INCLUSION OF ADVANCED FIELDS AND BOUNDARY CONDITIONS IN THE ANALYTIC THEORY FOR HIGH GAIN FELs

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Abstract

The efforts in realizing x-ray free electron lasers (FELs) and enhancing their performance has stimulated remarkable theoretical developments and experimental advances in the field. Yet, the successful operation of x-ray FELs based on the self-amplified spontaneous emission (SASE) principle which has made them a powerful new tool, has beckoned our attention for better understanding a comprehensive physical basis of the theory that has the potential to improve the temporal structure and spectral optimization of these sources. We have previously explained the advantages of including the coherent radiation reaction force as a part of the solution to the boundary value problem for FELs that radiate into "free space" (SASE FELs) and discussed how the advanced field of the absorber can interact with the radiating particles at the time of emission. Here we present the outline of our theoretical approach which follows from eigenmode analysis of optical guiding in FELs. We will also discuss in some detail the experimental setup that could verify and/or further our understanding of the the underlying physics of these devices.

INTRODUCTION

When formulated in the language of covariant action-at-a-distance, the solution to the boundary value problem corresponding to an oscillating particle within a spherical absorbing shell of arbitrary density is dominated by the interference of the retarded and advanced forces originating in the accelerated and absorbing particles [1]. This leads to a force on the accelerated particle exactly equal to that needed to match the power carried by radiation to the particles in the absorbing shell. Therefore a time-symmetric definition of electrodynamics provides non-diverging solution and origin for the radiation reaction field and satisfies Maxwell's energy integral [2]. In fact, it has also been shown that the action-at-a-distance formulation is not essential and the assumption of time-symmetry suffices for the conservation of energy [3]. The reliance of theory of SASE FELs on classical electrodynamics and their operational dependency on coherent radiation at femtosecond scale provides an excellent opportunity for the test and further study of the time-symmetric approach to electrodynamics.

DISCUSSION OF THE THEORY AND CONCEPT

Description of FEL interactions by Kimel and Elias [4] includes a viable model of the coherent radiation reaction in covariant form valid for radiation into free-space. In time-symmetric electrodynamics this can be introduced by taking in to account both the advanced and retarded field of the source and absorbing (non-emitting) particles. Applying that principle to the beam traveling in a SASE FEL, we start by considering both the advanced and retarded field/potential of both the absorber and the electrons (the emitter). It is important to note that in the absence of reflector/refractor/target in front of the electron beam traveling in z, the absorbers are the cosmological particles; and when including the effect of an absorber that far, the field and forces being considered approach the retarded field of experience). Now we must include half the retarded (outgoing) field of the emitters and the half the advanced (incoming) field of the target. The interaction of the advanced field of the target with the radiating electrons ensures energy conservation on the one hand, and on the other hand imposes the fields and forces initiated from the target on the source.

Non-reflecting Boundary Condition

The target mentioned above introduces a non-reflecting boundary condition to the system of the equations that must be solved. Here we refrain from calling the target an absorber to avoid confusion, since the role of the target is not absorbing the emitted photons but to be the origin of the advanced fields acting on the beam. For such signals carried on electromagnetic waves (advanced or retarded) the invariant interval $(cdt)^2 - dr^2$ between the emission of a wave and it's absorption at the non-reflecting boundary is always identically zero. So by that measure, which is the covariant statement of the distance in space-time separating transmitter and receivers, the emission and absorption of the retarded and advanced waves are all simultaneous. This has been illustrated in Figure 1. Note that the advanced wave of the non-reflecting boundary (mirror) co-propagates with the fields of the undulator acting on the electrons. (The characteristics of the mirror (partially reflecting) and why it was chosen for our experiment will be explained in the next section.)

Advanced Field and Evolution of Coherent Bunching in SASE

The SASE FEL starts from a randomly phased electron beam. After a few undulator periods the randomly phased electron beam gets bunched. The coherent radiation emitted by tightly bunched electron beams plays a critical role in the analysis and operation of free electron lasers. For conducting or reflecting (resonator mirror) FELs, a normal mode analysis of operation already includes the relevant boundary conditions. However, in order to arrive at a comprehensive first-principles field-based analysis of the intense radiation emitted into free-space by devices that work based on the

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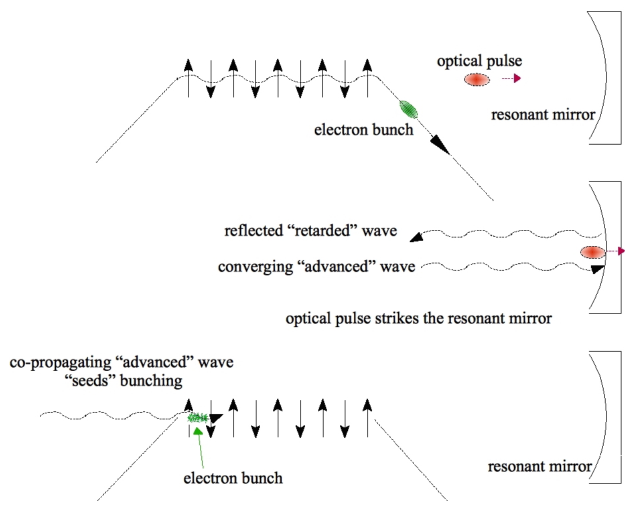

Figure 1: By the covariant statement of the distance in space-time separating transmitter and receivers, the emission and absorption of the retarded and advanced waves are all simultaneous. Presence of the mirror in front electron beam will introduce a co-propagation seed that can carry information that will improve the temporal coherence and the quality of the beam passing through the undulator.

SASE principle, including all target interactions, we also need to include the effect of boundary condition of the target.

EXPERIMENTAL TEST AND APPLICATION

The discussion of time-symmetric electrodynamics goes back to conversations between Wheeler, Feynman and Einstein. Since coherent emission is central to the operation of SASE FELs, not only could SASE FELs be instrumental in the study of this fundamental aspect of electrodynamics, their technology could potentially benefit from this previously un-utilized aspect of fundamental physics. The proposed setup provides just such opportunity.

Experimental Goal

Since SASE FELs operate based on the bunching induced by the co-propagating electrodynamic wave in these devices' undulator, they present the perfect set up for an experimental test of the model presented here. The goal of this experiment would be to evaluate the role of time-symmetric electrodynamics in operation of SASE FELs. We propose use of a multilayer bent-quartz crystal mirror at the end of the undulator to create the advanced field needed to alter the electron's bunching. This mirror will be curved (for the purpose of focusing) and be located in front of the bea and on it's transverse path right after the undulator. We ask the question: will the mirror's spontaneous response to the presence of the radiating beam (predicted by the time-symmetric electrodynamics) be able to carry phase information that will improve the temporal coherence and the quality of the beam passing through the undulator? The understanding gained from this experiment will be very important for both our current theoretical understanding and expanding the future applications of SASE FELS.

Set up and Required Parameters

Since the advanced co-propagating wave of the mirror will be acting as a seed laser the same condition that applies to the seeded FEL applies here: the seed power only has to exceed the spontaneous noise emitted into the coherent bandwidth and angle [5]. The wavelength of the radiation must match the spacing of the quartz lattice in one direction. Also the mirror must be located right after the undulator or beam dump to have access to a natural (unfiltered) SASE bandwidth. An optimal set of parameters for the experiment is presented in Table 1.

Table 1: Optimal Parametes for the Experimental Test

Radiation Beam	
Beam Wavelength	1 Angstrom
Bandwidth	Natural SASE BW
Pulse energy	1 mJ or less
Pulse Duration	70 fs
Spot size	micron range
Rep. Rate	1 Hz or Less

Bent Quartz Crystal Mirror

The bent quartz crystal mirror has two purposes. One is that only wavelengths that satisfy the Bragg condition pass through the mirror [6]. Second, the mirror will satisfy the non-reflecting boundary condition. Therefore if the radiation wavelength is tuned to the spacing of the lattice of the quartz we can expect that wavelength radiation to transfer phase information to the beam. Of course once the wavelength is shifted it will no longer be attenuated by the mirror and pass through. Significant improvement in the spectral coherence of the beam can be expected since the focusing (bent) multilayer quartz crystal (partially reflecting mirror) will simultaneously serve as a retroreflector and a filter. It is important to note that similar experiments have been conducted in the infrared regime with a dramatic improvement in the temporal coherence and spectral brightness of the emitted FEL radiation [7, 8]. Although the geometry of those experiments differ from what is proposed here, the physics is the same.

ANALYTICAL SOLUTION

An important question here is whether any of the currently used FEL theory or simulations can predict the effect of the advanced field on the FEL performance. In our study of simulations, we found that since we are looking for a previously un-utilized aspect of physics the result of the simulations could be unreliable. Therefore we looked for a method that gave us better physical insight.

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Review of Xie's Theory of Optical Guiding

In this section Equations 1-6 are from Reference [9] and demonstrate how the interaction of the field of the electron and FEL is studied by eigenmode analysis of optical guiding in FELs. We take the same approach and solve for the coefficient of expansion when the initial condition includes the advanced field. In Xie's method the paraxial equation for FEL is reduced to a compact form: a Schödinger equation with a non-Hermitian Hamiltonian,

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -L_1 & 1 \\ L_2 & 0 & 0 \end{bmatrix}, \tag{1}$$

where L_1 corresponds to the transverse Laplacian and detuning term and L_2 corresponds to the transverse density modulation of the paraxial wave equation for the FEL. The vector solution of this non-Hermitian Hamiltonian is

$$\psi = \begin{bmatrix} -i\frac{\partial \tilde{I}}{\partial \tau} \\ \tilde{a} \\ -L_2 \tilde{I} \end{bmatrix}. \tag{2}$$

Then the following initial condition is assumed

$$\psi(0) = \begin{bmatrix} 0 \\ \tilde{a}(0) \\ 0 \end{bmatrix}, \tag{3}$$

and the eigenmodes and eigenvectors are determined as:

$$\psi_n = V_n \exp(-i\lambda_n \tau)\,, V_n = \begin{bmatrix} \frac{1}{\lambda_n} \\ 1 \\ \frac{P_u(r_\perp)}{\lambda_n^2} \end{bmatrix}. \tag{4}$$

Then using the initial condition Equation 3 the expansion coefficient for the general solution and components of the field is solved.

$$\tilde{a} = \Sigma C_n V_n \exp(-i\lambda_n \tau) + \int C_\lambda g_\lambda \exp(-i\lambda_n \tau) d\lambda\,, \tag{5}$$

where the expansion coefficients for a discrete mode is

$$C_n = \frac{1}{N_n} < \tilde{a}(0) g_n >, \tag{6}$$

and the power coupling coefficient is

$$G_0 = \frac{|C_n|^2 < |g_n|^2 >}{< |\tilde{a}(0)|^2 >}. \tag{7}$$

Solution for an Initial Condition that Takes the Advanced Field into Account

As Xie states in [9] the three components of Equation 3 are, in order, the energy modulation of the electron beam, the amplitude of the radiation field and the density modulation of the electron beam. This complete picture allows for the direct inclusion of advance field. Due to the instantaneous propagation of the advanced field, initially the electrons in the beam would not have had the chance to see a density modulation. However, the advanced field could require energy modulation in the electron bunch. Therefore Equation 3 can be re-written for the case of including advanced field as:

$$\psi(0)_{adv} = \begin{bmatrix} \alpha(\nu,\tau)\tilde{a}(\tau_f) \\ \tilde{a}(0) + \tilde{a}(\tau_f) \\ 0 \end{bmatrix}. \tag{8}$$

Here τ_f is the focal distance of the quartz mirror and $\alpha(\nu, \tau)$ is a function relating the first two component of ψ_{adv}. For the simplest case, when the energy modulation is still small, expansion coefficients for the discrete mode now include a term that was contributed by the advanced field as shown here:

$$C_{n(adv)} = \frac{1}{N_n} < (\tilde{a}(0) + \tilde{a}(\tau_f)) g_n >, \tag{9}$$

therefore the power coupling coefficient will be stronger in this case. From this development we would expect that the initial condition would excite the lowest order optical mode which would couple with the electrons' field. Furthermore properties of the electron beam attributed to the higher mode will not appear in the advance field (due to the properties of the mirror) so they will not be amplified. A more detailed version of this calculation is presented elsewhere [10]. Initial result of a numerical study is shown in Figure 2 and Figure 3.

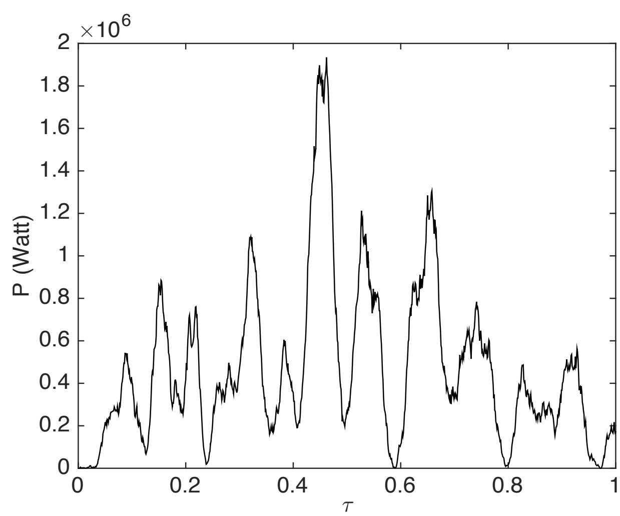

Figure 2: Typical SASE temporal profile with LCLS like parameters at 5 gain length without a non-reflecting boundary.

CONCLUSION

Here we discussed the importance and advantages of a comprehensive approach to electrodynamics for advance light sources like SASE FELs which is based on time-symmetric electrodynamics. A detailed picture of an experiment that would further our understanding in this area

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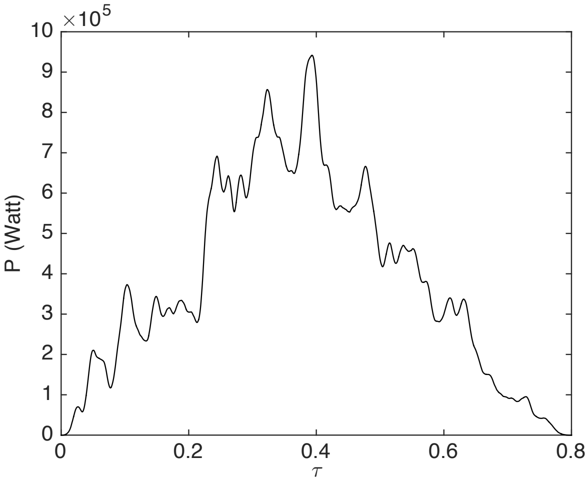

Figure 3: Typical SASE temporal profile with LCLS like parameters at 5 gain length with a non-reflecting boundary and including the advanced field.

was presented. Finally we took an analytic approach in order to determine exactly how inclusion of the advanced field in the analytic theory for high gain will change the solutions. We were able to demonstrate that an initial condition which takes the advanced field into account changes the expansion coefficients of a discrete mode. A numerical code in Matlab is being prepared which will be used for a more detailed study.

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