# DIVERGENCE REDUCTION AND EMITTANCE CONSERVATION IN A LASER PLASMA ACCELERATION STAGE

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### Abstract

In laser-plasma accelerators, very high acceleration gradients are reached, which makes them promising candidates for high-energy applications as well as as drivers for next-generation light sources. Yet, conserving the beam quality when coupling the beam into and extracting it from the plasma is very challenging. The concept presented here employs tapered matching sections to increase the matched beamsize at the plasma entrance and to adiabatically reduce the beam divergence at the plasma exit, which suppresses chromatic emittance growth.

### **INTRODUCTION**

The electric fields present in the wakefield of a driver laser or beam in a plasma accelerator [1,2] are far larger than in conventional cavities. Acceleration of electron bunches to GeV energies over cm-scale distances has been shown [3,4,5]. However, the focusing forces provided by the transverse field components are very strong.

An electron bunch whose transverse beam size is not matched to these focusing forces will perform betatron oscillations. The betatron frequency depends on the phase in the wake and on the bunch energy. Finite bunch length and (acquired correlated) energy spread then lead to emittance growth. The bunch size therefore needs to be matched, requiring extremely strong focusing optics both for injection into and extraction out of the plasma [6]. The combination of large divergence and large energy spread also leads to strong emittance growth in the drift following the plasma target [7,8].

## ADIABATIC MATCHING IN PLASMA ACCELERATORS

We include adiabatic matching sections at the start and end of a plasma stage to increase the beta function needed to match an external beam into the plasma and to reduce the divergence before the plasma-vacuum transition. Adiabatic profiles are characterized by the changes of focusing strength being slow enough so the bunch envelope can follow its changes and stay matched. Since no betatron oscillations are performed, the emittance is conserved. Here, we derive with simulations ideal profiles for both plasma density and driver laser evolution. We use the linear wakefield model with

$$E_z(r,\zeta) \propto a^2 k_p^2 \exp(-k_p^2 \sigma_z^2/2 - 2r^2/w^2) \cos \Psi,$$
  
 $E_r(r,\zeta) \propto -a^2 k_p r/w^2 \exp(-k_p^2 \sigma_z^2/2 - 2r^2/w^2) \sin \Psi,$ 

that we implemented in the particle tracking code Astra [9] and cross-checked for short plasma targets with the PIC code VSim (formerly Vorpal) [10]. Here,  $\Psi = k_n \zeta$  is the phase,  $\zeta = z - v_g t$  the co-moving variable,  $v_g^{\prime}$  the laser group velocity and  $k_p c = \sqrt{ne^2/m_e \varepsilon_0}$  the plasma frequency. The normalized vector potential a of the Gaussian laser pulse is given by  $a^{2} = a_{0}^{2} \exp(-2r^{2}/w^{2}) \exp(-\zeta^{2}/2\sigma_{z}^{2})$ . The model is valid for  $a_0 < 1$  and includes changes of bunch phase due to the laser group velocity and due to changes in density. It does not include pump depletion and changes of the wakefield caused by transverse density profiles.

We consider a plasma stage separated into injection, acceleration and extraction section and has a peak density of  $n_0 = 1 \cdot 10^{17}$  cm<sup>-3</sup>. An electron beam of 100 MeV kinetic energy, normalized emittance  $\varepsilon_{nx} = 1$  mmmrad and  $\sigma_z = 1 \mu$ m is injected externally. The laser with  $a_0 = 1.3$ , fwhm length 130 fs and a spot size of  $w_0 = 26 \mu$ m is focused at the start of the acceleration section. In the injection section it follows the Gaussian beam evolution while it is assumed to be guided throughout the acceleration.

The matching condition reads  $\beta_m = 1/\sqrt{K}$  for K the focusing strength of the wakefield given by

$$K = \frac{e}{\gamma m_e c^2} \frac{\partial E_r}{\partial r} \Big|_{r=0}$$
<sup>(1)</sup>

$$\propto -\frac{a^2k_p}{w^2}\exp(-k_p^2\sigma_z^2/2)\sin\Psi.$$

For a sharp plasma edge, i.e. no injection section, the laser focus and the electron focus coincide and a matched beta function of  $\beta_m = 0.5$  mm is required.

In an injection section, the bunch experiences slowly increasing focusing forces. This lensing effect leads to a decrease of bunch size till the start of the acceleration section. The bunch therefore can be injected with a virtual focus in the injection section that can be much larger than without this section. The section length is naturally limited by the Gaussian evolution of the laser leading to much weaker fields in the wake away from the focus.

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We choose a linear plasma up ramp as the simplest function and match the electron beam by backpropagating it from the start of the acceleration section using the differential envelope equation  $\sigma_x'' + K(z)\sigma_x - \varepsilon^2 \sigma_x^{-3} = 0$ . The beta function in the virtual focus depends on the length of the upramp  $l_{inj}$  as can be seen in Figure 1. It saturates for long plasma upramps when the laser evolution becomes the limiting factor.



Figure 1: The beta function in the virtual focus depends on the length of the linear density upramp  $l_{inj}$  and saturates for very long injection sections since here the contribution of the laser very far away from the focus becomes negligible.

In the acceleration section the density is constant and the laser is assumed to be guided. We end this section when the bunch has reached dephasing at  $k_p\zeta = -\pi/2$ , that is after 79 cm. The bunch acquires a large energy chirp of  $\Delta E/E/\sigma_z = 7$  %/µm which is mainly caused by off-crest acceleration combined with the finite bunch length compared to the plasma wavelength. After the acceleration, the bunch has a kinetic energy of 8.8 GeV. The emittance is conserved since changes of K are only caused by phase slippage which is a slow, adiabatic process.

In the extraction section, the divergence needs to be reduced for two reasons: Firstly, to relax the requirements on beam capturing optics behind the plasma and secondly to damps chromatic emittance growth in the drift following the plasma vacuum transition.

The general profile for the focusing strength [11]

$$K = \frac{K_0}{(1+gz)^4}$$
(2)

provides largest adiabatic bunch size increase over the shortest distance. Since the emittance is conserved due to adiabaticity, the divergence is reduced. Here, g is a taper parameter that has to fulfill  $g\beta_0 \ll 1$  with  $\beta_0$  the beta function at the start of the extraction section.

We derive profiles in terms of laser and plasma parameters for two extreme cases, separating the contributions of laser and density. Assuming constant density and neglecting slippage between laser and bunch,

the respect

equation (1) simplifies to  $K \propto a^2/w^2 \propto 1/w^4$ . The ideal transverse laser profile evolution is then

$$w(z) = w_0(1+gz).$$
 (3)

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Assuming the laser to be guided at constant  $w(z) = w_0$ , equation (1) gives  $K \propto -k_p \exp(-k_p^2 \sigma_z^2/2) \sin(k_p \zeta)$ . We find a fit to the numerical solution for  $k_p$  of this equation together with equation (2) as

$$n = \frac{n_0}{(1 + agz + bg^2 z^2)^2} \tag{4}$$

where a = 3.45 and b = 1.59 are fit parameters and g is the taper parameter. This fit is valid for starting the density downramp at dephasing.

Even though these profiles are optimized for the purpose of divergence reduction, their exact shape leaves room for variation as long as K is changed slow enough to still be adiabatic.

Astra simulations have been done to compare a plasma profile with a sharp end to a profile including the proposed ideal extraction section. Over a length of 53 cm K is tapered according to eq. (3) with  $g = 30 \text{ m}^{-1}$ . The plasma density is constant while the laser is assumed to be guided in such a fashion that its envelope increases linearly. As can be seen in Figure 2 the divergence is drastically reduced to only 15 µrad compared to 160 µrad for the sharp plasma end. This supresses emittance growth in the drift to only  $\Delta \varepsilon_{nx}/\Delta z = 0.06 \text{ mmmrad/m}$ , whereas the emittance increases by  $\Delta \varepsilon_{nx}/\Delta z = 33 \text{ mmmrad/m}$  without extraction section.

#### CONCLUSION

We have shown in simulations that by including dedicated matching sections for coupling a beam into and extracting it from a plasma stage, it is possible to drastically relax the requirements on beam optics before and after the stage. The beta function needed to match into the stage was increased by more than an order of magnitude, while the divergence after extraction was decreased by a similar amount. This minimizes chromatic emittance growth after the plasma and is of interest also for schemes employing internal injection.

Compared to PIC codes, the linear wakefield model used here is simplified in order to concentrate on the underlying beam dynamics, and does not include effects like beam loading or pump depletion.

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Figure 2: Top: Plasma profile (blue) and laser envelope (dashed red) for a case including a extraction section employing laser tapering (b) and a case with sharp plasma end (a) for comparison. Bottom: Normalized emittance (red) and beam divergence (black) of the electron beam throughout the complete stage. The extraction section drastically reduces the divergence and thus also suppresses chromatic emittance growth in the drift.

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