NUMERICAL CALCULATION OF DIFFRACTION LOSS FOR CHARACTERISATION OF A PARTIAL WAVEGUIDE FEL RESONATOR*

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Abstract

Waveguide is widely used in long wavelength Free-Electron Lasers to reduce diffraction losses. In this paper the amplitude and phase transverse distribution of light emission produced in a partial-waveguide FEL resonator is calculated by Fresnel principle. To acquire high power out-coupled and optimize resonator structure of HUST THz-FEL, the characterisation of reflecting mirror is discussed to reduce diffraction loss.

OPTICAL MODES OF WAVEGUIDE FEL

The geometry of HUST THz-FEL waveguide resonator is shown in Fig. 1. A rectangular waveguide made of good conductor is located at the middle of two reflecting mirrors. The cross section of the waveguide is a×b and the length is L. Two mirrors are at a distance from waveguide entrance, therefore the optical beam propagates in free space during waveguide and mirrors.

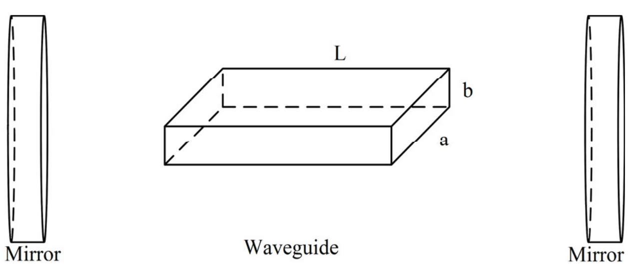

Figure 1: Layout of waveguide resonator.

To simplify this problem, the resistance of the metallic waveguide walls is neglected and the optical modes in the waveguide can be expressed as the solutions of a homogeneous Helmholtz equation [1]:

$$(\nabla^2 + k^2)\varphi(x, y, z) = 0 \qquad (1)$$

where k is the wavenumber of radiation.

Since this waveguide is adopted to a planar undulator with vertical periodic magnetic field, the eigenmodes in the waveguide is the pi modes with horizontal polarization, and it can be expressed as a combination of TE and Hermite-Gaussian mode:

$$E(x, y, z) = \cos\left(\frac{ny\pi}{b}\right)\exp\left(\frac{-x^2}{\omega^2(z)} + i\left(\frac{k_n}{2R(z)} - \frac{1}{2}\tan^{-1}\frac{z}{z_R}\right)\right) \qquad (2)$$

where $k_n = \sqrt{k^2 - \frac{n^2\pi^2}{b^2}}$, $\omega(z)$, $R(z)$ and z_R are the Gaussian beam x-radius, the x-radius of curvature of wave-fronts and the Rayleigh range. The distance z is measured with respect to the beam waist, in this case, the center of waveguide, then:

$$\omega(z) = \omega_0\sqrt{1 + \frac{z^2}{z_r}}$$
$$R(z) = z\left(1 + \frac{z_r^2}{z^2}\right) \qquad (3)$$
$$z_r = \frac{\pi\omega_o^2}{\lambda}$$

The solution describes an optical beam with a Gaussian amplitude distribution in horizontal direction and a cosine amplitude distribution in vertical.

DIFFRACTION LOSS

Optical beam is guided in waveguide so diffraction loss mainly occurs between waveguide and mirrors. In our case, the process starts from a given radiation field distribution out of waveguide, spreading to mirror and back to waveguide in free space. The shape of mirror work as diaphragm and the concave geometry will modulate the phase of optical wavefronts to focus the beam back into waveguide.

Propagation in Free Space

The Fresnel diffraction integral shows the electric field of diffraction pattern is given as a convolutional type function.

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$$E(x,y,z) = -\frac{ie^{ikz}}{\lambda z}\int_S E(x',y',0) e^{ik\frac{(x-x')^2+(y-y')^2}{2z}} dx'dy' = \psi_0(x,y) * h_z(x,y) \quad (4)$$

Analytical solution of this equation is difficult. For numerical computation, the wavefront of free space part is processed with 2D fast Fourier transformation.

The field can be calculated as a multiplication of initial optical field and a function related to free-space propagation length in the Fourier space [2].

$$F\{\psi_z(x,y)\} = F\{\psi_0(x,y) * h_z(x,y)\} = F\{\psi_0(x,y)\} \cdot F\{h_z(x,y)\} \quad (5)$$

where

$$h_z(x,y) = \frac{-ie^{ikz}}{\lambda z} e^{i\frac{k}{2z}(x^2+y^2)} \quad (6)$$

Reflecting from Mirrors

The effect of reflecting mirrors can be expressed as below:

$$R(x,y) = rM(\rho)D(x,y) \quad (7)$$

The constant r is the amplitude reflection coefficient, $M(\rho)$ is the modulation of phase which is related to the concave type and curvature radius of mirrors, $D(x,y)$ represents the geometrical shape of mirrors, which can be regarded as a diaphragm. $D(x,y)=1$ inside the mirror, and 0 outside.

For spherical mirror:

$$M(\rho) = e^{-ik\frac{x^2+y^2}{\rho}} \quad (8)$$

Because of guiding effect, the curvature radius of the wavefront is different in vertical plane and horizontal plane. A pair of toroid mirror may fit the wavefronts better and retain more optical energy.

For toroid mirror, phase delay on mirror is separated in 2 directions according to various curvature radius in x, y axis. Hence the modulation of phase is defined as

$$M(\rho) = e^{-ik\left(\frac{x^2}{\rho_x}+\frac{y^2}{\rho_y}\right)} \quad (9)$$

Diffraction Loss

Since the energy inside a diaphragm is defined by [3]:

$$I = \iint_S E(x,y)E^*(x,y)dxdy \quad (10)$$

The diffraction loss of propagation from waveguide to mirror, and then reflected back to the entrance of waveguide can be determined.

$$Loss\ Efficiency = 1 - I_{entrance}/I_0 \quad (11)$$

SIMULATION AND RESULT

Out of waveguide, the laser beam will propagate through a distance in the free space to the mirror then back to the entrance of waveguide. The simulation will be started with the eigen mode at waveguide entrance. The propagation and the optical field back to waveguide will be calculated by using Fresnel diffraction principle.

In the numerical simulation, which has been performed using HUST THz-FEL resonator structure, the loss efficiency at different wavelength is calculated to optimize the curvature radii and geometry of mirror. The configuration of HUST FEL is listed below:

Table 1: Configurations of Resonator

Configurations	Size
Waveguide horizontal size/a	40mm
Waveguide vertical size/b	10mm
Free propagation distance	905mm
Waveguide length/L	1130mm
Wavelength range	50μm-150μm
Mirror diameter	60mm

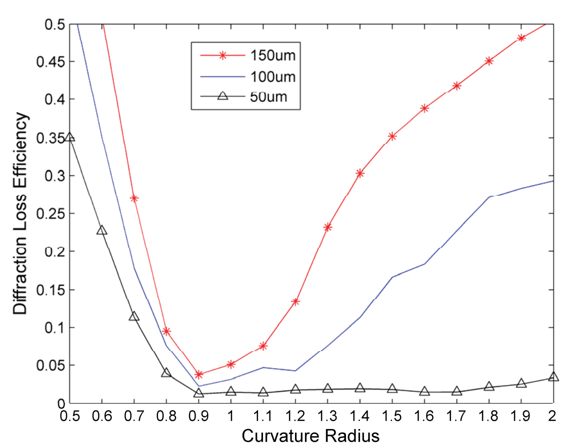

Figure 2: Diffraction loss efficiency as a function of curvature radius.

Here we take the first-order eigen mode at the opening of waveguide as initial field distribution, and calculate the diffraction loss efficiency between waveguide and reflecting mirror.

The diffraction loss of first-order eigen mode at different wavelength is plotted in Fig. 2 as a function of curvature radius of spherical mirrors. It is evident that diffraction loss is more significant at long wavelength range, which is the main reason why waveguide is widely adopted at long wavelength free electron lasers.

At various wavelength ranges, low loss situation occurs when curvature radius is 0.9m, and the loss efficiency at that spot is 3.8%. Diffraction loss at long wavelength is more sensitive, but loss efficiency at 50μm does not increase much while curvature radii rise above 0.9m.

In order to make further effort to reduce the diffraction loss, we also finished the computation of the performance of toroid mirrors. Since the radiation is guided in vertical direction and propagates freely in horizontal, the horizontal curvature radius of wavefronts would be larger than vertical ones to maintain more energy on the waveguide entrance.

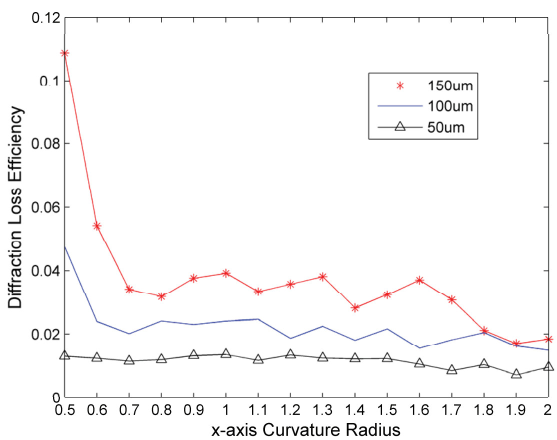

Figure 3: Vertical curvature radius at 0.9m, diffraction loss efficiency as a function of horizontal curvature radius.

Fig. 3 shows diffraction loss efficiency as a function of horizontal curvature radius when vertical curvature radius is 0.9m. The performance of toroid mirror in different wavelength is similar to spherical one: diffraction loss is higher and more sensitive at long wavelength.

The diffraction loss efficiency at different curvature radius in 2 directions is illustrated in Fig. 4.

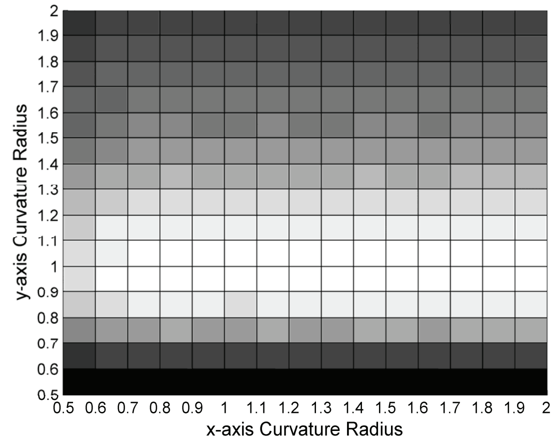

Figure 4: Wavelength at 150μm, diffraction loss efficiency as a function of 2-dimension curvature radius.

The best configuration at 150μm is a pair of toroid mirrors with R_x=1.9m in horizontal, and R_y=0.9m in vertical and the loss efficiency of that configuration is 1.7% , which is less than the loss with a pair of spherical mirror at R=0.9m. This calculation proved that the optimization of mirror in concave type and curvature radius can indeed reduce diffraction loss in HUST THz-FEL resonator.

CONCLUSION

In general, toroid mirror can achieve a better performance than spherical mirror. By optimizing mirror concave, it can lower nearly a half of diffraction loss and allows larger room for concave mechanical error.

However, in practice, adoption of toroid mirror is more expensive and time-consuming to process, and more importantly, it will increase the difficulty when using He-Ne lasers for alignment mirrors [4]. The laser spot can not be well-focused in both directions and form a round dot. Alignment accuracy might be effected.

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