INFLUENCE OF HORIZANTAL CONSTANT MAGNETIC FIELD ON HARMONIC UNDULATOR RADIATIONS AND GAIN

H. Jeevakhan, P. K. Purohit, Applied Science Department,NITTTR, Bhopal, India
G Mishra, SOP, DAVV, Indore, India

Abstract

Harmonic undulators has been analyzed in the presence of constant magnetic field along the direction perpendicular to the main undulator field. Effect of constant magnetic field magnitude on trajectory of electron beam, intensity of radiation and FEL gain at fundamental and third harmonics has been evaluated. Performance of harmonic undulator in the presence horizontal component of earth's magnetic field is the practical realization of the suggested scheme

INTRODUCTION

Current researches in science calls for an ultrafast, high brightness and X –ray light source . Fourth generation FEL systems are use to lase at X ray wavelength region [1, 2]. FEL is produced by interaction of relativistic electron beam, an electromagnetic wave travelling in the same direction and undulator. FEL differs from other conventional lasing systems in terms of operation mechanism and assembly as well [3, 4]. Novel design and error analysis of undulator are among the major and important part in FEL research. Concept of Harmonic undulator is given to use undulator assembly with slight modification and radiating lower wavelength with modest electron beam. Structure of undulator is optimized to enhance the output radiation and gain in FEL systems [5-9].

In this paper we have modeled an harmonic undulator with additional horizontal magnetic field. In real applications this component can be realized with earth's horizontal magnetic field component. In the related work, K Zhukovsky has given an analytical model and discus the effect of horizontal field constituent of undulator radiation and compare it with other factor such as energy spread in beam, emittance and focusing components [10]. N. O. Strelnikov et al presents experimental and modeling results concerning the effects of the interaction of Earth’s magnetic field with different types of Insertion devices [11]. In the previous paper [9], we have presented semi analytical results for the effect of constant magnetic field along the direction parallel to undulator field. In this paper we have added a constant field perpendicular to undulator field. The effect of additional field in horizontal direction on harmonic undulator radiations and gain has been analyzed.

UNDULATOR FIELD

We have considered a constant magnetic field in the direction perpendicular to planar undulator magnetic field encompass with harmonic field

$$B = [B_0\kappa, a_0B_0 sin k_u z + a_1B_0 sin hk_u z, 0] \quad (1)$$

where, $k_u = \frac{2\pi}{\lambda_u}$ undulator wave number, λ_u is undulator wave length, h is harmonic integer, B_0 is peak magnetic field, a_0 and a_1 controls the ratio of main undulator field to additional harmonic field κ is the magnitude of constant magnetic field. For practical purpose it is replica of horizontal component of earth magnetic field.

The velocity can be evaluated by using Lorentz force equation:

$$\frac{dv}{dt} = -\frac{e}{\gamma mc}(\vec{v} \times \vec{B}) \quad (2)$$

This gives

$$\beta_x = -\frac{K}{\gamma}\left[\cos(\Omega_u t) + \Delta\frac{\cos(h\Omega_u t)}{h}\right]$$

$$\beta_y = -\frac{K}{\gamma}[\kappa\Omega_u t] \quad (3)$$

$$\beta_z = \beta^* - \frac{K^2}{2\gamma^2}\left[\frac{1}{2}\cos(2\Omega_u t) + \frac{1}{2}\left(\frac{\Delta}{h}\right)^2\cos(2h\Omega_u t) + \left(\frac{\Delta}{h}\right)\cos(1+h)\Omega_u t + \left(\frac{\Delta}{h}\right)\cos(1-h)\Omega_u t + (\kappa\Omega_u t)^2\right] \quad (4)$$

where $K = \frac{a_0 e B_0}{\Omega_u m_0 c}$ is the undulator parameter and $\Delta = \frac{a_1}{a_0}$, and $\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2+K_1^2}{2}\right]$ with $K_1 = \frac{\Delta K}{h}$.

The electron trajectory along z direction is given by

$$\frac{z}{c} = \beta^* t - \frac{K^2}{8\gamma^2\Omega_u}\sin(2\Omega_u t) - \frac{K_1^2}{8\gamma^2 h\Omega_u}\sin(2h\Omega_u t) - \frac{KK_1}{2\gamma^2(1-h)\Omega_u}\sin(1-h)\,\Omega_u t - \frac{KK_1}{2\gamma^2(1+h)\Omega_u}\sin(1+h)\,\Omega_u t - \frac{K^2\kappa^2{\Omega_u}^2 t^3}{6\gamma^2} \quad (5)$$

The spectral properties of radiation can be evaluated from Lienard-Wiechart integral [12],

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\{\hat{n} \times (\hat{n} \times \hat{\beta}\} exp\left[i\omega(t - \frac{z}{c})\right] dt\right| \quad (6)$$

where the integration is carried over undulator length, $T = \frac{2N\pi}{\Omega_u}$ and ω is the emission frequency. Introducing the variables

$$z_1 = -\frac{\omega K^2}{8\gamma^2\Omega_u}, \qquad z_2 = -\frac{\omega K_1^2}{8\gamma^2 h\Omega_u},$$

$$z_3 = -\frac{\omega K K_1}{2\gamma^2(1-h)\Omega_u} \quad \text{and} \quad z_4 = -\frac{\omega K K_1 \kappa}{2\gamma^2(1+h)\Omega_u}.$$

The brightness expression is reduced to

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$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left(\frac{K}{\gamma}\right)^2 \left[\left|\hat{\imath}\int_0^T dt\left\{cos(\Omega_u t) + \frac{\Delta}{h}cos(h\Omega_u t)\right\} expi(\Phi t + \Psi t^3)J_m(0,z_1)J_n(0,z_2)J_p(z_3)J_q(z_4)\right|^2 + \left|\hat{\jmath}\int_0^T dt\{\kappa\Omega_u t\} expi(\Phi t + \Psi t^3)J_m(0,z_1)J_n(0,z_2)J_p(z_3)J_q(z_4)\right|^2\right] \quad (7)$$

$$\Phi = \frac{\omega}{\omega_1} - m\Omega_u - nh\Omega_u - p(1-h)\Omega_u - q(1+h)\Omega_u$$

$$\Psi = \frac{\omega K^2\kappa^2\Omega_u{}^2}{6\gamma^2}$$

and Eq. (7) can be further reduced to

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2T^2}{4\pi^2 c}\left\{|T_x|^2\ S(\Phi,\Psi) + |T_y|^2\frac{\partial S(\Phi,\Psi)}{\partial\Phi}\right\} \quad (8)$$

with

$$T_x = \frac{K}{2\gamma}\Big[\{J_{m+1}(0,z_1) + J_{m-1}(0,z_1)\}J_n(0,z_2)J_p(z_3)J_q(z_4) + \frac{\Delta}{h}\{J_{n+1}(0,z_2) + J_{n-1}(0,z_2)\}J_m(0,z_1)J_p(z_3)J_q(z_4)\Big]$$

$$T_y = \frac{2i\pi K\kappa N}{\gamma}$$

and $S(\Phi,\Psi) = \left|\int_0^1 \mathrm{e}^{(\Phi\tau+\Psi\tau^3)}\mathrm{d}\tau\ \right|^2$ and $\tau = {}^t/_T$ is unit interaction time.

FEL Gain

To calculate the small signal gain of the harmonic undulator with constant magnetic field, let us consider a radiation field as,

$$\vec{E} = E_0\cos(\psi)\hat{x} \quad (9)$$

where, $\psi = n_1k_1z - n_1\omega_1 t + \varphi$. n_1=1, 2, 3…. are the emission harmonics and φ is the phase of the electron with the radiation field. The change in energy of the electron is given by,

$$\frac{d\gamma}{dt} = -\frac{e}{m_0c}\left[\vec{E}.\vec{\beta}\right] \quad (10)$$

Eq. (3) & Eq. (9) are used to solve Eq.(10) and we obtain,

$$\frac{d\gamma}{dt} = \frac{eE_0K}{2m_0c\gamma}\left[\begin{matrix}\{\cos(\psi+k_uz)+\cos(\psi-k_uz)\}\\ +(\Delta/h)\{\cos(\psi+k_hz)+\cos(\psi-k_hz)\}\end{matrix}\right] \quad (11)$$

The electron longitudinal motion from Eq. (5) is expressed by,

$$z = \bar{z} + \Delta z$$

where

$$\bar{z} = \beta^* ct - \frac{K^2\kappa^2\Omega_u{}^2t^3}{6\gamma^2}$$

$$\Delta z = -\frac{K^2c}{8\gamma^2\Omega_u}sin(2\Omega_u t) - \frac{K_1^2c}{8\gamma^2h\Omega_u}sin(2h\Omega_u t) - \frac{KK_1c}{2\gamma^2(1-h)\Omega_u}sin(1-h)\,\Omega_u t - \frac{KK_1c}{2\gamma^2(1+h)\Omega_u}sin(1+h)\,\Omega_u t \quad (12)$$

Using Eq.(12) ,the phase terms appearing in Eq.(11) are simplified to read as,

$$\psi \pm (k_uz) = n_1\xi\ + \varphi + n_1k_1\Delta z - (n_1 \mp 1)k_uz - n_1k_hz - n_1(1-h)k_uz - n_1(1+h)k_uz$$

$$\psi \pm (k_hz) = n_1\xi\ + \varphi + n_1k_1\Delta z - (n_1 \mp 1)k_hz - n_1k_uz - n_1(1-h)k_uz - n_1(1+h)k_uz \quad (13)$$

where,

$$\xi = (k_1 + k_u + k_h + (1+h)k_u + (1-h)k_u)\bar{z} - \omega_1 t$$

Using, Eq. (13), Eq. (11) is simplified after averaging over the undulator to find ,

$$\frac{d\ \gamma}{d\tau} = \frac{eKE_0L_u}{2\gamma m_0c^2}[b_0 + (\Delta/h)b_1]\cos(n_1\xi+\varphi) \quad (14)$$

$$b_0 = \{J_{m+1}(0,z_1) + J_{m-1}(0,z_1)\}J_n(0,z_2)J_p(z_3)J_q(z_4)$$
$$b_1 = \{J_{n+1}(0,z_2) + J_{m-1}(0,z_2)\}J_m(0,z_1)J_p(z_3)J_q(z_4)$$

Expressing,

$$n_1\frac{d^2\xi}{d\tau^2} = \frac{4\pi N}{\gamma}\left[n_1 + \frac{3\Psi\tau^2L_u^2}{\Omega_uc^2}\right]\frac{d\gamma}{d\tau} - 6\Psi\tau\mathrm{T}^3 \quad (15)$$

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we get,

$$n_1 \frac{d^2\xi}{d\tau^2} = \frac{4\pi NeKE_0 L_u}{2\gamma^2 m_0 c^2}\left[n_1 + \frac{3\Psi\tau^2 L_u^2}{\Omega_u c^2}\right]$$
$$\left(b_0 + (\Delta/h)b_1\right]\cos(n_1\xi + \varphi) - 6\Psi\tau\mathrm{T}^3\Big) \tag{16}$$

Introducing the dimensionless optical field as,

$$a = \frac{4\pi NeKE_0 L_u}{2\gamma^2 m_0 c^2}\left[n_1 + \frac{3\Psi\tau^2 L_u^2}{\Omega_u c^2}\right][b_0 + (\Delta/h)b_1] \tag{17}$$

The pendulum equation is,

$$\frac{d^2\xi_s}{d\tau^2} = |a|\cos(\xi_s + \varphi) - 6\Psi\tau\mathrm{T}^3 \tag{18}$$

where, we have substituted $n_1\xi = \xi_s$.The wave equation for the vector potential $\vec{A}$ is written as

$$[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}]\vec{A} = -\frac{4\pi}{c}\vec{J} \tag{19}$$

where, the vector potential is calculated from $E = -\frac{1}{c}\frac{\partial A}{\partial t}$

Solving Eq. (19) for the transverse current density $\vec{J} = -nec\vec{\beta}$ where β is taken from Eq. (3) we get the free electron laser gain, for

$\Psi << 1$

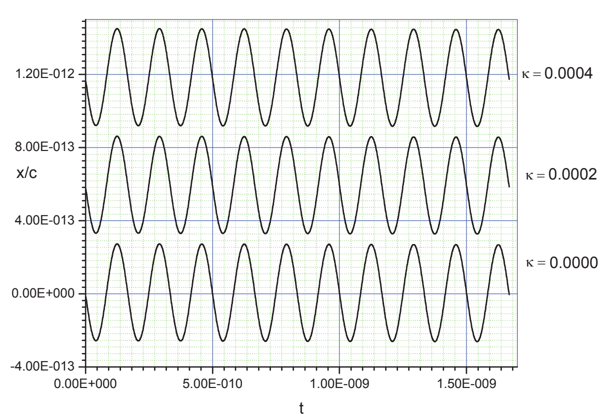
$$G = -\frac{j}{\nu_0^3}[2 - 2\cos(\nu_0) - \nu_0\sin(\nu_0)] \tag{20}$$

where,

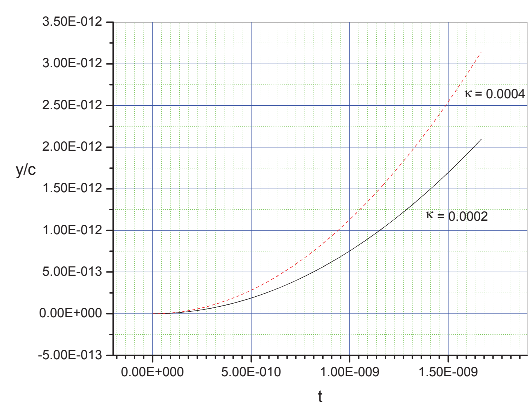
$$j = \frac{2\pi^2 Ne^2 K^2 L_u}{\gamma^3 m_0 c^2} n_1 [b_0 + \Delta_1 b_1 + \Delta_2 b_2]^2 .$$

RESULT AND DISCUSSION

The undulator field analyzed in the paper can be realized as the effect of horizontal components of earth's magnetic field on the harmonic undulator radiation. For beam and undulator parameters, $\gamma = 100, K = 1, \lambda_u = 5cm, h = 3$, we have find the effect on intensity of fundamental frequency and third harmonic frequency with the variation of κ. Figure 1 (a, b) shows the trajectory along 'x' and 'y' direction, demonstrates the shifting of beam from mean position (x=0; y=0) with κ . The emittance effect due non alignment of electron beam predicts the degradation of intensity of beam and modification in the spectrum.



(a)



(b)

Figure 1: Trajectory along 'x' and 'y' directions.

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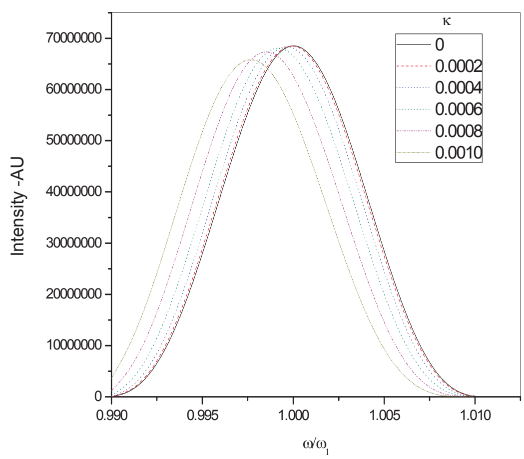
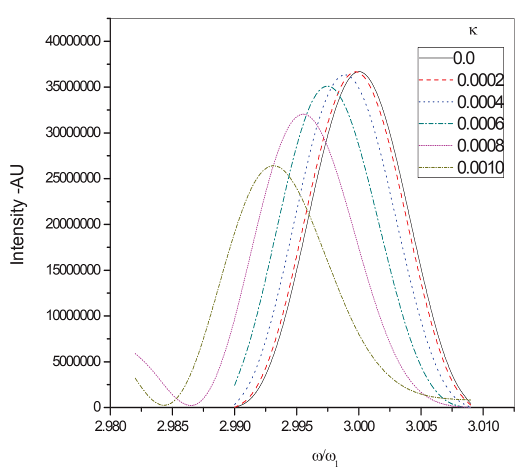
Figure 2 illustrates the case of undulator radiation for the case of $m=1,3,$ harmonic. For single peak beam energy distribution, the reduction of the intensity spectrum broadening is displayed. Figure 2 (a) and (b) reflects the intensity reduction for the fundamental and third harmonics respectively. The intensity reduction is proportional to the square of the harmonic number. So the reduction in intensity is substantial for harmonics $m=3$. However with harmonic field amplitudes, the intensity at $m=3$ the intensity loss is compensated.



(a)



(b)

Figure 2 : Intensity at fundamental and third harmonic.

The shift in resonance and reduction in intensity at third harmonic is more as compare to first harmonics. In Fig. 2a it is shown that for κ =0.0010 the resonance shift is 0.0025 and the intensity reduction is nearly 5 % at fundamental and for similar value of additional field there is shift of 0.0067 and intensity reduction of 28% .However the overall normalized shift at first and third harmonics is 0.2%. The additional field in the directional along the 'y ' direction shows similar results but at a very low magnitude of additional constant field [9].

In conclusion we have reported the intensity distribution at fundamental and third harmonics of harmonic undulator radiations of harmonic undulator field associated with very low magnitude constant magnetic field.

The constant magnetic field present in undulator due to horizontal component cause shifting of resonance frequency at fundamental and third harmonics and loss of intensity at resonance frequency. The shift in the resonant frequency is very low almost 0.1 % matters in very low gain band width system otherwise, it has adjusted for optimum output in high gain band width amplifier systems but the intensity degradation is the issue of concern as it is higher at third harmonic as compare to fundamental .

We have compared this effect at fundamental and third harmonics. The additional harmonic field with addition of shims in the planar undulator structure is helpful to overcome the loss in the intensity further effect the overall gain.

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