SIMULATION STUDIES OF HALO CREATION AND REGENERATION IN INTENSE CHARGED PARTICLE BEAMS

C. Papadopoulos *, I. Haber, R. A. Kishek, P. G. O'Shea, M. Reiser
University of Maryland, College Park MD, 20742

Abstract

We discuss the parametric resonance model of halo creation, from the viewpoint of the particle-core model and from self consistent particle-in-cell (PIC) simulations. Using a simple particle tracking algorithm, we distinguish the halo particles in phase space and we briefly discuss the issue of halo removal and regeneration in simulations.

INTRODUCTION

One of the main factors that need to be addressed for the reliable operation of high intensity, high brightness accelerators is beam halo. Although there does not exist a universally accepted, rigorous definition for beam halo, from a practical point of view particles are labeled as halo when they venture far from the beam center, compared to the rms beam radius. These particles can scrape the pipe wall, causing uncontrolled beam loss, as well as activation of the facility, at high energies, and electron cloud effects due to secondary electron emission, for hadron machines.

A proposed mechanism for halo formation in the case of mismatched beams, is a parametric resonance between envelope, or core, oscillations and single particles. This mechanism was initially studied by Gluckstern [1] in the smooth focusing approximation and explored further by Wangler et al [2] and Ikegami et al [3]. In this paper, we will briefly describe the particle-core model used by these authors and then move to comparisons with more accurate, particle-in-cell simulations.

PARTICLE-CORE MODEL

In the following, we will work within the smooth focusing approximation, in which the transport channel is modeled as an infinitely long, circular pipe and the beam is an infinitely long cylinder, confined by a constant gradient radial electric field for the whole length of the channel. The beam envelope equation in this case is given by Eq. 1, if the beam is assumed to have a Kapchinksi-Vladimirski distribution function [4].

$$\frac{\mathrm{d}^2R}{\mathrm{d}z^2} + k_0^2R - \frac{K}{R} - \frac{\epsilon^2}{R^3} = 0 \qquad (1)$$

where $R = \sqrt{< r^2 >}$ is the rms beam radius, k_0 is the betatron wavelength associated with the constant focusing electric field, $K = qI/2\pi\epsilon_0 m(\beta\gamma c)^3$ is the generalized perveance and ϵ is the beam emittance. The envelope equation 1 can admit oscillatory solutions, if the initial beam size R is not equal to the "matched" beam radius R_0, which can be calculated from the algebraic equation $k_0^2R_0 - K/R_0 - \epsilon^2/R_0^3 = 0$. For a single particle under the influence of both the external focusing force and the space charge field from the oscillating beam core we have Eq. 2.

$$\frac{\mathrm{d}^2x}{\mathrm{d}z^2} + k_0^2x - \begin{cases} \frac{K}{R^2}x, & x \geq R \\ \frac{K}{x}, & x < R \end{cases} = 0 \qquad (2)$$

where x is the position in the x-direction of the particle.

The parameter regime we explore is close to the one accessed by the University of Maryland Electron Ring (UMER). In particular, we use low energy electrons, at 10 keV, that form an intense beam with current 23 mA and emittance 48 mm-mrad. For a matched beam radius of 6.33 mm, the required external focusing force corresponds to a betatron wavelength of $k_0 = \sqrt{10}m^{-1}$, and the betatron wavelength depression ratio due to space charge is $k/k_0 = \sqrt{1 - K/(k_0^2a^2)} = 0.3785$. In order to launch the mismatch oscillations, we deliberately set the initial beam radius $R_i = 1.5R_0$ in Eq. 1.

In Fig. 1 we show a stroboscopic phase space plot of $r - r'$ that illustrates the halo creation mechanism. In particular, we solve numerically Eqs 1 and 2 with the initial conditions $R_i = 1.5R_0$ and $R_i' = 0$. At the minima of the core oscillations, we plot the distance r and transverse velocities $r' = \frac{\mathrm{d}r}{\mathrm{d}z}$ of single particles spread uniformly in phase space. In the resulting plot, we see that particles within the core of the beam perform small amplitude oscillations, while particles close to the beam edge can be driven to high amplitude oscillations, as they are trapped by the 2:1 resonance, corresponding to the large island seen in the figure.

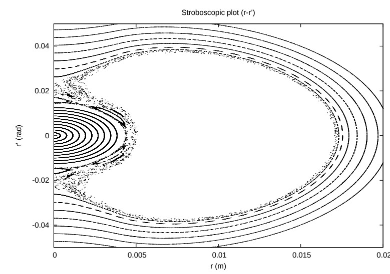

Figure 1: Stroboscopic plot of the single particle $r - r'$ phase space, showing the second order resonance island

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* email:papadopc@umd.edu

Comments on the Particle-Core Model

We note that the particles described by Eq. 2 do not couple into the envelope equation, and hence this model is not self-consistent, in the sense that the creation of a halo does not affect the beam core. Furthermore, the envelope equation assumes a constant emittance, something that is not realistic if the beam is transported for long distances. Within the context of the particle core model, removing the resonant particles should result in removing all of the halo, since there is no mechanism to drive new particles into the resonance.

SELF-CONSISTENT SIMULATIONS

The limitations of the particle-core model lead us to investigate halo formation by using long term, accurate and self-consistent simulation models of the beam. Namely, we use the WARP particle in cell (PIC) code, which is self-consistent within the electrostatic model, and has been widely used for the simulation of intense charged particle beams, showing good agreement with experiments in a lot of benchmarks [5].
We used the 2D model in WARP, and in order to ensure good statistics, we used a sufficiently high number of macroparticles, $N = 3.2 \times 10^5$, and a mesh size of 512×512. Special care was taken to avoid numerical collisions, and the simulation results were compared to runs with a higher number of macroparticles, showing excellent agreement.
We used two different initial distributions, namely the semi-gaussian (SG) and the thermal equilibrium (TE), which are described by equations 3 and 4 respectively [6].

$$f(\vec{x},\vec{v}) = Cn_0 \exp\left(-\frac{\vec{v}^2}{\sigma_v^2}\right) \quad (3)$$

$$f(\vec{x},\vec{v}) = f_0 \exp\left(-\frac{H(\vec{x},\vec{v})}{k_B T}\right) \quad (4)$$

In the case of the SG initial distribution, n_0 is the constant number density in configuration space and σ_v is the velocity spread, while C is a normalization constant. For TE, $H(\vec{x},\vec{v})$ is the Hamiltonian function, k_B is Boltzmann's constant, T the beam temperature and f_0 a normalization constant. We expect a beam coming out of a thermionic gun to have a distribution close to the semi-gaussian, while, for thermodynamical reasons, it is thought that the TE distribution is close to the final, stationary state of the beam.
In order to compare with the particle-core mode, we use the same parameter values as before, and again launch a symmetric mismatch by setting $R_i = 1.5R_0$. After propagating the beam for 100 m (roughly 400 envelope oscillation periods), we show the projections of the final beam distribution in the $r - r'$ plane in Fig. 2, for both the TE and SG initial distributions.
We can see that the coarse features of the resonance structure are the same for both distributions, and agree well with what we expect based on the particle core model shown in Fig 1. This is not wholly surprising, since the halo particles are, by definition, far from the beam core. Thus, they are influenced by the gross features of the beam, such as current and size, but not so much by its detailed structure.

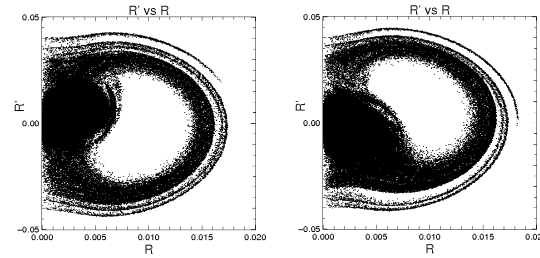

Figure 2: Final state with halo for initial TE (left) and SG (right) beam distributions

Particle Tracking

As discussed before, we use a simple and practical definition of halo, namely that any particle that ventures far from the beam center compared to the rms beam radius will be labeled as a halo particle. This is quantified in Eq. 5:

$$\frac{x_p^2}{4X_{rms}^2} + \frac{y_p^2}{4Y_{rms}^2} > \rho_0 \quad (5)$$

where x_p, y_p are the x- and y- positions of the particle, X_{rms} and Y_{rms} the rms beam size in x and y and ρ_0 is an adjustable parameter, that we determine empirically so that as many as possible halo particles are included, but none of the core particles.

IDEAL COLLIMATION

In the section describing the self-consistent simulations, we saw that beam halo is formed according to a parametric resonance mechanism, that, at least in its basic features, is described well by the particle-core model.
The most common mechanism of dealing with the halo is by employing collimators, which are foils placed at certain locations that scrape the halo particles. For the collimators to be useful, it is of course vital that the halo is not regenerated after it is scraped.
In order to study the regeneration of halo, we employ an ideal collimation scheme. In particular, since we track the halo particles in the simulations, we can remove all of them, at any point, without causing wakefields or other undesirable phenomena. Furthermore, this ideal scheme also removes particles that are within the core, but have high transverse velocities that will eventually lead them in the halo.
We employ the ideal collimation scheme for the TE initial distribution in two cases, one at the beginning of the run,

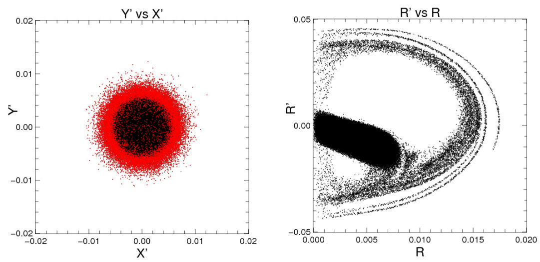

Figure 3: Projection in $x' - y'$ of the initial beam distribution, with the halo particles in red. In the right, the $r - r'$ projection after 100 m shows the regenerated halo

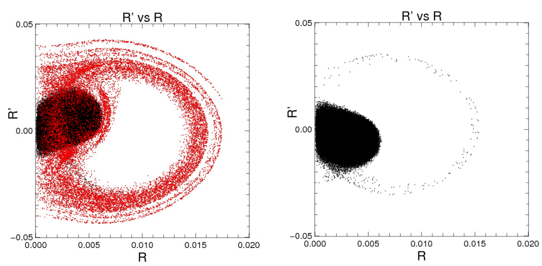

Figure 4: Projection in $r - r'$ of the beam distribution at z=100 m, with the halo particles in red. In the right, the $r - r'$ projection at z=200 m shows a faint halo

before the halo is developed, and one at the end of the run, when the halo is fully formed and has roughly stabilized in its extent. The first case is shown in Fig. 3, where we plot the $x' - y'$ projection of the initial beam distribution and we color the particles that will form the halo with red. After those particles are removed, we run again for 100 m, and we see that the halo is regenerated. The second case is shown in Fig. 2, where we plot the $r - r'$ projection of the beam distribution at z=100 m, again coloring the halo particles red. If we remove these particles, and propagate the beam for another 100 m, we can plot the $r - r'$ projection at z=200 m. As we see, the halo population is now significantly smaller.

As discussed before, the particle-core model does not predict the regeneration of halo, since all the particles that satisfy the resonance condition are trapped by the resonance, and there is no mechanism to shift particles into or out of the resonance. On the other hand, the self consistent simulations take into account collective phenomena within the beam that damp the core oscillations and redistribute the beam in phase space, and can pump new particles into the resonance. The damping of core oscillations in particular explains the difference between collimation at z=0 m and z=100 m, since in the latter case the mismatch oscillations have significantly lower amplitude, and thus the parametric resonance is not as strong.

CONCLUSIONS

In our comparisons of the particle-core model with self consistent PIC simulations, we saw good agreement, at least for the gross features of the halo. On the other hand, we also observed halo regeneration in the case of ideal collimation, something that was not predicted from the particle-core model. This can complicate realistic collimation schemes, since collective phenomena within the beam core will always pump particles into the parametric resonance causing halo, as long as the beam performs oscillations due to mismatch.

ACKNOWLEDGMENTS

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