DISPERSION OF CORRELATED ENERGY SPREAD ELECTRON BEAMS IN THE FREE ELECTRON LASER

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Abstract

The effect of a correlated linear energy chirp in the electron beam in the FEL, and how to compensate for its effects by using an appropriate linear taper of the undulator magnetic field have previously been investigated considering relatively small chirps. In the following, it is shown that larger linear energy chirps, such as those found in beams produced by laser-plasma accelerators, exhibit dispersive effects in the undulator, and require a non-linear taper on the undulator field to properly optimise.

INTRODUCTION

In the FEL, it is well known that an energy spread correlated with the temporal bunch coordinate, or an energy chirp, in the electron beam can be compensated for by using an appropriate taper of the undulator magnetic field [1]. For the case of a linear energy chirp, it was previously derived that a linear taper is necessary, with gradient proportional to the gradient of the chirp, and this result was derived considering small variations in energy due to the chirp.

However, with the increased interest in novel accelerator concepts as FEL drivers, *e.g.* use of plasma accelerators [2–4] or the synthesis of broadband beams from linacs as in [5], the case of larger chirps has become more relevant. In this regime, dispersive effects can no longer be ignored, and the beam current and energy spread are a function of propagation distance through the undulator. Consequently, the gain length of the FEL is then itself a function of distance. In addition, dispersion due to the chirp will cause the gradient of the chirp to vary upon propagation, meaning that the taper necessary to compensate the chirp is also a function of undulator propagation length, and will not be linear.

FEL codes which employ ‘slices’ with periodic boundaries to model the electron beam [6–9] cannot model this dispersion properly, as the electrons cannot travel between slices, and so cannot model any current redistribution through the undulator. In addition, the Slowly Varying Envelope Approximation (SVEA) [10] means that they cannot model a broadband range of frequencies produced by large energy differences due to the chirp and/or a large taper. So-called ‘unaveraged’ FEL codes [11–15] are free of these limitations.

In the following, a general case of a large chirp which can be fully compensated with a taper is identified, which reduces to the previous, well known case only when dispersive effects are neglected. This simple case allows an analytic prediction for the variation in the gain length at a fixed frequency, which is compared to results from the unaveraged FEL code Puffin [11].

REVISITING THEORY IN SCALED NOTATION

Using the scaled notation of [11], the propagation distance through the undulator is scaled to the 1D gain length, and the temporal coordinate in the stationary radiation frame is scaled to the 1D cooperation length, so that, respectively,

$$\bar{z} = \frac{z}{L_g} \tag{1}$$

$$\bar{z}_2 = \frac{ct - z}{L_c}. \tag{2}$$

The scaled axial velocity of the jth electron is defined as

$$p_{2j} = \frac{d\bar{z}_{2j}}{d\bar{z}} = \frac{\beta_{zr}}{1-\beta_{zr}} \frac{1-\beta_{zj}}{\beta_{zj}}, \tag{3}$$

where $\beta_{zj} = v_{zj}/c$ is the z velocity in the undulator normalised to the speed of light. The subscript r denotes some reference velocity, which is usually sensible to take as the mean velocity of the beam, but which in general may be any velocity, as the model presented in [11] allows a broadband description of both the radiation field and the electron energies. The ‘r’ denotes the resonant condition for this reference velocity, so that the reference resonant frequency is denoted by

$$k_r = \frac{\beta_{zr}}{1-\beta_{zr}} k_w, \tag{4}$$

and the electrons with $p_{2j} = 1$ are resonant with the reference frequency.

Tapering is achieved by varying $\alpha(\bar{z}) = \bar{a}_w(\bar{z})/\bar{a}_{w0}$, which is the relative change in the magnetic undulator field from its initial value, as defined in [16].

The gradient of an electron beam chirp may then be defined as

$$\frac{dp_2}{d\bar{z}_2} \approx -\frac{2}{\gamma_r}\frac{d\gamma}{d\bar{z}_2}, \tag{5}$$

assuming small deviations in energy, a small chirp so that

$$\frac{dp_2}{d\bar{z}_2} \ll 1, \tag{6}$$

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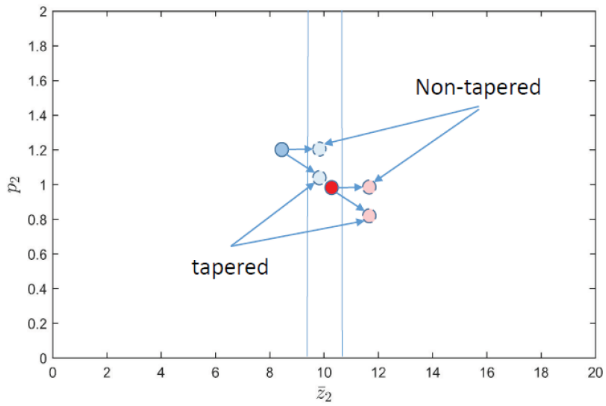

Figure 1: Showing the manipulation of p_2 by variation of the undulator magnetic field α. By altering the magnetic field, one may guide the blue electron to the correct value of p_2 to be resonant with the radiation in the blue slice indicated.

and small deviations in the undulator magnetic field, $\alpha \approx 1$.

Rewriting the formula for the taper required to compensate the detuning effect [1] from a chirp in the above notation, we obtain

$$\frac{d\alpha}{d\bar{z}} = -\frac{1+\bar{a}_{w0}^2}{\bar{a}_{w0}^2}\frac{1}{\gamma_r}\frac{d\gamma}{d\bar{z}_2}. \tag{7}$$

DISPERSIVE AND BROADBAND EFFECTS

To take into account dispersive effects, it is convenient to describe the system using the p_2 phase space. p_{2j} is the scaled velocity of the jth electron, and so describes, linearly, how the beam will disperse. It also linearly measures the resonant frequency of the electron; from Eq. (3)

$$p_{2j} = \frac{k_r}{k_j}, \tag{8}$$

so it is the inverse of the frequency scaled to the reference frequency.

Relaxing the constraint on the energies - once again allowing large energy changes - then Eq. (7) is no longer correct. In the 1D limit, and using a helical wiggler, from Eq. (3), p_{2j} may be defined as a function of α and γ as

$$p_{2j}(\bar{z}) = \frac{\gamma_r^2}{\gamma_j^2}\left(\frac{1+\alpha(\bar{z})^2\bar{a}_{w0}^2}{1+\bar{a}_{w0}^2}\right), \tag{9}$$

under the approximation that $\gamma_j, \gamma_r \gg 1$, ignoring any transverse velocity spread (1D limit), and ignoring any interaction with the radiation field (in the planar wiggler, one obtains the equivalent expression for p_{2j} averaged over the wiggle motion).

Using this definition, Figure 1 shows the effect of tapering in the $(\bar{z}_2, p_2)$ phase space, and shows what occurs when

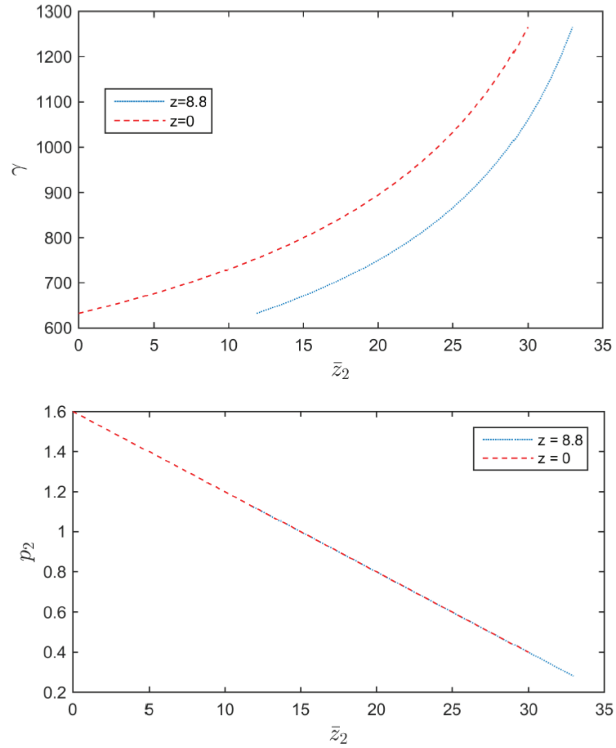

Figure 2: Top: The electron beam mean energy γ as a function of scaled temporal coordinate $\bar{z}_2$ at the start (red) and end (blue) of the undulator. Bottom: Same beam, now plotting the mean p_2 of the beam. The conversion from p_2 to γ can be obtained from Eq. (9). This is the stationary radiation frame, and the head of the beam is to the left, so the beam slips backwards through the field from left to right.

compensating for energy changes correlated in $\bar{z}_2$. The red electron, initially in the slice indicated, emits radiation at frequency k_r before slipping back to the right. Recall this is the stationary radiation frame, and the head of the pulse is to the left. The blue electron, slipping back into the thin slice, finds itself interacting with radiation it is not resonant with. By varying, or tapering, the magnetic field α, the value of p_2 of the blue electron can be manipulated, and reduced to the red electron's original value of p_2; therefore it is now resonant with the radiation in the slice originally emitted by the red electron.

Consequently, if an electron beam has an initial linear chirp in p_2, so that

$$\left.\frac{dp_2}{d\bar{z}_2}\right|_{\bar{z}=0} = m, \tag{10}$$

then the correct magnetic field taper to ensure the beam stays resonant should cause each electron to follow the line of the chirp defined by m. Figure 2 shows this. It plots the mean energy of a beam, and the corresponding mean p_2, as a function of $\bar{z}_2$, at the start ($\bar{z} = 0$) and end of an undulator tapered to compensate for the chirp. The taper may be derived from Eqs. (9) and (3), forcing $dp_{2j}/d\bar{z} = m$

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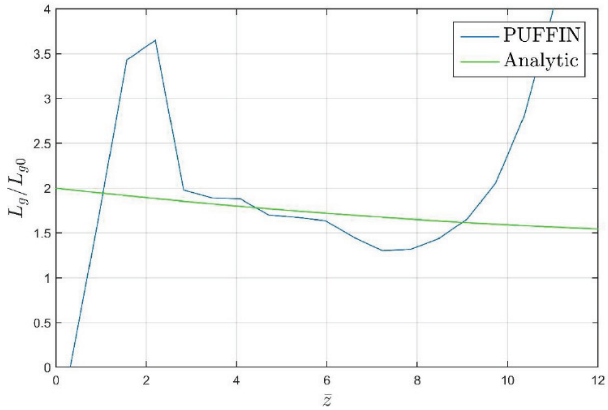

Figure 3: Variation in gain length as a function of distance through the undulator due to dispersive effects. Analytic from Eq. (19) (green) compared to numerical result from Puffin (blue).

and $d\gamma_j/d\bar{z} = 0$, and solving for α. The solution is found to be:

$$\alpha = \frac{1}{\bar{a}_{w0}}\sqrt{\exp(m\bar{z})(1+\bar{a}_{w0}^2) - 1}, \tag{11}$$

which reduces to the solution of Eq. (7) only when

$$|m\bar{z}| \ll 1 \tag{12}$$

and

$$\frac{\bar{a}_{w0}^2}{1+\bar{a}_{w0}^2} \sim 1. \tag{13}$$

For magnetic undulators, where $\bar{a}_{w0}^2 \gtrsim 1$, condition (13) is satisfied.

To measure the beam compression or decompression from this linear p_2 chirp, remembering that p_2 is the velocity of the electron in $\bar{z}_2$, then the change in the pulse width σ_{z2} is

$$\frac{d\sigma_{z2}}{d\bar{z}} = m\sigma_{z2}(\bar{z}). \tag{14}$$

From this, a stretch factor S is defined as

$$S(\bar{z}) = \frac{\sigma_{z2}(\bar{z})}{\sigma_{z20}} = \exp(m\bar{z}). \tag{15}$$

From this, it is seen that condition (12) is the limit of negligible dispersion in the undulator. This is different from the limit of a small chirp as previously identified in Eq. (7), which is simply

$$|m| \ll 1. \tag{16}$$

For a typical SASE FEL, $\bar{z} \approx 10-15$, so the dispersive condition is more restrictive by around an order of magnitude.

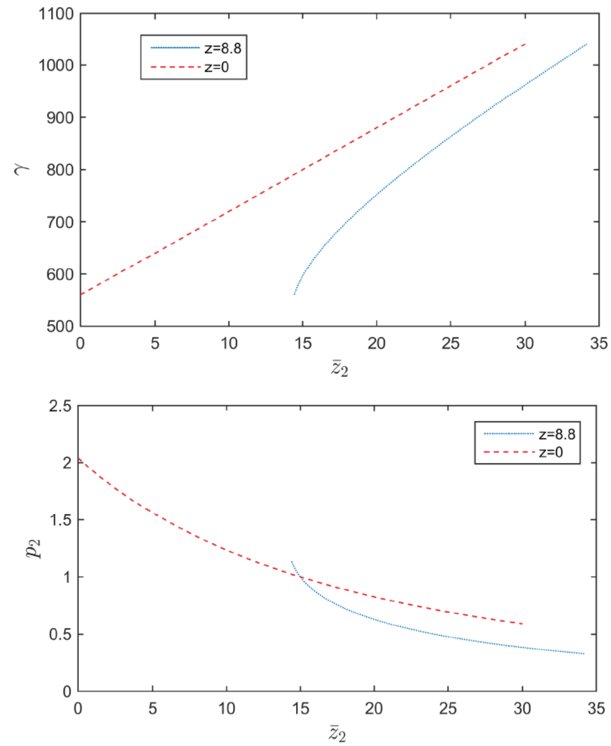

Figure 4: Same as Figure 2, now using an initially linear chirp in energy γ. The undulator is now tapered to try to keep the mean electron beam p_2 constant at $\bar{z}_2 = 15$. In this case, so that it is resonant with the reference frequency k_r, $p_2 = 1$ at $\bar{z}_2 = 15$. The undulator taper is calculated numerically and is not linear.

MEASURING THE EFFECT ON THE GAIN LENGTH

The dispersion has an effect on the '3D' gain length [17], as the compression/decompression will cause a change in the peak current and energy spread of the beam. The change in peak current can be analytically estimated very simply by

$$I(\bar{z}) = \frac{I_0(\bar{z}=0)}{S(\bar{z})}. \tag{17}$$

The dispersion will also alter the localised, or 'slice' energy spread of the beam. However, in this case, when using a linear chirp in p_2 with the taper in Eq. (11), every electron follows the line with gradient m in the $(\bar{z}_2, p_2)$ phase space (see Figure 2), so the slice p_2 spread does not change despite the compression/decompression. This will cause a corresponding variation in the transverse velocity spread, which will affect the gain length. Here, only the 1D case is considered, so the increased transverse spread has no effect.

The other consideration is that the gain length is different for each frequency; here, the frequency is linearly correlated with $\bar{z}_2$, and, because the taper is compensating perfectly, this correlation is fixed across the full undulator. Again refering to Figure 2, the mean p_2 at an instantaneous point in $\bar{z}_2$ remains constant, but the corresponding mean energy (from

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the top plot) is very different. Picking a coordinate initially in the center of the beam, $\bar{z}_{2c}$, with corresponding beam energy γ_c, which is a function of $\bar{z}$, then the normalized energy of the electron resonant with the fixed frequency is given by

$$\Gamma = \frac{\gamma_c}{\gamma_{c0}} = \left(\frac{1+\alpha^2\bar{a}_{w0}^2}{1+\bar{a}_{w0}^2}\right)^{1/2}, \qquad (18)$$

where $\gamma_{c0} = \gamma_c(\bar{z}=0)$.

From the definition of the FEL parameter, the gain length then varies as

$$L_g(\bar{z}) = \frac{S(\bar{z})^{1/3}\Gamma(\bar{z})}{\alpha(\bar{z})^{2/3}} L_{g0}, \qquad (19)$$

where L_{g0} is the gain length at $\bar{z} = 0$, and the gain length as referred to here is the M. Xie gain length, with only the energy spread parameter included.

A comparison of this analytic expression with the unaveraged FEL code Puffin is shown in Figure 3. Relevant parameters used are $\rho = 0.01$, $\bar{a}_{w0} = 2$, $\gamma_r = 800$ and $m = -0.04$, and slice spread of $\sigma_\gamma/\gamma_r = 1\%$. The gain length from Puffin is measured numerically from the radiated energy narrowly filtered around the frequency at $\bar{z}_{2c}$, and compares well with the analytic result. Note that the exponential gain region is $\bar{z} \approx 3$ to ≈ 8; before this is the startup regime where there is no gain, and after this the system is in saturation. There is good agreement in the exponential gain regime.

By using a linear chirp in *energy*, the beam compresses asymmetrically, and it is not possible the compensate for the detuning effect for all frequencies. Figure 4 plots the same quantities as Figure 2, but with a linear energy chirp, and the taper is calculated numerically to keep the reference frequency at $\bar{z}_2 = 15$ interacting with electrons resonant with it (so, in this case, keeping $p_2 = 1$). The same can be done for any frequency emitted, so it is possible to preferentially compensate for certain frequencies, but it is not possible to properly compensate for all frequencies.

However, this does not necessarily result in a higher power at that frequency. Other factors, such as the energy and slice spread, change differently for each frequency. Only the detuning effect is being compensated for; the other quantities (*e.g.* current), varying asymmetrically across the bunch, may result in less or more gain at other frequencies when all effects are accounted for.

Consequently, there is a large range of tapers which can be considered ‘optimum’. But the detuning effect can only be completely removed across the whole bunch when the beam has a linear chirp in p_2, and using the taper described in Eq. (11). In that case, the effect on the gain length can be easily predicted.

Note that, in the above, only 1D effects have been taken into account. There is no examination of the change in diffraction parameter, beam divergence parameter *etc.* (from [17]) occuring as a result of the dispersion. Preliminary work suggests that when these effects are included the impact on the gain can be more severe.

CONCLUSION

We have shown that the beam dispersion in the undulator is an important effect, and the constraint on when it appears is actually an order of magintude tighter than the condition of a ‘small’ chirp. A simple model was presented to take into account the dispersion, which allows an analytic solution for a matched taper to eliminate the detuning effect, and allows one to isolate the effects of the dispersion and measure them. It is shown that the unaveraged code Puffin agrees with this result.

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