OPTICAL CAVITY LOSSES CALCULATION AND OPTIMIZATION OF THz FEL WITH A WAVEGUIDE*

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Abstract

The optical cavity with waveguide is used in most long wavelength free electron lasers. In this paper, the losses, gains and modes of a terahertz FEL sources in Huazhong University of Science and Technology (HUST) are analysis. Then the radii of curvature of the optical mirrors and shapes of the waveguide are optimized.

INTRODUCTION

Considering wide applications of high power coherent THz radiation sources on biology, imaging and material science etc., a prototype compact terahertz FEL oscillator is proposed at Huazhong University of Science and Technology (HUST), which is considered to generate 50-100μm terahertz radiation. The concept design of the compact THz FEL oscillator is composed of an independently tunable cell (ITC) thermionic RF gun, a linac booster, a planar undulator and a near concentric waveguide optical cavity[1].

The main design parameters of this FEL oscillator are listed in Table 1.

Table 1: Parameters of the THz FEL Oscillator

Parameter	Value
Radiation wavelength λs	50 - 100μm
Beam energy	8.1-11.7 MeV
Energy spread	0.3%
Normalized Emmittance	15πmm·mrad
Charge per pulse	≥200pC
Bunch length	5ps (5-10ps)
Macro pulse length	4-6μs
Undulator parameter K	1.25 (1.0-1.25)
Undulator Period Number	30
Undulator wavelength λu	32 mm
Optical cavity length Lcav	2.940m
Waveguide width	2a=40mm
Waveguide height	2b=10mm
Waveguide length	Lwg=1130mm

In this paper the losses, gains and modes of the THz FEL source with waveguide in HUST are analysis. Then the radii of curvature of the optical mirrors and shapes of the waveguide are optimized.

MODES IN WAVEGUIDE

Compared to the conversional FEL, new issues arise in a long wavelength THz FEL due to the short electron pulse and the radiation diffraction loss. The vacuum duct of undulator is used as the waveguide, which offers the advantage of small wiggler gaps and small mode area is helpful to improve gains and decrease losses in FEL cavity.

In conventional lasers, the oscillating modes are determined by both the radii of curvature of the two mirrors and their separation.

While in a waveguide FEL, the modes of propagation in the waveguide are different from the Gaussian beam. Each end of the waveguide may be considered as a radiating source, each mirror acting as a feedback element coupling the radiation back into the allowed waveguide mode. A part of the radiation is being scattered out of the guide aperture [2].

Consider the simplified waveguide resonator structure shown in Fig.1, in which toroidal mirrors of curvature radius Rx and Ry are positioned at a distance d from the waveguide end, which coincides with the plane z =0. The vacuum duct in the undulator is acted as a rectangular waveguide, and its cross-section is 2a*2b.

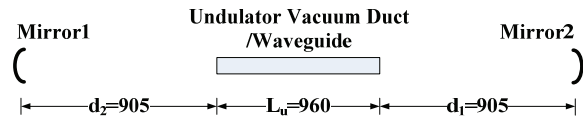Fig.2 gives the coordinate system used in loss calculation.
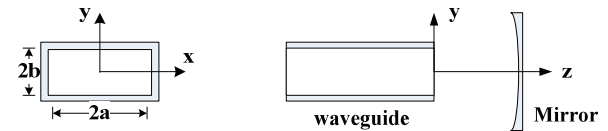

Figure 1: Structure of the waveguide resonator.

Figure 2: Coordinate system used in calculations.

The transverse dimensions of waveguide in a practical FEL waveguide are usually much larger than the guide wavelength; therefore we are only interested in the overmode, low-mode and low-loss solution.

For convenience, two sets of linearly independent solutions are identified.

The magnetic field of the first class modes is almost completely polarized in the x direction, denoted as: E_{pq}^{σ} $\left(H_x^{\sigma} \geq H_y^{\sigma}, H_z^{\sigma}\right)$; that of the second class is polarized in the y direction , denoted as: E_{pq}^{π} $\left(E_x^{\pi} \geq E_y^{\pi}, E_z^{\pi}\right)$. With

boundary conditions $\{E^{\pi}_{x,z}(x,0,z)=0$, $E^{\pi}_{x,z}(x,b,z)=0\}$ and $\{H^{\sigma}_{y}(x,0,z)=0$, $H^{\sigma}_{y}(x,b,z)=0\}$, respectively.

For the spatial symmetry reasons, both the two sets of E^{π}_{pq} and E^{σ}_{pq}, where p, q are odd, are ideally for FEL.

In a planar undulator FEL, where the electrons are wiggling in the x direction, the electric field is x polarized, the E^{π}_{pq} modes are excited efficiently with low losses.

The intensity of the field (E_x or H_y) of the E^{π}_{pq} mode at the rectangular waveguide is proportional to: [3]

$$E_{x,pq}(x,y) = Z_{EH} H_{y,pq}(x,y)$$

$$\propto F_x(x)F_y(y) = \left\{ \begin{matrix} \dfrac{1}{\sqrt{a}}\cos\left(\dfrac{p\pi x}{2a}\right) \\[2mm] \dfrac{1}{\sqrt{a}}\sin\left(\dfrac{p\pi x}{2a}\right) \end{matrix} \right\} \left\{ \begin{matrix} \dfrac{1}{\sqrt{b}}\cos\left(\dfrac{q\pi y}{2b}\right) \\[2mm] \dfrac{1}{\sqrt{b}}\sin\left(\dfrac{q\pi y}{2b}\right) \end{matrix} \right\}$$

where the upper and lower terms are used with odd and even values of the integers p and q, respectively.

EH_{11} is the lowest linearly polarized mode.

LOSSES AND GAIN WITH WAVEGUIDE

The ohmic losses on the mirrors, the waveguide losses experienced by the EH_{11} mode, and the coupling losses for this mode are considered, respectively. Then the roundtrip losses and the gain of the FEL cavity are calculated.

Ohmic Loss on the Mirrors

In the view of low absorbing, high reflectivity mirrors are required in a low gain FEL oscillator. Copper mirrors coating with gold are used in our optical cavity system.

The ohmic loss of our mirrors with gold coating is given as [3]:

$$C_{mo} = 10^{-2}\left(0.71 - 1.2\lambda s[mm]\right)$$

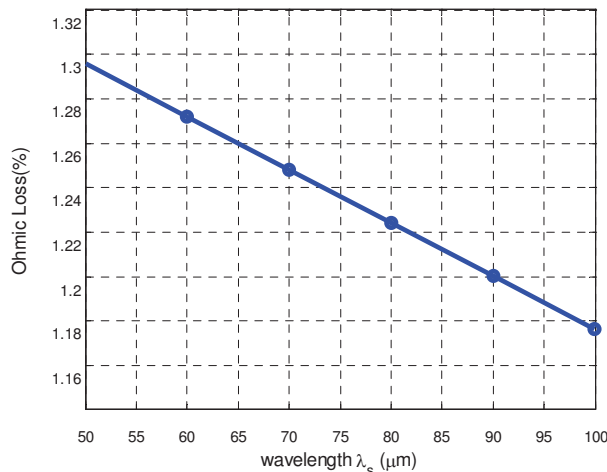It is small in the terahertz region as shown in the Fig.3.

Figure 3: Ohmic loss on mirrors coating with gold

Ohmic Losses on the Waveguide Wall

The attenuation of the transmitted power per unit of length due to the losses in the walls having finite conductivity can be calculated using the standard approach of computing the mean power flowing into the metallic walls per unit length and the total power flow in the axial z direction [4].

As shown in the Fig.4, the waveguide wall losses experienced by the E^{π}_{11} modes are much smaller than that of the E^{σ}_{11} modes, so the E^{π}_{11} modes will be excited efficiently in FEL with waveguide and its waveguide wall loss is negligible.
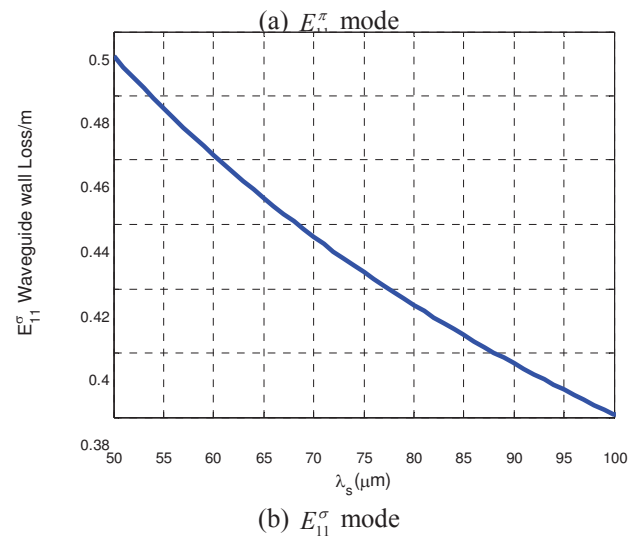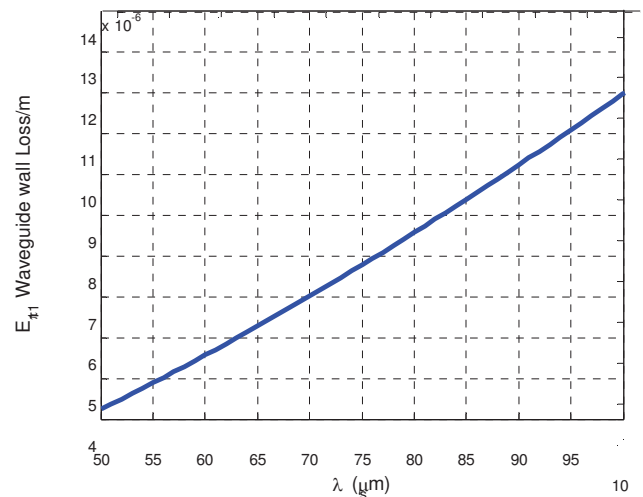

(a) E^{π}_{11} mode

(b) E^{σ}_{11} mode

Figure 4: Ohmic loss on waveguide wall.

Coupling Loss of Mode E11

The coupling efficiency is calculated by propagating the radiation field at the output of the waveguide to the mirror and back and calculating the projection of the resulting field on the EH_{11} mode. Only modes of same polarization can be coupled.

when 2a>> λ, The radiation travelling from the waveguide end in a section between this end and a mirror

and back again, can be represented by a Fourier series employing a complete system of axially symmetric normal free-space Gaussian modes.

$$F_x(x) = \sum_m A_m \Psi_{xm}(x)$$

$$A_m = \int_{-\infty}^{+\infty} F_x(x)\Psi'_{xm}dx = \int_{-a}^{+a} F_x(x)\Psi'_{xm}dy$$

$$\Psi'_{xm}(x) = K_m H_m\left(\sqrt{2}\frac{x}{w_{0x}}\right)\exp\left[-\left(\frac{x}{w_{0x}}\right)^2\right]$$

$$K_m = \left(\frac{2}{\pi}\right)^{1/4}\left(2^m m! w_{0x}\right)^{-1/2}$$

The coupling efficiency C_{pq} with which a waveguide mode couples back to itself is given by[5]:

$$C_{pq} = \int_{-a}^{+a} F_x(x)F_x'(x,z)dx\int_{-b}^{+b} F_y(y)F_y'(y,z)dy$$

$$L_{pq} = 1 - \left|C_{pq}\right|^2 \text{ gives the coupling loss.}$$

Where F_x' and F_y' are functions of the field which returns to the waveguide end after refection from the mirror.

$$F_x'(x,z) = \sum_m A_m \Psi_{xm}(x)\exp[2j\Phi_{xm}(z)]$$

$$F_y'(y,z) = \sum_m B_m \Psi_{ym}(y)\exp[2j\Phi_{ym}(z)]$$

Assumed that the waists of all the normal Gaussian modes are located at the waveguide end where the radiation emerging from the waveguide also has a plane wavefront. If at any position from the end of the waveguide, the curvature radiuses of the mirror coincide with the wavefront of the optical beam, the coupling loss will be minimal.

To get low loss cavity, a toroidal mirror is adopted, curvature radius Rx and Ry is optimized that the mirror focus the beam on the waveguide end. The curvature of the mirrors is chosen as:

$$ROC_x = \frac{d}{1 + (Z_{r,x}/w_{0,x})}$$

$$ROC_y = \frac{d}{1 + (Z_{r,y}/w_{0,y})}$$

$$w_{0x} = 0.70a$$

$$w_0 = 0.70b$$

where d is the distance between the waveguide end and the mirror, $Z_{r,x}/Z_{r,y}$ is the Rayleigh length, and w_{0x}/w_{0y} are the beam waists.
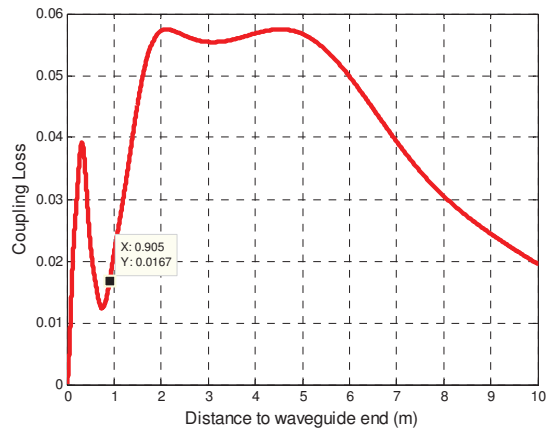
Change the distance of the mirrors to the waveguide end, the coupling loss in a rectangular waveguide (2a=40mm, 2b=10mm) is illustrated in Fig.5. It seems that If the wavelength is 100μm and mirror is placed at z1=0.376m or z2=6.40m, the coupling loss is decreased to about 1.23%. A long separation between the waveguide and mirror will contribute to low losses.

Due to space limitation, the distance between the waveguide end and the mirror is 905mm, and then the coupling loss is 5.5% at λs=100μm.

While the coupling loss is only 1.2% with the same distance to waveguide end d=905mm at the shorter wavelength λs=50μm.

(a) λs=100μm
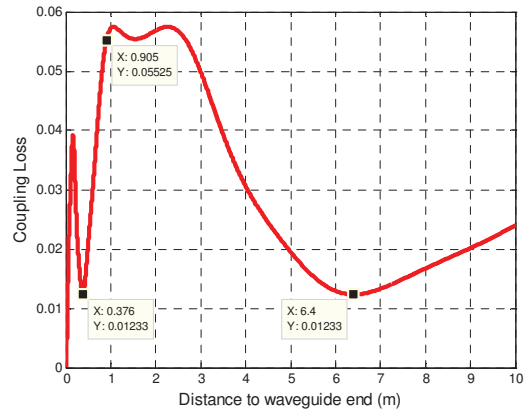
(b)λs=50μm

Figure 5: Coupling loss vs. the distance between the mirror and waveguide end (λs=100μm).

Change the waveguide dimension ratio of width to height, the coupling loss is presented in Fig.6. It seems that when a/b>10, the coupling loss is lowed to the value of parallel plane waveguide. Considering the size of the bend magnetic, ratio a/b=4 is chosen.
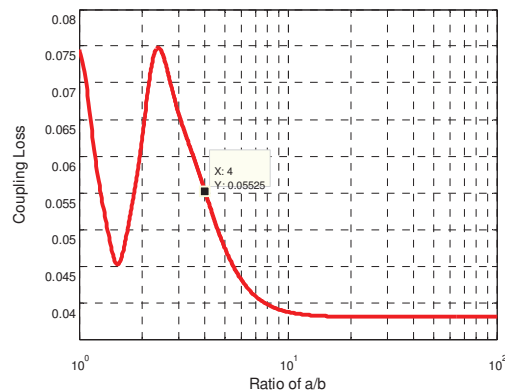

Figure 6: Coupling loss vs. waveguide dimension ratio a/b.

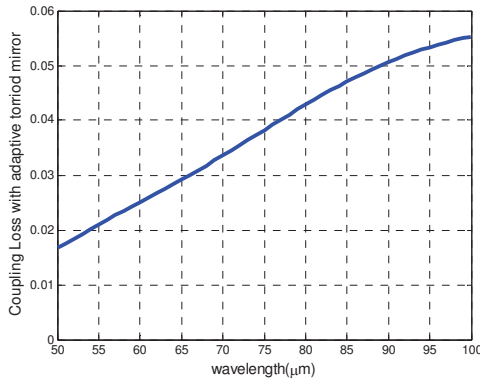The coupling loss with adaptive toroidal mirrors is presented in Fig.7, with d=905 and a/b=4.

Figure 7: Coupling loss with adaptive toroidal mirrors vs. wavelength.

The coupling losses for E_{mn}^{π} mode are dominated in the optical cavity compared to other losses in the entire FEL wavelength range.

Total Roundtrip Loss

Assumed that the optical beam pass through the rectangular waveguide, and then are coupled out, reflected by the mirror and coupled in the waveguide end twice per round trip. The resonator losses for round trip are calculated, as shown in Fig.8.
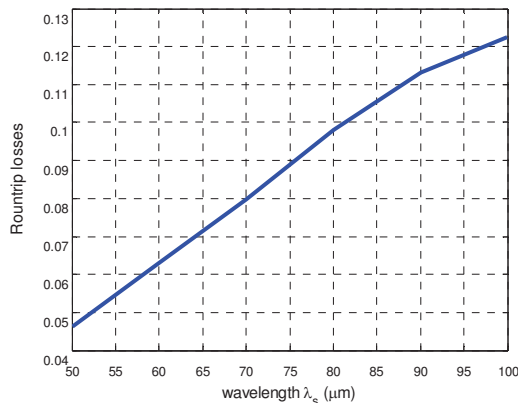

Figure 8: Total round trip loss vs. wavelength.

Gain and Saturation Time of the Cavity

The maximum single pass gain G_{max} is significantly influenced by the slippage distance and the filling factor with waveguide, which changes the longitudinal coupling and the transverse coupling between the electron beam and optical beam.

The maximum single pass gain is calculated with the parameters listed in Table 1.As shown in Fig.9, higher gain is achieved in the long wavelength with smaller waveguide height 2b.
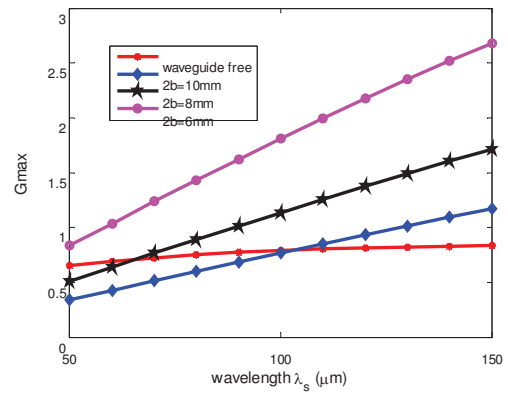

Figure 9: Max single pass gain Gmax vs. wavelength with different 2b (undulator parameter K=1) .

CONCLUSION

In a planar undulator FEL with rectangular waveguide, where the electrons are wiggling in the x direction, the electric field is x polarized, the E_{mn}^{π} modes are excited efficiently with low losses.

The ohmic losses on the mirrors coating with gold is small in terahertz spectrum region. The waveguide wall losses experienced by the E_{mn}^{π} mode is negligible, which is much smaller than the E_{mn}^{σ}, then the E_{mn}^{π} mode will be excited efficiently in FEL with waveguide.

The coupling losses for E_{mn}^{π} mode are dominated in the optical cavity. To get low loss cavity, a toroidal mirror is adopted, curvature radius Rx and Ry is optimized that the mirror focus the beam on the waveguide end. When the mirror is placed at z1=0.376m or z2=6.40m, the coupling loss is decreased to 1.23%. When a/b>10, the coupling loss is lowed to the value of parallel plane waveguide. Limited to the space, a/b=4 is chosen.

With designed parameter, the gain is more than twice of the round trip loss ~ 15%. Higher gain is achieved in the long wavelength with small waveguide height 2b.

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