TOWARDS HIGH FREQUENCY OPERATION WITH A MULTI-GRATING SMITH-PURCELL FEL

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Abstract

Three-dimensional simulations and experiments have shown that, for a grating equipped with sidewalls, copious emission of coherent Smith-Purcell (SP) radiation at the fundamental frequency of the evanescent surface wave is possible. Since the underlying theory is scale invariant, the wavelength emitted is reduced in proportion to a uniform rescaling of the grating. In order to increase our 5 GHz to 100 GHz, the grating surface would be reduced by a factor of 400, which would lead to greatly reduced power. In addition, the required beam might be hard to generate. To avoid this, we propose to use several gratings in parallel with no overall reduction in the total width and the same beam as in our microwave experiment. For this scheme to succeed, it is essential that the bunching in the different gratings be coherent. . Simulations suggest that this occurs for as much as a ten-fold scale reduction. To test this idea, an experiment is using several gratings is being performed.

INCREASE FREQUENCY BY DOWNSIZING

A demonstration experiment in the microwave domain showed that a Smith-Purcell (SP) free electron laser (FEL), with conducting sidewalls placed at the ends of the grating's grooves, emitted intense radiation at the frequency of the surface wave on the grating [1]. In single shot operation, the ratio of emitted power to beam power exceeded 10 %, and was in reasonable agreement with simulations performed previously [2]. An earlier experiment, on a grating without sidewalls [3], had demonstrated emission of coherent SP radiation at the second harmonic of the surface wave, thereby confirming the scenario proposed in the two-dimensional (2-D) model of Andrews and Brau [4]. But the efficiency was only of order 0.1 %. The sidewalls cause a modification of the dispersion relation for the surface wave [5]. The intersection of the beam line with the new dispersion relation may occur at an allowed SP frequency, which can't happen in the 2-D theory of Reference [3]. Thus the use of sidewalls produces much greater power, albeit at half the frequency. It must be emphasized that the sidewalls protrude only a small distance above the top of the grating, so that the radiation may be emitted in any direction above the grating.

The expressed aim of Andrews, Brau and their collaborators [6] was to develop an intense, compact, and tunable source of THz SP radiation. Theory indicates that a uniform reduction in size of all grating parameters, at constant beam energy, leads to an equal reduction in the wavelength of the SP radiation. This is emitted at the same angle, according to the usual SP formula [7],

$$\lambda = L(1/\beta - \cos\theta_{SP})/|n|.$$

Here λ denotes the wavelength, L the grating period, n the order of diffraction, θ_{SP} the angle with respect to the beam, and β the relative velocity of the electron. Obviously, if all parameters of the grating were reduced by a scaling factor, the wavelength would be reduced by the same factor, at constant beam energy. Such a reduction in size might be expected to greatly reduce the output power. In addition, the beam would have to propagate in a very narrow channel.

MULTI-GRATING ARRAYS

We propose to partially compensate this power loss by superposing in parallel several such gratings with sidewalls, while keeping our flat beam of width a few cm. We have performed 3-D particle-in-cell (PIC) simulations using the code "MAGIC" [8] to study the effect of diminishing the size of the grating, but making a planar array of them so as to maintain a quasi-constant beam width. In order to avoid the problem of beam height above the grating, we simulated a 1 mm-thick beam whose lower edge is flush with the top of the grating, regardless of the individual grating width. Our previous experience with the set-up convinces us that this is feasible. A magnetic field is used to impose approximately linear trajectories on the electrons. The essential question is whether the radiation from the individual channels remains coherent, so that the peak power emitted will scale as the square of the number of channels. For this to happen, the spatial and temporal beam bunching in each channel must be nearly identical. We can use the tools furnished by MAGIC to verify that fields and bunching in the different channels do indeed display the necessary coherence. In these simulations, the number of periods remains fixed (25), so that the reduction in scale leads to successively shorter gratings. In the simulations we have studied reductions of 2, 4, 6, 8 and 10 in size, each compensated by increasing the number of gratings by the same factor. The same beam of 40 A, 80 keV (beam power 3.2 MW) and 1 mm thickness is used in all simulations. The height of the sidewalls above the grating top and their thickness are also scaled down by the same factor. The sidewalls intercept some fraction of the beam, which will make CW operation impossible. The separation of the inner faces of the outermost sidewalls is thus 44-4/N mm, where N denotes the number of channels, and the beam width is equal to this.

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In Figure 1 is shown a sketch of our grating in its original form, and also in the reduced form with ten channels. The cathode, gratings with sidewalls, and the beam dump are shown, along with our choice of axes.

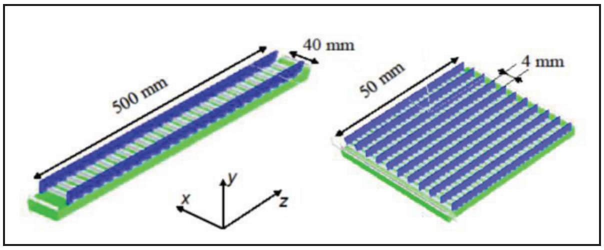

Figure 1: Sketch of the 3-D “MAGIC” simulation geometry for the full-scale grating and the superposition of 10 tenfold reduced copies, indicating the axes choice.

Our simulated gratings all had 25 periods, with both groove depth and width equal to half the period. For the full-size grating, the period was 2 cm. The nominal operating point (intersection of the beam line with the cold grating dispersion relation) corresponds to 5.36 GHz, the wavelength = 5.6 cm, and the Smith-Purcell angle = 143.7°. Because of space charge and plasma effects, the observed operating frequency is about 5.2 GHz.

ANTENNA ARRAY THEORY

In order to furnish a framework for the results of our simulation, we invoke dimensional arguments, and rely upon the theory of antenna arrays. If we follow the reasoning of SP that the radiation they discovered is essentially dipole radiation, we expect that P, the total power radiated, should follow the standard formula [9] $P = C\omega^4|\vec{p}|^2$, where C is a constant, ω is the frequency and $\vec{p}$ denotes the electric dipole moment of the radiating system. If all dimensions of the grating were rescaled, i.e., $L \to L/N$ we expect

$$\vec{p} \to \vec{p}/N^3, \omega \to N\omega,$$

which imply

$$P \to P/N^2.$$

Thus for a tenfold reduction in scale, the power of an individual grating is expected to decrease by a factor of 100. Of course, the input power would be reduced by a factor of 10, so that efficiency would also decrease by 10.

In order to describe the angular distribution to be expected from an array of parallel gratings, we refer to the well-known theory of antenna arrays [10] The set-up we simulate may be described as a broadside array of N identical antennas, in which each antenna emits radiation in a narrow cone of opening angle θ_{SP}. Provided they are all in phase, the resulting signal in the radiation-zone is given by

$$R(\theta,\phi)\left(\frac{\sin(N\pi b\sin\theta\cos\phi)}{\sin(\pi b\sin\theta\cos\phi)}\right),$$

where $R(\theta,\phi)$ denotes the field of a single isolated grating, the dimensionless quantity b is the spacing of adjacent gratings in units of wavelength, θ denotes the polar angle with respect to the beam direction, and ϕ the azimuthal angle ($\pi/2$ for the normal to the grating plane). The quantity $R(\theta,\phi)$ will have support only in a narrow range of θ, centered about θ_{SP}. When $\phi = \pi/2$, the result becomes $NR(\theta,\pi/2)$. But since the field of the individual grating scales as $1/N$, the peak field is independent of the number of gratings in the array. However, the numerator vanishes whenever

$$Nb\sin\theta\cos\phi = \pm 1. \text{ (or any integer)}$$

If $Nb\sin\theta_{SP} \gg 1$, almost all of the radiation is emitted in a narrow range

$$\pi/2 - \Delta < \phi < \pi/2 + \Delta$$

where $\Delta = 1/(Nb\sin\theta_{SP})$. If we assume that $R(\theta,\phi)$ is slowly varying in this interval, and that

$$\cos\phi \cong \tfrac{\pi}{2} - \phi$$

one can integrate over ϕ. Writing $\psi = \frac{\pi}{2} - \phi$, we find

$$\int_{-\Delta}^{\Delta} d\psi \frac{\sin^2(N\pi b\sin\theta\psi)}{\sin^2(\pi b\sin\theta\psi)}$$
$$\frac{2}{\pi b\sin\theta}\left[\pi + \sum_{p=1}^{N-1}\left(\frac{N}{p}-1\right)\sin\left(\frac{2p\pi}{N}\right)\right].$$

The quantity in brackets is accurately fitted by the expression 0.1173+1.4147 N for $N \geq 3$.

We can then estimate the power radiated into the main lobe, for our value of $b = 0.772$, as

$$P_{Lobe} = (0.1 + 1.17\,N)\int_0^{\pi} d\theta |R(\theta,\pi/2)|^2.$$

Since the power radiated by the individual grating scales as $1/N^2$, the total power in the main lobe decreases as $1/N$. However, the solid angle of the main lobe decreases as $1/N$, so the average $dP/d\Omega$ in the main lobe is independent of N.

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SIMULATIONS OF MULTI-CHANNEL GRATINGS

In order to reduce the computer time needed for a simulation we used a high-current electron beam of 40A. The initially continuous beam was emitted uniformly across the cathode, with energy 80 keV. In “MAGIC” simulations, electrons that strike a conducting surface are removed from the simulation at that time. Since the space charge is quite large, the electrons lose some of their initial energy, especially for the deeper gratings. The simulations depend only on one parameter, which sets both the overall scale and the mesh size. The only departures from strict scaling are that the beam was 1 mm thick in all simulations, and the overall distance between the outermost sidewalls was (44-4/N) mm. The simulations assumed a uniform magnetic field of 1 T along the z-direction.

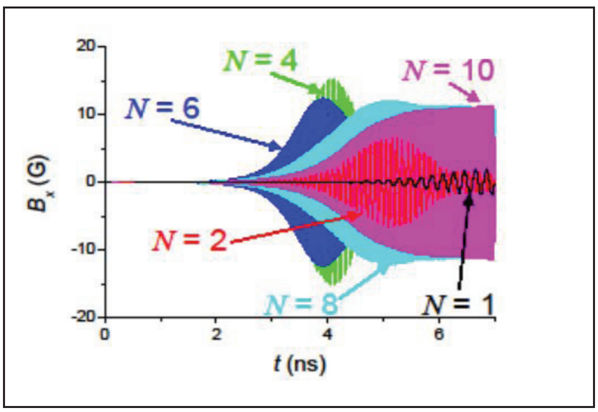

Figure 2: The time histories of the magnetic field component B_x for the superposition of N N-fold reduced gratings, with N=1, 2, 4, 6, 8, and 10.

In Figure 2 we show the time variation of B_x at six points situated at x = 0, at distance d/N from the center of the grating with d = 714 mm, and at angle θ= 143°. The signals are for 1, 2, 4, 6, 8 and 10 channels. The order of magnitude is several gauss which corresponds to intensities of order kW/cm^2. But the ratio of distance to grating length is only 1.4, which means that we are not in the far-zone. The shorter gratings tend to produce a significant signal more rapidly, but we remark that the time needed to reach the downstream end of the longest grating is almost 2 ns.

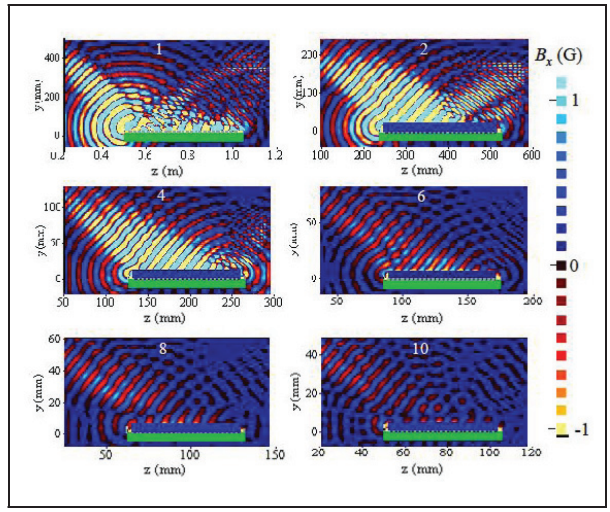

Figure 3: “MAGIC” generated contour maps of the magnetic field component B_x in the median y-z plane for the six different configurations.

In Figure 3 we show six instantaneous contour maps of B_x in the median y-z plane. The times were chosen such that the maximum value of B_x in the upper left corner was approximately 1 G. The emission of the fundamental in the backward direction is a striking feature of all maps. One observes that the angular distributions in this plane are similar, as expected from the scaling theory. Emission of the second harmonic at about 60° is clearly visible.

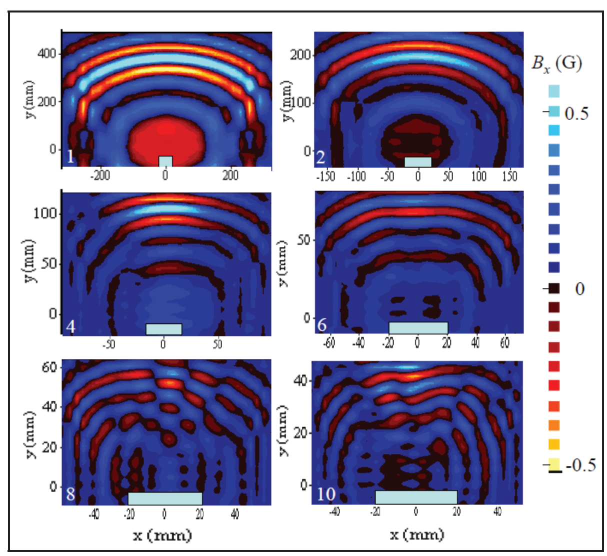

Figure 4: “MAGIC” generated contour maps of the magnetic field component B_x on the most upstream x-y planes for each of the six different configurations.

In Figure 4, we show six instantaneous contour maps of B_x in the rear-most x-y plane. In contrast to Figure 3, where all the maps are quite similar except for scale, these maps clearly show a progressive concentration about the azimuthal angle ϕ = $\pi/2$ as N increases. For larger values of N, almost all of the power is concentrated in a small angular region. This supports the arguments

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based on the theory of antenna arrays. It may also of practical importance, since a small but well-placed mirror could gather most of the emitted radiation.

Figure 5 shows six instantaneous contour maps of B_x in x-z plane at the level of the tops of gratings. If there were total coherence, all of the N channels in each map would be identical. While this is not strictly true, there is considerable coherence among the channels, which suggests that broadside antenna theory may be valid. This is a key point, since without coherence among the channels, the angular distribution of radiation might become complex and hard to concentrate.

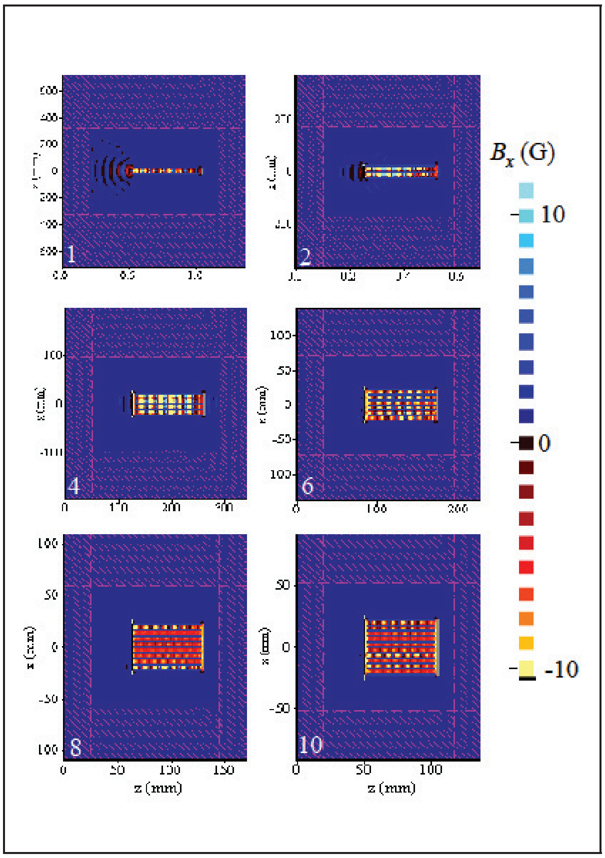

Figure.5. "MAGIC" generated contour maps of the magnetic field component B_x on the x-y plane at the level of the grating tops for the six different configurations.

Table: Summary of "Magic" Results

N	f(GHz)	P(kW)	ΔT(keV)	G (ns^{-1})	η(%)
2	10.2	380	12.7	2.2	11.8
4	21	200	7.8	2.6	6.3
6	31.6	100	4.8	2.5	3.2
8	42.2	40	2.4	2.1	1.3
10	52.8	30	1.9	1.7	1

A summary of some numerical values are shown in the Table for different N. The observed frequencies are indicated: P is the peak power crossing the rearmost plane of the simulation volume, ΔT is the mean kinetic energy lost by an electron at the end of the grating, G denotes the imaginary part of the frequency ω, and η is the efficiency. These results are roughly consistent with the theory presented here.

CONCLUSION

Although an experiment is needed to support our claim that miniature multi-grating SP FELs can reach higher frequencies while still maintaining adequate power levels, we believe that the simulations presented here are rather encouraging. Since the antenna theory we discussed is based on phase coherence among the individual gratings, it remains to be seen whether this can be achieved in practice. To test this, we plan to perform an experiment with a seven-fold reduction in size. This should allow us to see what sort of power levels are attainable around 35 GHz.

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