SIMULATION STUDY OF TRANSVERSE COHERENT INSTABILITIES IN INTENSE BUNCHES WITH SPACE CHARGE AND IMAGE CURRENTS

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Abstract

The head-tail instability is a well known intensity limitation for hadron bunches in synchrotrons. The instability has been observed in several synchrotrons and storage rings. Also for the FAIR synchrotrons the head-tail instability represents a potential intensity limitation. In the SIS-18 and SIS-100 synchrotrons space charge effects together with image currents play an important role for the determination of the instability threshold. In this work we study head-tail modes using 3D particle simulations for SIS-100 beam parameters. The unstable modes are driven by the resistive wall impedance. The possibility to include space-charge and image currents into long-term simulations, which are necessary for head-tail instability studies, is investigated using the HEADTAIL code and the PATRIC code.

INTRODUCTION

The beam intensity and the beam quality in the FAIR synchrotrons [1] can be limited by transverse collective instabilities. The head-tail instability is a potential danger for bunched beams in the future SIS-100, especially during the ∼1 s accumulation phase at the injection energy of 200 MeV/u.

Using the classical approach by Sacherer [2], the complex frequency shift of a head-tail mode induced by an external impedance can by described using

$$\Delta\Omega_m = \frac{\omega_0}{Y_0}\frac{1}{1+m}\frac{C}{L_\mathrm{b}}\frac{\sum_p (-i)Z^{\perp}(\omega_p)\, h_m(\omega_p-\omega_\xi)}{\sum_p h_m(\omega_p-\omega_\xi)}\ , \quad (1)$$

where the transverse impedance is probed at frequencies

$$\omega_p = (p+Q_0)\omega_0 + m\omega_\mathrm{s} \quad (2)$$

with the oscillation spectrum h_m, and the chromatic frequency $\omega_\xi = \omega_{\beta 0}\,\xi/\eta$. The resistive-wall impedance is the main instability source for low-m modes in SIS-100. The power spectra of the head-tail modes h_m extends over frequencies where the skin depth is larger and smaller than the pipe thickness, thus both resistive-wall impedance regimes (thick-wall and thin-wall) should be taken into account. Table 1 summarizes the relevant machine- and beam parameters for the reference U^{28+} operation in SIS-100. The Sacherer formula predicts as the most dangerous mode $m=4$ with the growth time $\tau \approx 70\,\mathrm{ms}$.

However, the special beam conditions in the SIS synchrotrons can change these predictions, especially the following three factors which are not included in the Sacherer approach:

Table 1: Basic parameters of the SIS-100 synchrotron and nominal parameters for the reference beam at the injection energy

$C = 1083\,\mathrm{m}$	γ_t = 23.14
Q_vert = 18.73	Q_s = 0.0017
U^{28+} ions	$N_\mathrm{b} = 0.625\times10^{11}$
$\mathcal{E}_\mathrm{beam} = 200\,\mathrm{MeV/u}$	$\delta p/p = 3.0\times10^{-4}$
$\beta = 0.566$	$\gamma = 1.21$
$f_0 = 156\,\mathrm{kHz}$	$\eta = -0.67$
$\epsilon_\mathrm{v} = 13\,\mathrm{mm\,mrad}$	$\epsilon_\mathrm{h} = 36\,\mathrm{mm\,mrad}$

1. Direct (self-field) space charge, with a tune shift $\Delta Q_\mathrm{SC} = -0.25$. In contrast to a coasting beam, this tune shift inevitably provides a tune spread along the bunch, since in the middle (longitudinally) of the bunch the tune shift reaches the maximum ΔQ_SC, but at the ends it is zero. To the authors' knowledge, no comprehensive theory has been proposed to describe this effect on the weak head-tail instability, especially for $\Delta Q_\mathrm{SC} \gg Q_\mathrm{s}$.
2. Image charges from the conducting wall, with the tune shift $\sim 1/5$ of ΔQ_SC. Note that, in contrast to space charge, image charges shift the coherent frequency at thus their effect is strongly different. Image charges also cause a coherent tune spread along the bunch. There is lack of analytical models for image charges, too; again our beam conditions correspond to $\Delta Q_\mathrm{coh} \gg Q_\mathrm{s}$.
3. During the accumulation phase the bunches will be relatively long in SIS-100, $L_\mathrm{b} = 61\,\mathrm{m}$, which means a phase half-length of $\approx 100°$. This causes a spread in the synchrotron frequency due to its dependency on the synchrotron amplitude. No adequate model has been proposed to describe the detailed effect of the synchrotron frequency spread on head-tail modes, although, some authors suggest that nonlinear rf can stabilize head-tail modes for certain chromaticities [3].

In this work, we present first attempts to study these effects using particle tracking simulations. Realistic studies represent a numerical challenge, since complete 3D nonlinear simulations with a long computing time are necessary. The latter is due to the fact that the growth time of the weak head-tail instability cannot be arbitrarily scaled down, since at the certain impedance or particle number the strong head-tail (or TMCI) is excited. In order to deal with this challenge, we use two different numerical tools. The first one, the PATRIC code [4, 5], is optimized for relatively short-term simulations with well-resolved betatron oscillations and self-consistent space charge. The second tool is the HEADTAIL code [6, 7], which is developed for one-turn maps and long-term simulations of short bunches with a simplified space charge model.

In both codes the following wake-field module for the resistive wall has been implemented. The momentum kick for a beam slice due to a wake field can be represented as an integral over the slices in front,

$$\Delta p_x = \int_0^{L_{\mathrm{ahead}}} \Psi(s)\, W_{\mathrm{RW}}(z) \mathrm{d}z \,, \tag{3}$$

where $z = s_1 - s_2$ is the distance between slices and $\Psi = k\, n_{\mathrm{p}} \overline{x}$ is the normalized dipole moment; n_{p} is the slice particle number and $\overline{x}$ is the slice transverse offset. Consider now a wake-field kick at an interaction point, see Fig. 1 for a scheme. The slice '1' leaves behind the wake

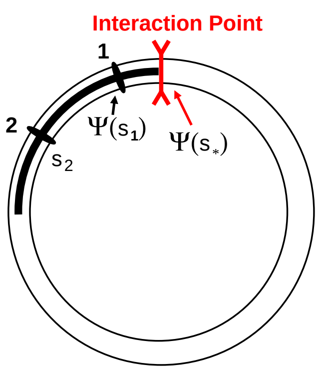

Figure 1: Illustration scheme for the principle to compute the transverse momentum kick due to a wake field.

which kicks the slice '2', but the dipole moment of the first slice does not correspond to its instantaneous value at the position s_1. Instead, the dipole moment at the interaction point s_* should be considered,

$$\Delta p_{12} \;=\; \Psi(s_*)\, W_{\mathrm{RW}}(z) \,, \tag{4}$$

and thus the phase advance for each slice should be taken into account.

In the present simulations, the resistive-wall wake-field (thick-wall regime) is used,

$$W_{\mathrm{RW}}(z) \;=\; -\frac{c L_{\mathrm{wall}}}{b^3}\left(\frac{\beta}{\pi}\right)^{3/2}\sqrt{\frac{Z_0}{z\,\sigma_{\mathrm{wall}}}} \,. \tag{5}$$

RESULTS

First attempts were made to simulate head-tail modes with image charges, space charge, or nonlinear rf taken into account. Here, preliminary results of the simulations are presented.

First, we consider the effect of image charges from a conducting wall, which cause the coherent tune shift ΔQ_{coh}. Figure 1 presents results of HEADTAIL simulations for SIS-100 beam parameters (with a scaled resistive wall length in simulations). We observe a stabilizing ef-

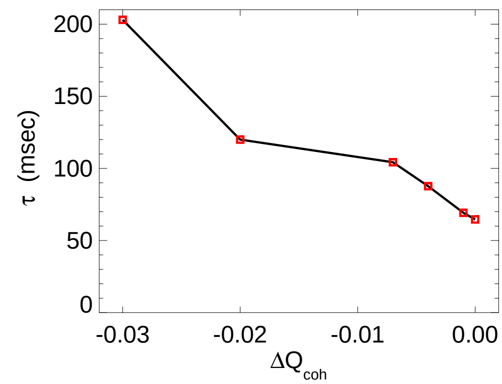

Figure 2: Growth time of the most unstable head-tail mode in dependence on the coherent tune shift due to image charges.

fect of image charges, the growth time increases for larger ΔQ_{coh}, which is given for the bunch middle. Here the smallest values of ΔQ_{coh} are comparable with the synchrotron tune $Q_{\mathrm{s}} = 0.0017$, but for the rest $\Delta Q_{\mathrm{coh}} \gg Q_{\mathrm{s}}$ applies.

It is often useful to take a look at the coherent spectrum of instabilities, which is presented in Fig. 2. In these

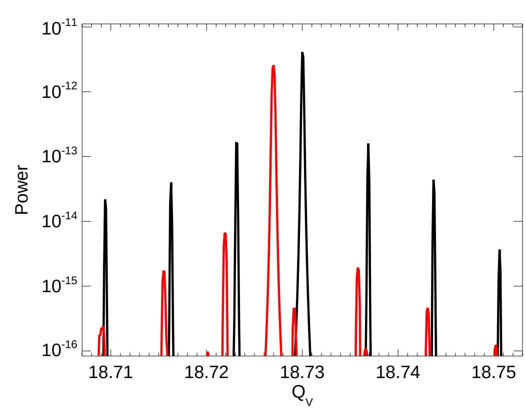

Figure 3: Spectrum of coherent bunch oscillations with (the red line) and without (the black line) image charges.

simulations we have assumed a larger synchrotron tune $Q_{\mathrm{s}} = 0.0068$ and $\Delta Q_{\mathrm{coh}} = -0.004$. In Fig. 2 the spectrum without image charges including the the synchrotron satellites (black curve), and the simulation result with image charges (red curve) are plotted. Consistently with the e.g. model in Ref. [8], the central line $m = 0$ is shifted the strongest. However, as an important disagreement with the discussion in [8], no strong head-tail modes (TMCI) appear at $\Delta Q_{\rm coh} \sim Q_{\rm s}$ in our simulations.

As a first attempt to include direct space charge, the beam-beam model was used in simulations with the code HEADTAIL, and the 'frozen electric field' model [5] in simulations with the code PATRIC. However, significant discrepancies have been observed between two codes, with the PATRIC results suggesting a damping effect and HEADTAIL results showing a much weaker effect. Figure 4 presents the coherent spectrum from a HEADTAIL simulation, which indicates broader lines, but without coherent shifts. These results should be further clarified and

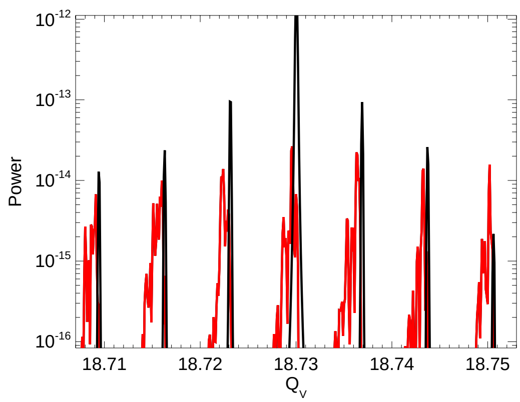

Figure 4: Spectrum of the coherent bunch oscillations with (the red line) and without (the black line) space charge.

the effect of space charge on head-tail instabilities should be further studied.

Finally, an attempt was made to include the effect of nonlinearity in synchrotron oscillations, but a discrepancy was obtained between results of two codes. Further studies are in progress, here we present (Fig. 5) a coherent spectrum from a PATRIC simulation for linear/nonlinear rf.

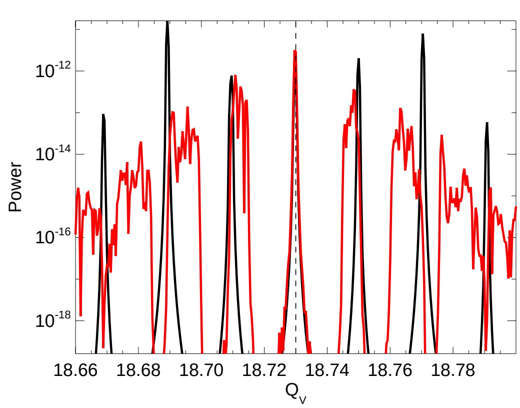

Figure 5: Spectrum of the coherent bunch oscillations with linear (the black line) and nonlinear (the red line) synchrotron oscillations.

In conclusion, first attempts to include direct space charge, image charges and nonlinear rf into long-term simulations of head-tail instabilities have been made. These effects may strongly change the growth times of the head-tail modes in SIS synchrotrons. Two different particle tracking codes, HEADTAIL and PATRIC, are used to ensure a correct treatment of this numerical challenge. Image charges seem to have a stabilizing effect on bunches, without a build-up of TMCI at $\Delta Q_{\rm coh} \sim Q_{\rm s}$. This should be clarified by further simulations and comparisons with mode-coupling theories. First simulations with direct space-charge (in both codes the corresponding model is non-self-consistent) have shown unexpected discrepancies between the codes. Also the role of nonlinearity in synchrotron oscillations could not be understood right away, further simulations and comparisons are in progress. In is worth to stress that an adequate dispersion relation, which correctly describes these effects, would greatly support our studies but, to the authors' knowledge, has not been suggested so far.

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