected phase and beam spatial profile, and consequently accelerate the electrons using the electric and magnetic fields of the propagating beam. The optimum operating wavelength of the accelerator has been often a topic of elaborate discussions in the community. On one side, the available efficient sources limit the operation wavelengths to specific ranges and on the other the amount of charge needed for the application of interest and the overall efficiency of the device give priorities to other wavelength ranges. Linear accelerators in radio frequency (RF) range has been widely studied and matured in the last century, owing to the availability of high power RF klystrons [2]. By the emergence of high power lasers and the techniques in ultrafast optics, research on laser-plasma acceleration received substantial attention [3]. The RF regime often leads to devices with large dimensions whereas the laser acceleration exhibits acceleration in very small sizes with very limited aptitude in terms of charge amount. In this regard, high power short THz pulses available from optical rectification [4, 5] show strong potentials for realizing compact accelerators with large charge capacities compared to optical counterparts [6, 7].

This paper focuses on the design and implementation of a scheme in which electrons are accelerated by a THz beam. For this purpose, a THz waveguide is designed considering the required specifications for the electron acceleration and the beam requirements. Subsequently, the interaction between the electron and the guided beam is investigated. The pros and cons of THz acceleration of an electron bunch is examined based on the theoretical simulations. In the following sections, the step by step analysis and optimization is presented with the goal of achieving an optimal performing THz accelerator.

DIELECTRIC LOADED METAL WAVEGUIDES

To accelerate charged particles using an electromagnetic wave a longitudinal electric field is always required for transferring energy to the electrons. One approach to realize such an acceleration gradient is coupling the electromagnetic radiation into a waveguide. The phase velocity of the wave is synchronized with the velocity of the electron beam, which is always less than the speed of light or in case of relativistic beams is very close to it. However, the phase velocities of electromagnetic waves in empty uniform waveguides always exceed the speed of light in vacuum. Therefore, this parameter must be slowed down to the particle velocity to achieve considerable acceleration. This can be accomplished by loading waveguides with dielectrics as is done in linear accelerators based on dielectric loaded metal waveguide [8].

Consequently, the structure to analyze is a multilayer dielectric-loaded metallic waveguide (Fig. 1) and the goal of the analysis is evaluating the propagation constants of the modes and the acceleration properties of the waveg-

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† arya.fallahi@cfel.de

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uide. Particularly, the main focus and interest is the TM_{0n}

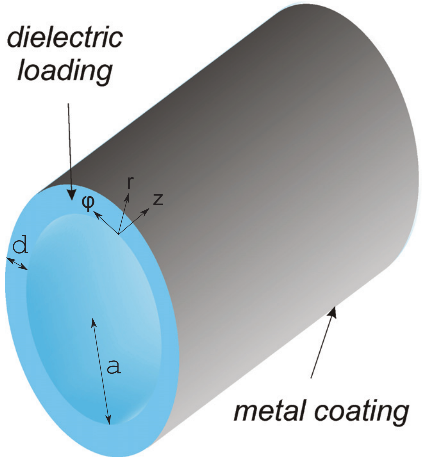

Figure 1: Schematic illustration of the considered geometry for the dielectric loaded metal waveguide

modes owing to the existing uniform longitudinal electric field which is the main feature for accelerating the particles. The general solution for the electric and magnetic field components for the TM_{0n} mode in each of the layers reads as:

$$\begin{aligned} E_{zi}(z,r,\phi) &= (A_i J_0(k_i r) + B_i Y_0(k_i r))\, e^{i(\omega t-\beta z)} \\ E_{ri}(z,r,\phi) &= -\tfrac{j\beta}{k_i}\, (A_i J_1(k_i r) + B_i Y_1(k_i r))\, e^{i(\omega t-\beta z)} \\ H_{\phi i}(z,r,\phi) &= -\tfrac{j\omega\epsilon}{k_i}\, (A_i J_1(k_i r) + B_i Y_1(k_i r))\, e^{i(\omega t-\beta z)} \end{aligned} \tag{1}$$

where $k_i = \sqrt{k_0^2 - \beta^2}$ with $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ as the wave number in vacuum, $J_n(x)$ is the n'th-order Bessel function of the first kind and $Y_n(x)$ is the n'th-order Bessel function of the second kind (Neumann function). As seen in the equations five unknowns should be found including the two coefficients A_i and B_i for each layer and the propagation constant of the mode. There is in fact one degree of freedom which emanates from the incident energy. Thus, the coefficients should be evaluated in a normalized scheme, meaning that the four coefficients are written in terms of one and the remaining value is determined based on the incident beam. The boundary condition at the vacuum-dielectric interface, and the boundary condition on the metal coating provide three equations which together with the non-singularity condition of the field at the center ($B_1 = 0$) provide the required equations to obtain the field solutions.

Usually the boundary condition on the metal layer is obtained by considering zero tangential electric field on the surface ($E_z = 0$). However, in THz regime metals no longer behave as a perfect metal and there exists a considerable loss due to the finite conductivity of the metallic layer. The boundary condition can then be written as:

$$Z_c H_\phi = E_r \tag{2}$$

where Z_c is the surface impedance of the metallic layer and is found from the *skin effect* assumption according to $Z_c = \sqrt{\omega\mu/2\sigma}(1+i)$ with σ being the conductivity of the metal.

Using the aforementioned boundary conditions, the propagating field inside the waveguide can be computed at different frequencies. The time domain accelerating field of a THz short pulse propagating in the accelerator is then approximated by the following equation:

$$\begin{aligned} & E_z(z,r,t) = E_0 J_0(\sqrt{k_0^2-\beta^2}\,r)e^{-\alpha z} \\ & \Re\left\{ \frac{\tau e^{-2\ln 2\frac{(t-z/v_g)^2}{\tau^2 - j4\ln 2\cdot \mathrm{GVD}\cdot z}}}{\sqrt{\tau^2 - j4\ln 2\cdot\mathrm{GVD}\cdot z}} e^{j(\omega t - kz + \psi_0 + \psi_{\mathrm{GVD}})} \right\} \end{aligned} \tag{3}$$

where GVD is representing the group velocity dispersion in the waveguide and ψ_{GVD} is the effect of this dispersion on the field phase. Other components of the electromagnetic fields, i.e. E_r and H_ϕ, have similar dependence on the time and longitudinal components with a change in the transverse profile corresponding to (1). Once the propagating fields in the waveguide is formulated, they will be used to study the capability of the device in accelerating and compressing electron bunches.

WAVEGUIDE OPTIMIZATION FOR ACCELERATION

In order to investigate the amount of energy an electron bunch can gain from a guided mode, optimization needs to be performed on all the involved parameters to find the optimum acceleration scenario. For this purpose, like any optimization study a suitable parameter, i.e a *fitness function*, should be defined which not only is computed straightforwardly but also directly relates to the optimum final energy of a bunch. The considered function in this work is the final energy of one electron entering the waveguide at the central axis. This function is calculated with a low computation cost and gives an appropriate measure of the waveguide power to accelerate a bunch. To evaluate this function, we solve the equation of motion using the central difference method:

$$\begin{aligned} \beta_{n+1} &= \beta_{n-1} - \frac{2eE_z(z,0,t)\delta t}{mc\gamma_n^3} \\ z_{n+1} &= z_{n-1} + 2\beta_n c\delta t \end{aligned} \tag{4}$$

where β_n is the normalized velocity of the electron.

Several parameters, including $v_p = \omega/k$, v_g, GVD, α, z_0, and ψ_0 need to be optimized to find the maximum acceleration achievable in the waveguide. v_p as the phase velocity of the guided mode should be constant and equal to the speed of light to prevent phase slippage of the particles with respect to the accelerating signal. The parameters v_g, GVD, and α are functions of the waveguide dimensions and are optimized by sweeping over the dimensions, namely a and d in Fig. 1. The dependence of optimum acceleration point on z_0 and ψ_0 is very complex and should indeed consider the small phase slippage of the electron during the pulse. We use the nonlinear optimization toolbox in MATLAB to find these optimum values, meaning

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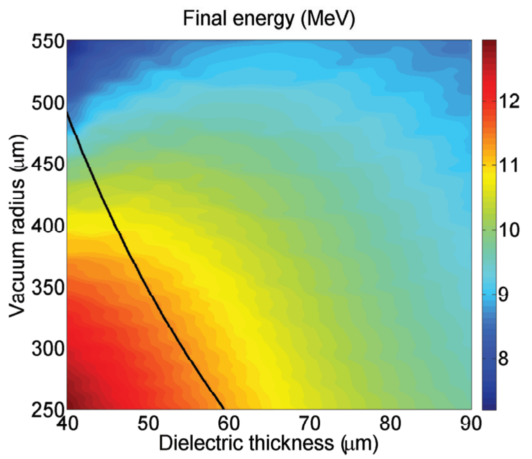
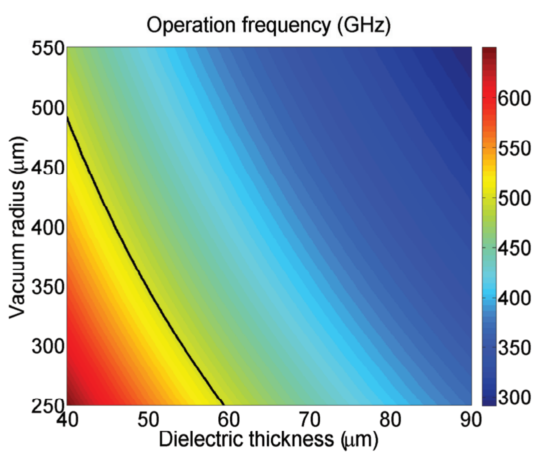
that for each dimension pairs a and d of the sweep, the maximum final energy is found.

Figure 2a shows the color map of final energy in terms of the waveguide dimensions. We consider a quartz loaded copper waveguide with a 20 mJ 10-cycle THz pulse centered at the waveguide operation frequency for accelerating one electron with initial energy equal to 1 MeV. The operation frequency of the waveguide in terms of its dimensions is illustrated in Fig. 2b. As observed from the



(a)



(b)

Figure 2: (a) Maximum final energy and (b) the operation frequency of the electrons for different dimensions of the waveguide.

color map, the final achievable energy of the particles increase for smaller waveguide dimensions, which occurs due to higher fields produced by the 20 mJ pulse. Nonetheless, the operation frequency of the waveguide varies correspondingly, whereas the value is strongly limited by the THz source. In addition, in small wavelengths, there are tight limitations on the amount of charge that can be accelerated by the guided fields. Provided that an optical rectification source with Lithium-Niobate is utilized for producing the required power, the optimal operation frequency is $f = 500$ GHz. Therefore, we evaluate over the line of the

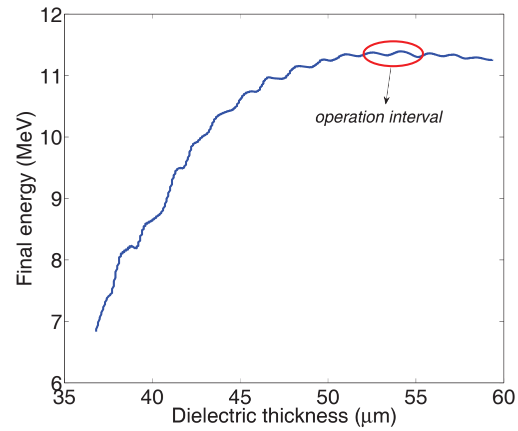


Figure 3: Maximum final energy of the injected electron bunch in terms of the thickness of the dielectric loading at 0.5 THz operating frequency. The oscillations observed in the curve appear due to the interpolation method used for increasing the resolution of the sweep.

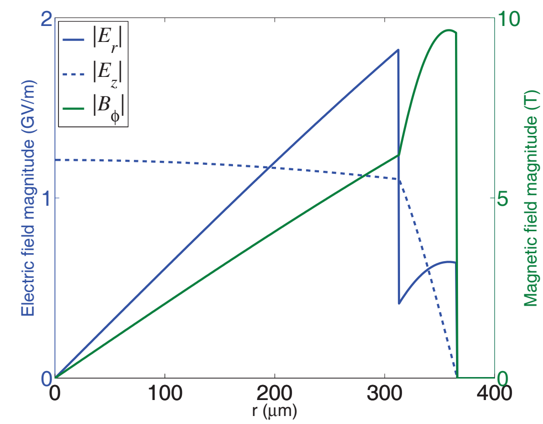


Figure 4: (a) Maximum final energy and (b) the operation frequency of the electrons for different dimensions of the waveguide.

desired operation frequency and read the optimum final energy (Fig. 3). The optimum accelerator is found by locating the maximum in the obtained curve, which represents a waveguide with vacuum radius $a = 313\,\mu$m and quartz thickness $d = 53\,\mu$m. The field profile of the 20 mJ THz pulse inside the waveguide is shown in Fig. 4.

BUNCH ACCELERATION INSIDE THE DESIGNED WAVEGUIDE

After optimizing the waveguide for the acceleration of one electron, the study of the acceleration of an electron bunch is necessary to figure out the device performance. The waveguide parameters for the optimum structure is obtained as $d = 53\,\mu$m, $a = 313\,\mu$m, $v_p = c$, $vg = 0.7c$, $\tau = 20$ ps, $\alpha = 6.36 \times 10^{-4}$ 1/m, GVD $= 5.93 \times 10^{-22}$. We consider that a 1.6 pC electron bunch with 1 MeV en-

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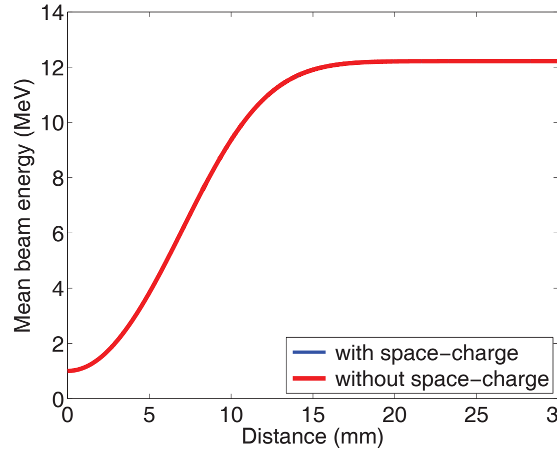
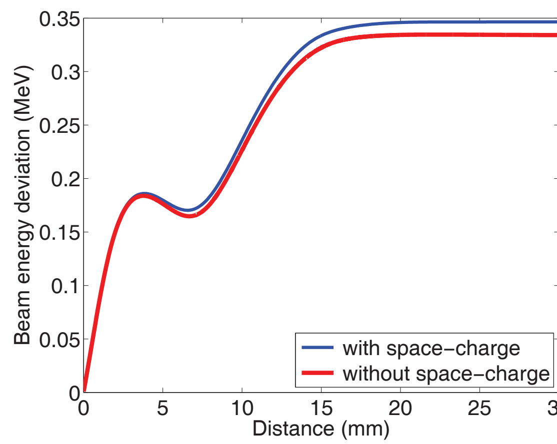
ergy, spatial spread $\sigma_x = \sigma_y = \sigma_z = 30\,\mu$m and energy spread $\sigma_{\gamma_x} = \sigma_{\gamma_y} = \sigma_{\gamma_z} = 0.006$ is injected into the THz pulse at $\psi_0 = -0.3\,$rad and $z_0 = -33.3\lambda\,$rad. In fact, producing the assumed $100\,$fs bunch is a challenge for RF guns. However, the new techniques based on laser-induced field emission have shown promises for providing such short bunches. Notably, the THz acceleration is an efficient approach only when a short bunch is going to be accelerated, otherwise the field variation in the range of a THz wavelength yields a large energy spread in the electron energies. The bunch is simulated with 10000 macro-particles, Gaussian-distributed in every dimension of phase and space. For the temporal update of the equation of motion for each macro-particle, the 4'th order Runge-Kutta method is employed .

An important aspect in the analysis of bunch acceleration is the internal interactions between the particle, the so-called space-charge forces. We use a point-to-point approximation for calculating this effect and as usual the time retardation is neglected in the calculation. This is justified by noting that the retardation effect becomes significant in large distances where these forces are almost negligible. For more details on the equations considered for space-charge simulation, the reader is referred to [6]. In the illustrated results, we try to show the space-charge effect on the acceleration by comparing the simulation results with and without considering this effect.

Figure 5 shows the simulation result for the mean bunch energy and its energy spread versus distance. As observed from the curves, the energy gain of the bunch from the signal is around $11\,$MeV and the space-charge effect on the total energy spread is around 6%. The $11\,$MeV occurs in only $20\,$mm length of the waveguide, which corresponds to $550\,$MeV/m acceleration gradient. This is by one order of magnitude larger than the conventional RF accelerators. In contrast to the acceleration gradient, the relative energy spread of 3% is very prohibitive, but we believe that it can be improved by using shorter ellipsoidal bunches. As mentioned before, due to sharp variations of the field strength compared to RF regime, THz acceleration is more suitable for short bunches. Hence, if we start with a short enough ellipsoidal bunch we should be able to accelerate it without its energy spread deteriorating significantly. The spatial spread of the bunch as well as the momentum spread in each direction are illustrated in Fig. 6a-f, respectively. Interestingly, the electron bunch is compressed in the longitudinal direction and simultaneously focused in the transverse directions. Nevertheless, this has been accompanied by a significant increase in the energy spread. In all the cases, space-charge is causing considerable changes in the transverse distribution whereas smaller influences is observed on the longitudinal ones. Beam emittance is also an important parameter of any electron beam which quantitatively shows the electron beam divergence. In some applications like electron diffraction imaging and inverse Compton scattering, this parameter plays a major role to obtain high quality outcomes. The beam emittance in each



(a)



(b)

Figure 5: (a) Mean bunch energy and (b) the energy spread of the $1.6\,$pC electron bunch versus the traveled distance in the waveguide.

three direction versus distance is depicted in Fig. 6g-i. The significant increase in the longitudinal emittance caused mainly by the large energy spread in the bunch is the main drawback of the introduced acceleration scheme. In contrast, the transverse emittance is increased by only a factor of two which is a benefit of this scheme.

CONCLUSION

The existing obstacles in the acceleration using RF fields, such as large dimensions and low beam qualities were often the motivation for designing and implementing accelerators in high frequency regimes. In this regard, laser acceleration recently attracted much interest. However, studies showed that this scheme suffers from small bunch capacities. These problems inspire the idea of THz acceleration to be a promising candidate and trade-off for obtaining relativistic pico-Coulomb bunches. The recent works on realizing high energy short THz pulses additionally prompt the application of THz technology for electron acceleration. In this regime, electrons experience higher

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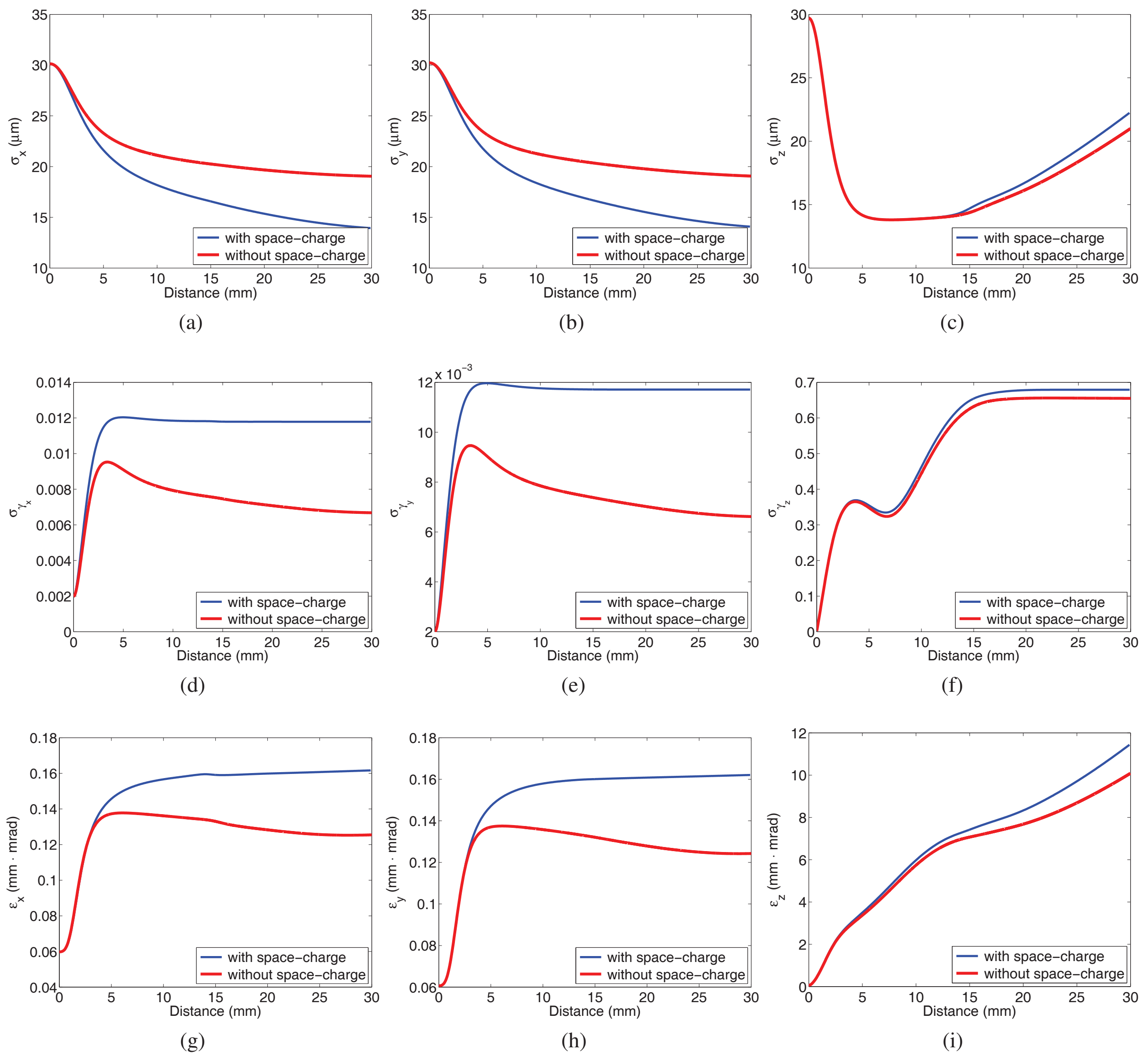

Figure 6: (a) σ_x, (b) σ_y, (c) σ_z, (d) σ_{γ_x}, (e) σ_{γ_y}, (f) σ_{γ_z}, (g) ϵ_x, (h) ϵ_y and (i) ϵ_z of the 1.6 pC electron bunch versus the traveled distance in the waveguide.

accelerating fields over a shorter time, which in turn allows for much better quality of the electron beams. This paper demonstrated through a numerical simulation that 20 mJ 10-cycle pulses with the center frequency at 0.5 THz coupled into a quartz loaded cylindrical copper waveguide can accelerate an electron bunch from 1 MeV to around 12 MeV in the 20 mm waveguide length. This corresponds to 550 MeV acceleration gradient being one order of magnitude larger than the conventional RF accelerators. The obtained results show the promise of coherent THz pulses in realizing compact electron acceleration.

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