CHIRPED AND MODULATED ELECTRON PULSE FREE ELECTRON LASER TECHNIQUES

J. R. Henderson[1], L.T. Campbell[1,2,3,4], and B.W.J. McNeil[1]

[1]SUPA, Department of Physics, University of Strathclyde, Glasgow, UK

[2]ASTeC, STFC Daresbury Laboratory and Cockcroft Institute, Warrington, UK

[3]Center for Free-Electron Laser Science, Notkestrasse 85, Hamburg, Germany

[4]Institut für Experimentalphysik, Universität Hamburg, Hamburg, Germany

ABSTRACT

A potential method to improve the free electron laser's output when the electron pulse has a large energy spread is investigate and results presented. A simplified model is the first given, in which there are a number of linearly chirped beamlets equally separated in energy and time. By using chicanes, radiation from one chirped beamlet is passed to the next, helping to negate the effect of the beamlet chirps and maintaining resonant interactions. Hence the addition of chicane allow the electrons to interact with a smaller range of frequencies ($\Delta\omega < 2\rho\gamma_r$), sustaining the FEL interaction. One method to generate such a beamlet structure is presented and is shown to increase FEL performance by two orders of magnitude.

INTRODUCTION

Free Electron Lasers are already important research tools and have started to unlock many new areas of science in diverse fields such as; Warm-Dense matter studies, short pulse protein diffraction and medicine/surgery. Current Free Electron Lasers rely on linear accelerators to provide the electron bunch, for an x-ray FEL the accelerator can be kilometres long. The potential for plasma-wakefield accelerators to drive the Free Electron Laser has been of theoretical and experimental interest for many years. Plasma accelerators generate accelerating gradients on the order of 10^3 times greater traditional linear accelerators, which offers the potential to reduce the total length of the FEL. Electron pulses used in free electron lasers can exhibit a large energy chirp (greater than 1 % of mean electron beam energy) which can degrade the FEL interaction. Linear energy chirps have been previously studied in [1] the results of this work have been recreated here using Puffin [2] an unaveraged 3D parallel FEL simulator. The results of these chirped pulse simulations are in good agreement with [1] showing the flexibility of Puffin. Electron pulses from plasma accelerators are limited by a large energy spread, this is also issue with older accelerators were energy spread is sacrificed for a larger rho (a measure of FEL efficiency) and higher pulse energies. A method that may allow the free electron laser to operate with a large energy spread is proposed, simulations were performed using Puffin. In this method a chirped electron pulse is split in a number of chirped electron beams or beamlets. To sustain the FEL interaction radiation is passed from beamlet to beamlet by applying a series of chicane slip-page sections. By making the slippage in undulator-chicane module equal the beam separation the radiation pulse will continuously interact with electrons within the same energy range. One method to generate a similar beamlet structure, the beamlet method, is presented. In the beamlet method a modulator-chicane section is used to generate a set of beamlets which have a smaller local (slice) energy spread than the initial electron pulse. Radiation is then passed from these areas of reduced energy spreads to sustain the FEL interaction. This method shows an approximate two-fold improvement in the radiation field intensity and a four-fold improvement when the radiation field is filtered around the resonant frequency.

SINGLE CHIRPED PULSE

When an electron pulse is given an energy chirp, the effects can be both beneficial and detrimental to the FEL interaction depending upon the gradient of the chirp [1].
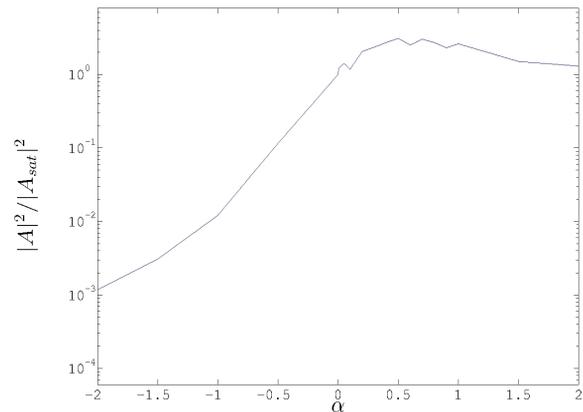
Figure 1: Chirped electron pulse: the scaled saturation power $|A|^2$ is plotted as a function of the energy chirp, the energy chirp parameter [1] is given by $\hat{\alpha} = -\frac{2}{\rho\gamma_r}\frac{d\gamma}{d\bar{z}_2}$, were $|A_{sat}|^2$ is the saturation intensity at $\hat{\alpha} = 0$. This agrees with Figure 2 of [1] where matching parameters parameters have been used. $|A|^2/|A_{sat}|^2$ is equivalent to η from [1].

The results of [1] are reproduced using Puffin. Puffin uses the scaled notation of [3, 4], were $\bar{z}_2$ defines a position in the electron bunch and is given by $\bar{z}_2 = (ct - z)/l_c$ where the cooperation length is defined as $l_c = \lambda_r/4\pi\rho$. The scaled radiation is field is given by, $A_\perp = \frac{eu\bar{a}_u l_g}{2\sqrt{2}\gamma_r^2 mc^2\rho}E_\perp$, where u

describes the undulator polarization (i.e. $u = 1$ planar or $u = \sqrt{2}$ helical), which in this case is helical. At saturation in the free electron laser exhibits temporal spikes of the order $A_\perp A_\perp^* = |A|^2 = 1$.

Figure 1 shows the normalised saturation power ($|A|^2/|A_{sat}|^2$) against energy chirp parameter $\hat{\alpha}$, where $|A_{sat}|^2$ is the average radiation field at saturation for the unchirped electron pulse. The resonant energy is defined as the initial average energy $\gamma_r = <\gamma_j>|_{\bar{z}=0}$ and the gain length is defined as $l_g = \lambda_u/4\pi\rho$. The saturation length for various chirps was estimated from Figure 1 of [1]. It should be noted that extending the simulation along a long undulator can generate large power spikes of $|A|^2 = 25$ (see Figure 2). This result was not included in [1]. The electron pulse's energy chirp is beneficial, with a flat-top (non-chirped) electron pulse, the electron pulse loses energy as the radiation field grows after a number of undulator periods the electrons are no longer resonant with the initial radiation field (i.e. the electron pulse can now only amplify radiation at a lower frequency). With a chirped pulse the interaction can be sustained, when the radiation pulse propagates through the electron pulse it will find electrons at a higher energy (which are not resonant with it) and electrons that have lost energy which are now resonant with this radiation pulse. This technique is similar to undulator tapering, wherein the undulator's magnetic field is longitudinally tapered to maintain a resonant interaction along the undulator. The resonant FEL interaction continues as the radiation pulse propagates through the electron pulse allowing the generation of large radiation spikes.
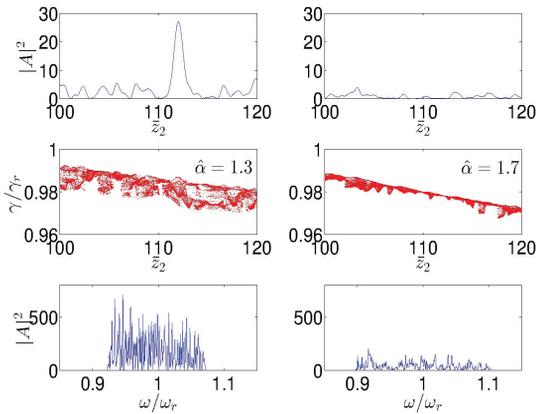
Figure 2: Comparison of chirped electron pulses with different gradients at $\bar{z} = 25$, for $\hat{\alpha} = 1.3$ larger radiation spikes are generated. The energy chirp is beneficial to the FEL interaction because the energy chirp is matched to the rate of energy loss by the electrons so maintaining resonance as the spike propagates through the electron pulse. Larger radiation spikes are present when $\hat{\alpha} = 1.3$ because as the radiation pulse as propagates through the electron pulse it will interact with more electrons within it's FEL bandwidth than it will with a larger energy chirp.

MULTIPLE CHIRPED PULSES - BEAMLETS

In this Section multiple chirped pulses equally spaced in energy are presented. We call this a system of 'beamlets'. Five beamlets are linearly chirped in energy and then equispaced in energy. A schematic of the chirped beamlets is shown in Figure 3 The energy separation of $\Delta\gamma = 2.5\rho\gamma_r$ with a gradient $\frac{d\gamma}{d\bar{z}_2} = \rho\gamma_r$.

In Puffin each macroparticle is given a macroparticle charge weight χ_j defined as $\chi_j = n_j/n_p$ [2], n_j is the macroparticle charge density and n_p is peak macroparticle density of the electron beam. In the beamlet model here, the chi-value of each macroparticle is given by $\chi_n = \chi_j/N_b$ where N_b is the number of electron beams and χ_j is the chi-value if only one beam were modelled. Therefore if all the electron pulses have the same resonant energy (i.e. $\Delta\gamma = 0$) and then the pulses would be indistinguishable.

Table 1 lists all the relevant simulation parameters.

The Model - Multiple Chirped Pulses
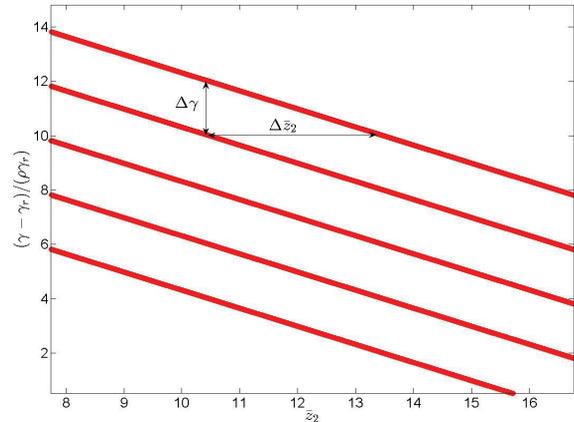
Figure 3: This diagram shows the initial electron pulse phase space. Five cold electron pulses with a linear energy chirp are overlapped and separated in energy by $2.5\rho\gamma_r$. The chirp gradient is $\frac{d\gamma}{d\bar{z}_2} = \gamma_r\rho$ and the temporal separation is $2.5\,l_g$.

The basic principle of this model is to pass radiation from beamlet to beamlet. This is achieved by using a undulator-chicane lattice, the range of electron energies experienced by a radiation pulse can be controlled by changing number of undulator periods per undulator-chicane module.

Standard mode-locking free electron laser theory [5,6] states that an undulator-chicane lattice will amplify modes which are separated by $\Delta\omega/\omega_r = 4\pi\rho/\bar{s}$, where $\bar{s}$ is the scaled slippage in an undulator-chicane module $\bar{s} = s/l_c$. The undulator-chicane slippage is the sum of slippages in the undulator $\bar{l}$ and chicane $\bar{\delta}$ i.e $\bar{s} = \bar{l} + \bar{\delta}$. The beamlets are are separated by $\Delta\gamma$ in energy which can be rewritten as a frequency difference of $\Delta\omega/\omega_r \approx 2\Delta\gamma\gamma_r$ using the resonance condition. To lock the modes these two frequencies are equated; $2\Delta\gamma/\gamma_r \approx 4\pi\rho/\bar{s}$. The chirp gradient is given

by the ratio of the beamlets energy and temporal separation. In this case the temporal separation is given by $\bar{s}$ to ensure that the radiation always interacts with electrons of the same energy range. Therefore the electron chirp gradient is given as,

$$\frac{d\gamma}{d\bar{z}_2} = \frac{\Delta\gamma}{\Delta\bar{z}_2} = \frac{4\pi\rho\gamma_r}{2\bar{s}}\frac{1}{\bar{s}}$$
$$\frac{d\gamma}{d\bar{z}_2} = \frac{2\pi\rho\gamma_r}{\bar{s}^2}. \tag{1}$$

In Figure 3 the initial electron pulse phase space is shown, the cold beam approximation is made, the beamlets are separated by $2.5\rho\gamma_r$ and have energy chirp such that $\frac{d\gamma}{d\bar{z}_2} \leq \rho\gamma_r$. This condition ensures that for $1l_g$ (gain-length) of interaction the radiation field only interacts with electrons within the FEL bandwidth, a value of $\frac{d\gamma}{d\bar{z}_2} = \rho\gamma_r$ was chosen. Temporally the beams are now separated by $2.5l_c$. By using an undulator-chicane lattice radiation can be passed from one beamlet to the next. The undulator-chicane slippage is set equal to the temporal separation of the beamlets, by doing so the radiation pulse at the start of each undulator module can interact with electrons of the same energy. This allows the radiation field to continuously interact with electrons of the same energies sustaining the FEL interaction.

An electron pulse with a linear energy chirp will have an energy dependent slippage length, s_γ, because electrons travelling at different speeds will arrive at the end of the undulator at different times. Therefore the separation of the modes amplified by an undulator-chicane lattice will be energy dependent $\Delta\omega/\omega_r = 4\pi\rho/\bar{s}_\gamma$, where $\bar{s}_\gamma = \bar{l}_\gamma + \bar{\delta}_\gamma$. Here $\bar{l}_\gamma$ and $\bar{\delta}_\gamma$ are the energy dependent slippages in the undulator and chicane respectively. The energy dependent slippage can be written as,

$$\bar{s}_\gamma = (\bar{l} + D)p_{2j} - D + \bar{\delta} \tag{2}$$

were $p_{2j} \approx 1 - 2(\gamma_j - \gamma_r)/\gamma_r$, or in terms of γ_j,

$$\bar{s}_\gamma = 2\left(\frac{\gamma_r - \gamma_j}{\gamma_r}\right)(\bar{l} + D) + \bar{s} \tag{3}$$

.

As the modal separation of the undulator-chicane modes is now energy dependent this can reduce the mode visibility for large energy chirps and long undulators. It is possible to overcome this by applying a negative dispersion in the chicane , when $D = -\bar{l}$ the effect of the energy dependent slippage is negated (i.e. $\bar{s}_\gamma = \bar{s}$). Equation 3 also explains why using a long undulator (large $\bar{l}$) can destroy the mode-locking by increasing the energy dependence of the slippage length. Similarly a large dispersion D also increases the effect of the energy dependent slippage.

Results - Multiple Chirped Pulses

The simulations that follow were performed using Puffin with the parameters stated in Table 1. Numerous simulations were performed to demonstrate the effects of changing the

Table 1: Simulation Parameters for Multiple Chirped Pulses

Parameter	Value	description
Q	3E-12 C	charge per electron pulse
ρ	0.001	FEL parameter
γ_r	176.2	mean energy of beamlets
l_b	80 l_c	bunch length
a_w	0.511	undulator parameter
$\Delta\gamma$	2.5 $\rho\gamma_r$	beam separation in energy
$\Delta\bar{z}_2$	2.5	beam separation in time
$\frac{d\gamma}{dz_2}$	$-\rho\gamma_r$	linear electron pulse chirp
α	2	Saldin chirp parameter

undulator-chicane configuration. The effect of the energy dependent slippage is demonstrated, by destruction of the modes when increasing $\bar{l}$ and D (see equation 3). It is will also be shown that modes normally destroyed by a long undulator (large $\bar{l}$) can be recovered by applying a negative dispersion such that each undulator-chicane module has zero dispersion, i.e. $D = -\bar{l}$, in accordance with equation 3. In addition to this previous unknown chicane modes are generated after the undulator-chicane modes are destroyed. The additional modes are generated by a slippage only chicane.
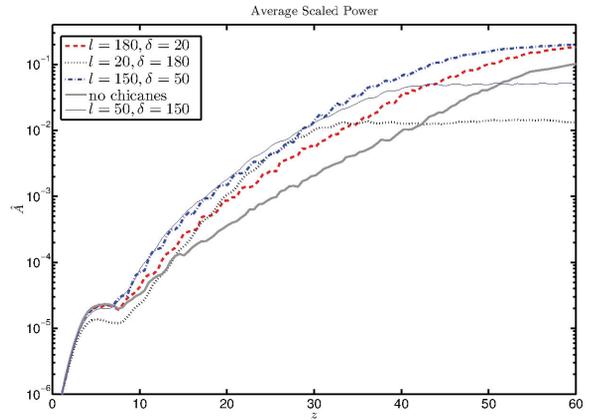
Figure 4: Comparison of a simulation using chicane slippage sections to a simulation without chicane sections. The average radiation power is plotted against $\bar{z}$ (i.e. undulator position). For a large section of the FEL interaction the average radiation power is a factor of ten greater when chicane slippage sections are used.

Figure 4 shows that the average power from beamlets is dependent on the undulator-chicane lattice step-up, increasing the undulator length (while decreasing the chicane slippage) produces higher radiation powers, as happens in normal mode-locking free electron lasers.

In Figure 5 five beamlets are sent through an undulator-chicane lattice. This lattice amplifies modes that are separated by $\Delta\omega/\omega_r = 4\pi\rho/\bar{s} = 0.005$ where $\bar{s}$ is the scaled slippage in an undulator-chicane module. Electron pulses with an energy chirp have an energy dependent slippage (equation 3), i.e. higher energy electrons slip less than low
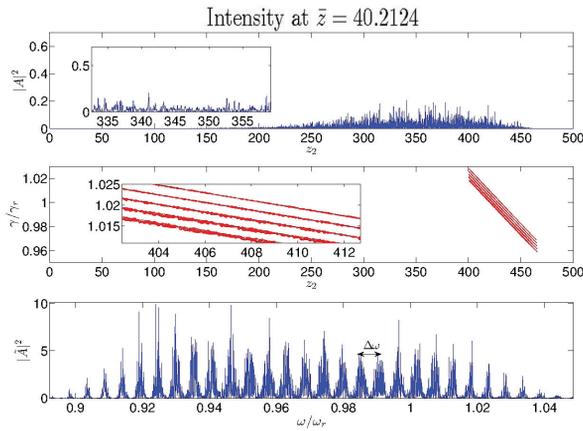
Figure 5: At $\bar{z} = 40$ the radiation is high (top panel), electron microbunching is well developed (middle), the spectrum of the field is shown in the bottom panel. In this case $\bar{l} = 0.2513, \bar{\delta} = 2.2619$. Giving a modal separation is given by $\Delta\omega/\omega_r = 4\pi\rho/\bar{s} = 0.005$, which matches the numerical result well.
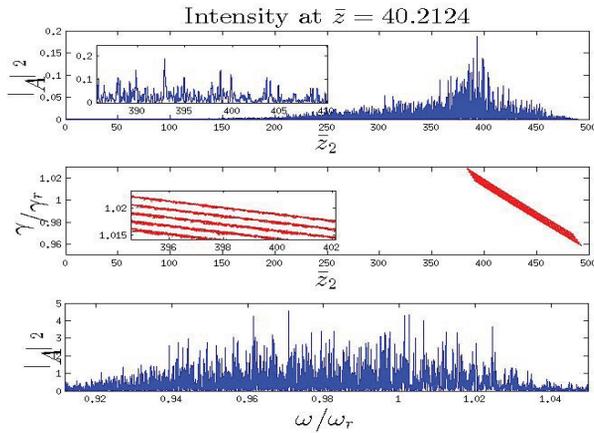
Figure 6: This simulation is identical to Figure 5 , expect that the chicanes now apply a small dispersion to the electron pulse which disrupts mode formation. The reason for this is that an electron pulse with such a large energy chirp will have an energy dependent slippage length. For example the slippage of a low energy electron will be greater than that of a higher energy electron. Using a dispersive chicane will increase the difference in slippage lengths between high and low energy electrons. Therefore modes generate at the head of the pulse will have different $\bar{s}$ than at the tail and hence a different $\Delta\omega_\gamma/\omega_r = 4\pi\rho/\bar{s}$ (see Eq. 3), such that the modes start overlapping each other. However using a non-dispersive chicane means that the difference in modal separation is due solely to undulator dispersion, which is small enough to allow the modes to form.

energy electrons. The result of this is that any mode-locking that occurs at the high energy part of the pulse will involve a different set of modes than low energy part of the pulse. The energy dependent slippage (equation 3) can counteracted

by using a short-undulator and a slippage only chicane as is seen Figure 5. However using a dispersive chicane will increase the effect of this energy slippage (equation 3), as higher higher electrons will propagate more than low energy electrons increasing the difference in their slippage lengths. This difference in slippage lengths means that radiation field cannot be passed to the same energy range from one electron pulse to the next. The combination of these effects destroys the modes as is shown in Figure 6. The modes are also destroyed when using a long undulator (Figure 7), as the undulator dispersion over a large number of periods is significant enough to disrupt the mode formation.

In Figure 7 a dispersionless chicane generates an extra set of modes separated by $\delta\omega = 4\pi\rho/\bar{\delta} = 0.02$, these modes are generated because the beamlets will produce a radiation pulse before and after the dispersionless chicane that are similar (nearly identical), therefore the only modes
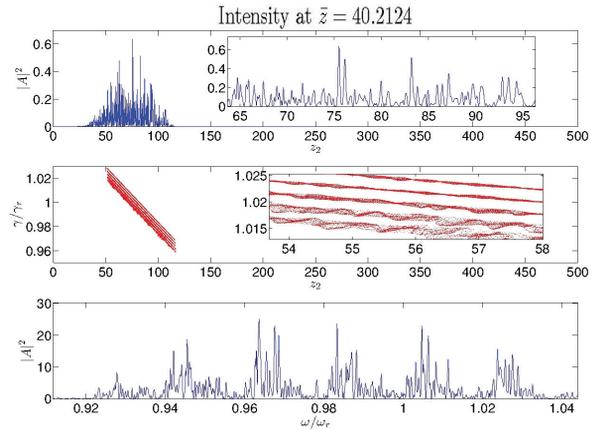
Figure 7: In this simulation longer undulator modules are used, $\bar{l} = 1.885, \bar{\delta} = 0.6283$ and slippage only chicanes. Using a longer undulator will produce more dispersion in the electron pulse, this makes the energy dependent slippage between electron of various energies greater. Therefore the radiation modes now overlap. However the slippage only chicanes will produce modes separated $\delta\omega/\omega_r = 4\pi\rho/\bar{\delta}$, this because the electron pulse will produce similar (nearly identical) radiation pulse before and after the slippage only chicane. Therefore the only modes that can survive the U-CS-U section are given by $\delta\omega/\omega_r = 4\pi\rho/\bar{\delta} = 0.02$.

that can survive this chicane slip are those separated by $\delta\omega = 4\pi\rho/\bar{\delta} = 0.02$. These modes can be replaced by the original modes by applying a negative dispersion in the chicanes (Figure 8), the dispersion is in a chicane is such that the total dispersion by an undulator-chicane module is now zero. Having zero-dispersion undulator-chicane modules negates the effects of the energy dependent slippage (see equation 3); as energy dependent slippage and dispersion are the same thing. Using zero-dispersion undulator-chicane modules hinders the development of the bunching, reducing the radiation intensity. The modes of Figure 7 can also be generated by using a short undulator (U), a dispersion only

chicane (CD) and a slippage only (dispersionless) chicane (CS). For this simulation (Figure 9) of an undulator-chicane lattice was constructed from modules of U-CD-U-CS, where each undulator has half the number of periods (10 periods) of the undulator used in Figure 5. The dispersion only chicane will supply the equivalent of 130 periods of undulator dispersion. This gives a total dispersion of 150 periods per module just as in Figure 7 and then the slippage only chicane generates modes separated by $\delta\omega = 4\pi\rho/\bar{\delta} = 0.02$.
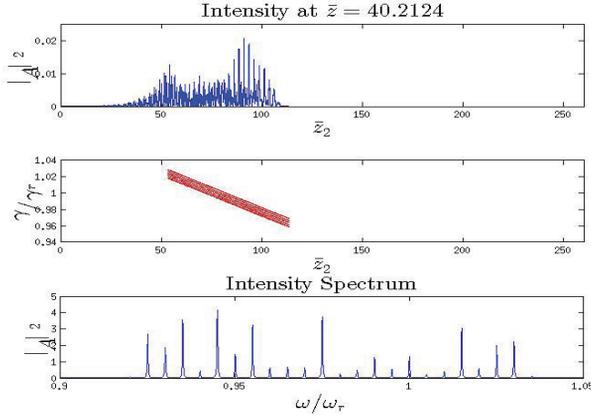
Figure 8: The modes that where destroyed in Figure 7, can be restored by using a chicane that applies a negative dispersion. The magnitude of dispersion applied is equal to the amount dispersion experienced in the preceding undulator section, therefore in an undulator-chicane module there is effectively zero dispersion. In this simulation $\bar{l} = 1.885, \bar{\delta} = 0.6283$. The modes are more clear in this case because of the zero dispersion undulator-chicane modules, where as in Figure 7 the undulator dispersion causes a differential slippage whereby the mode separation varies with energy. Note the reduced radiation intensities, which is due to the negative dispersion chicanes that prevents the formation of microbunches.

A full understanding of the interaction of multiple electron pulses (beamlets) is important when modelling more advanced 'novel' FEL schemes. The effect of undulator length, energy dependent slippage and dispersion has been analysed, and will impact different FEL schemes where large variation in electron energies are present.

BEAMLETS

In this section a technique to generate chirped electron pulses from a single electron pulse is presented. The beamlet technique is a two stage method which involves an undulator and a chicane. The electron pulse is modulated in energy and then dispersed. This generates a series of beamlets with reduced local energy spreads, passing radiation from beamlet to beamlet can sustain the FEL interaction.

The Model - Beamlets

The electron pulse is first modulated in an undulator and then dispersed by a chicane section, these transformations
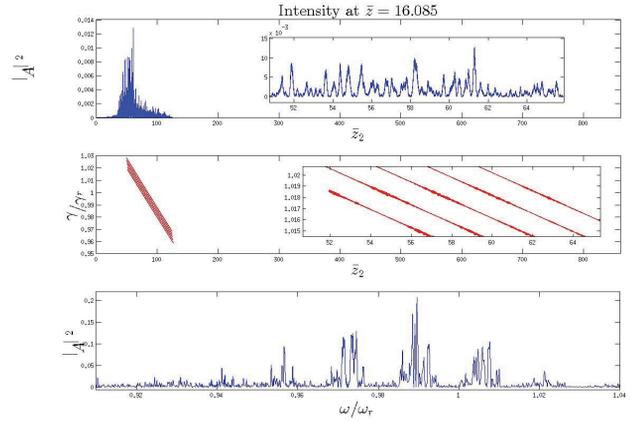
Figure 9: Using a unique undulator-chicane lattice, consisting of dispersion only chicanes (CD) and slippage only chicanes (CS). The lattice is constructed from blocks of U-CD-U-CS arranged in series. In the undulator $\bar{l} = 0.1257$ (10 periods), in the dispersion only chicane (CD) the equivalent of $\bar{\delta} = 1.6336$ (130 periods) of dispersion is applied and in the slippage only chicane (CS) a slippage of $\bar{\delta} = 0.6283$ (50 periods) is used. Therefore the dispersion is the same per module as it is in Figure 7. As in Figure 7 the slippage only chicane generates modes given by $\delta\omega/\omega_r = 4\pi\rho/\bar{\delta} = 0.02$.

are performed by applying the point-transforms given below,

$$\gamma_j = \gamma_j + \gamma_m \sin\left(\frac{\bar{z}_{2j}}{2\rho}\right) \qquad (4)$$

$$\bar{z}_{2j} = \bar{z}_{2j} + 2D\left(\frac{\gamma_j - \gamma_r}{\gamma_r}\right). \qquad (5)$$

Upon exiting the chicane the electron pulse has a unique phase-space structure. In Figure 10 this phase space structure is shown and is similar to series of chirped electron pulse or beamlets. After the undulator-chicane section the beamlets are passed through an undulator-chicane lattice, this allows radiation to be passed from beamlet to beamlet, similar to the chirped electron pulses method present earlier, sustaining the FEL interaction throughout the electron pulse. The slippage in a undulator-chicane module is therefore equal to the period of energy modulation. A modulation amplitude of $\gamma_m = 0.04\gamma_r$ and dispersion of $D = 200$ was selected.

Results - Beamlets

An electron pulse with a large energy spread and Gaussian current profile is initially generated ($\sigma_\gamma \approx 2.5\gamma_r\rho$, were $\rho = 0.01$). Applying a Gaussian distribution in energy and space will reduce the charge density at the electron pulse corners (in phase space) see Figure 10. Macroparticles with such a low weight are eliminated by Puffin, consequently the electron pulse's phase space is rounded. The electron pulse's Gaussian current profile generates coherent emission at the lower frequencies consequently this radiation has been filtered out. Due to the electron pulse's large energy spread
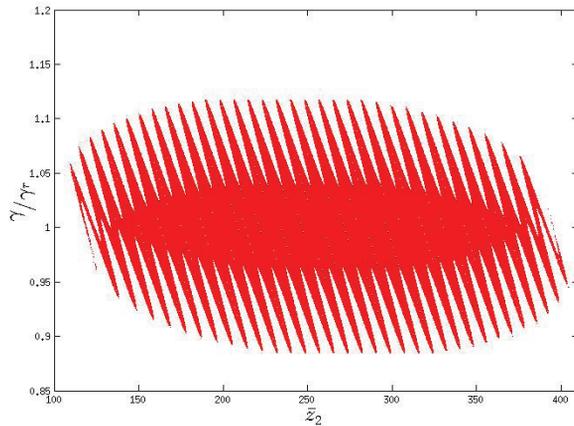
Figure 10: The EEHG beamlet scheme, an electron pusle of a large energy spread is energy modulated ($\gamma_m = 0.04\gamma_r$) and dispersed ($D = 200$) by applying transform 4 and 5. This results in the formation of beamlets with a reduced local energy spread. Radiation can then be passed from beamlet to beamlet sustaining the FEL interaction. The electron pulse was given gaussian distribution in space to cut down on coherent emission from the pulse edges. The macroparticle model of Puffin eliminates marcoparticles whose weight is below a certain threshold, as such the particles at the corners of the pulse (in phase space) have the lowest weight and are eliminated. This leaves the outer beamlets less dense and therefore less able to contribute to the FEL interaction.

it cannot produce and amplify FEL radiation to useful intensities, at $\bar{z} = 40$ generally consider to beyond saturation significant radiation is not present (Figure 11). However by applying equations 4 and 5 the electron pulse is transformed into beamlets. These beamlets (at $\bar{z} = 40$) show an approximately two-orders of magnitude improvement as can be seen in Figures 12 and 11. For these simulations $\rho = 0.01$ and the undulator-chicane lattice had 50 undulator periods per module ($\bar{l} = 6.2832$) and 18 chicane ($\bar{\delta} = 2.2619$) slippage periods. In Figure 13 a comparison is made between the beamlet case and the original beam, the radiation field has been filtered around the resonant frequency $0.8 < \omega/\omega_r < 1.2$.

CONCLUSION

The interaction of multiple electron pulses has been analysed and this understanding should prove useful when designing more novel FEL techniques. A potential scheme for generating radiation from an electron pulse of large energy spread has been demonstrated. However many improvement (optimisations) should be possible. Using dispersive chicanes further along the undulator-chicane lattice may help improve the FEL interaction. Due the energy dependent slippage of the beamlets, it may be useful (when using large undulator sections) to match the slippage of various sections of the beam. For example decreasing the slippage per module to counteract the increased slippage for lower en-
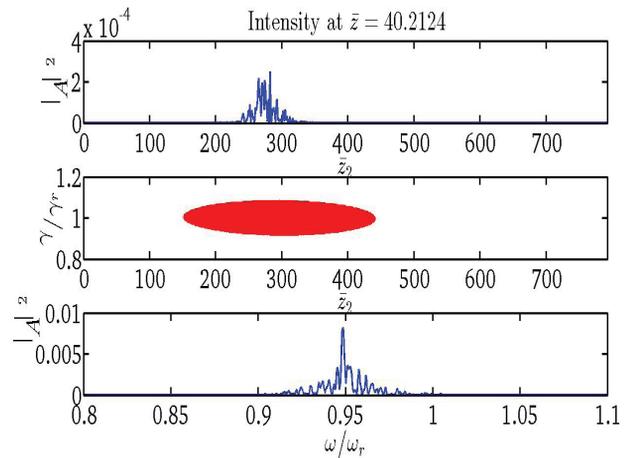
Figure 11: An electron pulse with a large energy spread that exceeds the FEL operational range. This pulse also has a gaussian distribution in space, this was done to minimize coherent spontaneous from the pulse edges.
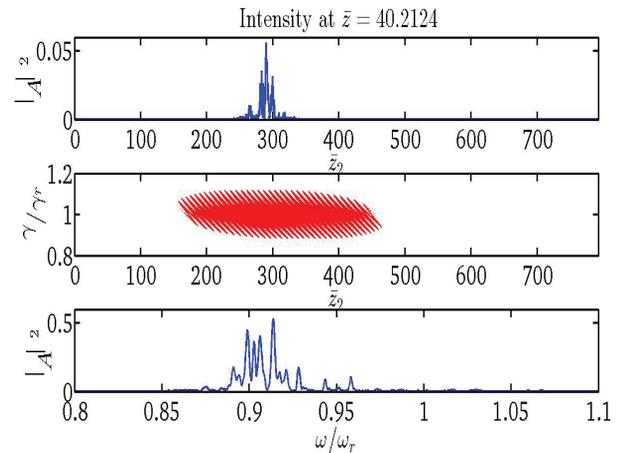
Figure 12: Beamlets near saturation, the radiation field is approaching saturation and the electron microbunching highly developed. Comparing with Figure 11 shows an increase in peak field intensity of 200.

ergy electrons may prove beneficial. The modulation and dispersion parameters of the beamlets scheme can also be optimised and requires further study. The use of the two-colour FEL technique to preferentially amplify modes from the high and low energy regions of the pulse may produce higher radiation powers.

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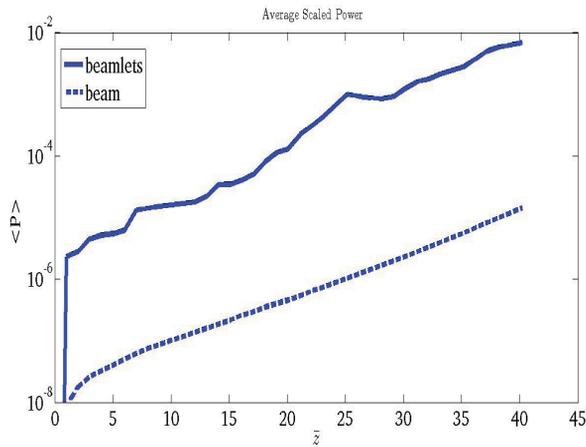
Figure 13: A comparison of the average filtered power of beamlet and original beam is given above. The radiation is filtered around the resonant frequency, $0.8 < \omega/\omega_r < 1.2$. Around 2-3 orders of magnitude improvement has been achieved and may be further optimized.

REFERENCES

[1] Saldin, E., Schneidmiller, E., and Yurkov, M. (2006). Self-amplified spontaneous emission FEL with energy-chirped electron beam and its application for generation of attosecond x-ray pulses. Physical Review Special Topics - Accelerators and Beams, 9(5), 050702.

[2] Campbell, L. T., and McNeil, B. W. J. (2012). Puffin: A three dimensional, unaveraged free electron laser simulation code. Physics of Plasmas, 19(9), 093119. doi:10.1063/1.4752743.

[3] R. Bonifacio et. el. Physics of the High-Gain FEL and Super-radiance (1990).

[4] R. Bonifacio et. el. New Effects in the Physics of High-Gain Free-Electron Laser (1992).

[5] N. R. Thompson and B. W. J. McNeil Phys Rev Let, **100** 203901 (2008).

[6] B. W. J. McNeil, N. R. Thompson D. J. Dunniing and B. Sheehy J. Phys. B: At. Mol. Opt. Phys. **44** (2001) 065404.