COMPUTATION OF SPACE-CHARGE EFFECT IN ALLISON SCANNER AND ITS APPLICATION TO THE MEASUREMENT OF EMITTANCE

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Abstract

The space charge effect has an impact on emittance measurement of low energy H^- ion beam injected into the SNS RFQ. This paper presents numerical investigations of the space charge effect of the beam on transverse emittance measurement using an Allison style scanner attached to the front-end test stand at SNS. The investigations are based on mathematical modelling the emittance measurement by the scanner taking into account space charge of the beam. We present a method of emittance data analysis that includes the modelling and allows more accurate measurements of the emittance.

INTRODUCTION

The transverse emittance is one of the most important characteristics involved in H^- beam studies of SNS LINAC. The emittance measurement of the beam, injected into the SNS RFQ is realized by an Allison emittance scanner attached to front-end test stand. Emittance parameters such as rms emittance and Twiss parameters are normally measured from two-dimensional distribution of current data obtained by the scanner. In common case algorithm of data processing involves a first order model of the Allison scanner [1, 2]. Unfortunately, model of realistic measurement device is affected by higher order processes such as noise and space charge and slit scattering and other other factors yielding an error of measurements. The effect of noise and bias can be eliminated by both self–consistent exclusion analysis and suppressing the effect by design modification of the scanner for minimizing the sensitivity to higher order process [3-8].

The beam of H^- ions injected into the SNS RFQ has relatively small energy 65 keV and high current 30-70 mA [10]. Simple estimations show that at these parameters and typical parameters of the scanner space charge effect can significantly develop in the space between the two slits of the scanner. In turn it will lead to perturbation of the obtained two-dimensional current distribution and the measured emittance parameters. We present a method of calculation of error of measured results relative to true results by taking into account space charge effect in a model of emittance measurements with Allison scanner. In that way we introduce model-based correction into the algorithm of processing the data obtained by the scanner.

The space charge effect will be negligibly small at relatively high energies of the ion beam but geometrical effect of the entrance and exit slit width will always be the same. In this case we will present analytic expressions for estimation of error of measurements in this case.

SLIT WIDTH EFFECT

In this section we will separately discuss the slit width effect leading to distortion of current distribution obtained by two-slit scanner. The space charge effect is considered to be negligible i.e. trajectories of ions are straight for operation of Allison scanner. One can read about Allison scanner or two-slit scanner in papers [1-8]. For our estimations we will need the following parameters: Δa, Δb – widths of the entrance and exit slit respectively, L_{eff} – distance between two slits (see Fig. 1).

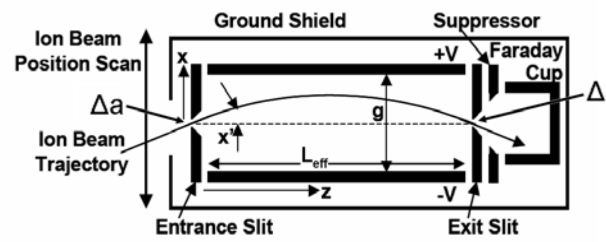

Figure 1: Schematic Allison emittance scanner [2].

We will consider transverse emittance distribution in phase space as Gaussian throughout the paper and equal along x and y axes. The investigated beam is considered to have an axial symmetry. For x axis this distribution can be written in the following form:

$$\psi_t(x,x') = \frac{1}{2\pi\varepsilon_t} e^{-\frac{x^2+(\alpha_t x+\beta_t x')^2}{2\beta_t\varepsilon_t}} \qquad (1)$$

Here α_t, β_t, and ε_t are Twiss parameters and rms emittance respectively. ”t” denotes true distribution and values that we want to measure. If we have this distribution just before the entrance slit of the scanner then we obtain two dimensional set of part currents propagated through the exit slit:

$$I_m(x_i,x'_j) = \int_{x_1}^{x_2}\int_{x'_1}^{x'_2} \psi_t(x,x')dxdx' \qquad (2)$$

Here, $x_{2,1}=x_i\pm\Delta a/2$ and $x'_{2,1}=x'_j+(x_i-x)/L\pm\Delta b/2L$ are position and angle limits of integration and ”m” denotes the measured distribution. At rather dense set of positions x_i and x'_j distribution of currents (2) can be considered as continuous and indexes *i, j* can be omitted. At finite parameters Δa and Δb normalized distribution $I_m(x,x')$ will not be the same as $\psi_t(x,x')$. It is reasonable to present the

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function $I_m(x,x')$ in form of the first approximation of Taylor series for parameters Δa and Δb. Finally we apply formulas from paper [9, 11] for rms emittance analysis written in integral form and obtain relations between the measured and the true parameters of emittance. Expression for rms emittance has the following form:

$$\varepsilon_m = \left[\varepsilon_t^2 - \frac{\varepsilon_t\alpha_t\Delta a^2}{6L} + \frac{\Delta a^2\Delta b^2}{144L^2} + \frac{\varepsilon_t\beta_t(\Delta a^2+\Delta b^2)}{12L^2} + \frac{\varepsilon_t\Delta a^2(1+\alpha_t^2)}{12\beta_t}\right]^{\frac{1}{2}} \qquad (3)$$

One can obtain the expressions for $\alpha_m(\alpha_t, \beta_t, \varepsilon_t)$ and $\beta_m(\alpha_t, \beta_t, \varepsilon_t)$ in the similar way. Numerical simulations of emittance measurements carried for typical parameters of the scanner are in good agreement with formula (3). Thus by measuring the apparent emittance parameters we can recalculate true parameters of the beam.

For the next sections of the paper we will need the following expression obtained similarly (3):

$$R_m = \sqrt{R_t^2 + \frac{\Delta a^2}{6}} \qquad (4)$$

where R_t is rms radius of the beam defined for axially symmetric beams as: $R_t=[(1/I)\int r^2 j(r)\cdot 2\pi r dr]^{1/2}$, $j(r)$ and I are current density and full current of the beam respectively. One can also derive that $R_t^2=2\varepsilon_t\beta_t$.

SPACE CHARGE EFFECT

Here, we present numerical investigations of space charge effect on emittance measurement of the H^- ion beam of low energy, injected into the SNS RFQ. The low energy of the beam results in a long time of flight between the two slits. In turn it leads to a development of space charge effect in this region of the scanner. Typical parameters of the scanner and the beam to be measured are listed in Table 1:

Table 1: Parameters of simulation

Parameter	Name	Value
L	Length between two slits	120.95 mm
Δa	Entrance slit width	0.5 mm
Δb	Exit slit width	0.5 mm
I	Current of the beam	50 mA
T	Energy of the beam	65 keV
R_t	rms radius of the beam	2 mm
α_t	Twiss parameter of the beam	-1
ε_t	Non-normalized rms emittance	20 mm·mrad

Below, we list the results of numerical investigations for these parameters. Allison emittance scanner measures the beam emittance in section, located just before the entrance slit. Figure 2 shows the distributions in this section. Figure 2a presents current density of the beam with axial symmetry and Fig. 2b presents the distribution of Gaussian emittance in phase space obtained by Monte-Carlo method using formula (1).

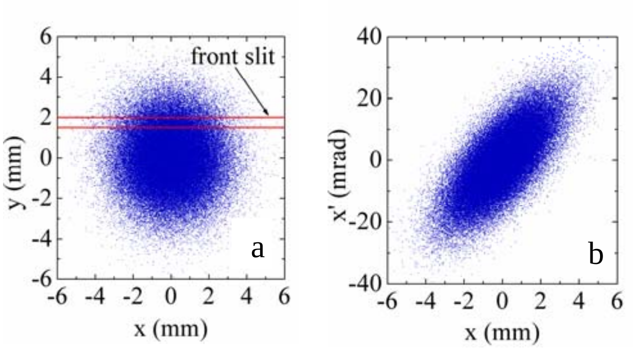

Figure 2: Distribution of current density of the beam **a**, and emittance of the beam **b**.

The method for simulation of emittance measurements includes calculation of the static distribution of current density of the beam between the two slits. The current density of the beam just after propagation of the beam through the entrance slit is shown in Fig. 2a as limited by the two lines. Finally two dimensional current distribution in phase space is obtained by calculation of the part current propagated through the exit slit.

Figure 3 shows the calculated distribution. Figure 3a and 3b present the emittance distributions not taking and taking into account space charge into the simulations. Scans are calculated for 60×60 positions of the entrance and exit slit. The measurement of non-normalized rms emittance yields 20.5 mm·mrad for 3a and 23.5 mm·mrad for 3b.

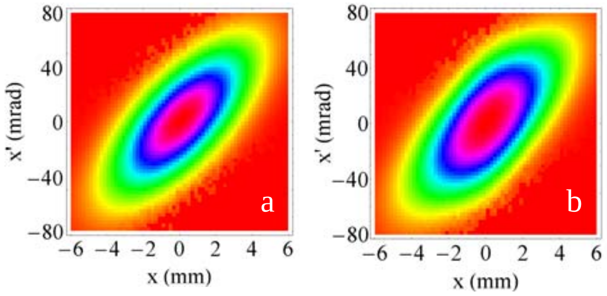

Figure 3: Calculated emittance distribution not taking into account space charge **a,** taking into account space charge **b**.

Figure 4 shows the measured emittance increase (in percent) relative to the true emittance in a wide range keeping fixed all other parameters of the beam listed in Table 1. The normalized rms emittance can be obtained by multiplying of the non-normalized one by βγ=0.0115.

Solid and dashed lines show the dependences at different parameters of slit width taking and not taking into account space charge in computations respectively. Numerical dependences presented by dashed lines are in good agreement with formula (3).

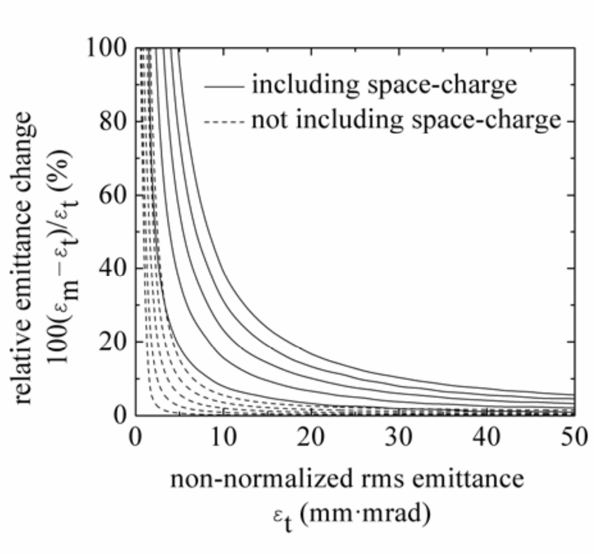

Figure 4: The measured emittance increase as a function of true emittance keeping fixed all other parameters of the beam. The lines following upwards respect to different slit width: 0.1, 0.2, 0.3, 0.4, and 0.5 mm for dashed and solid lines.

At high energies of the beam space charge can be neglected and formula (3) can be used for the estimation of error of measurement.

METHOD OF THE ANLYSIS OF THE EMITTANCE DATA

By using the results of previous section it is possible to develop the algorithm of reconstruction of true emittance parameters via the measured parameters. For reconstruction of the true emittance it is necessary to know true parameters β_t and α_t for calculation of dependences similar to that in Fig. 4. In turn parameters β_t and α_t can not be measured from a scan because they are also distorted.

For this reason it is more suitable to introduce two parameters that can be obtained from the measured emittance scan: rms radius of the beam R_t and orientation angle of emittance ellipse θ. These parameters have the following relations with β_t and α_t: $R_t^2=2\varepsilon_t\beta_t$ and $(1+\alpha_t^2-\beta_t^2)\cdot\mathrm{tg}(2\theta)=2\alpha_t\beta_t$ [1]. The true parameter of rms radius can be measured from the scan and reconstructed by formula (4) because it is measured quickly by the front slit and it is not affected by space charge. In measurement of emittance distribution only the angular distribution is affected by space charge in the region between the entrance and the exit slit. The orientation angle θ can be measured directly from the scan without recalculations because the edges of ellipse used for the measurement are distorted negligibly relative to its centre. Thus, after the measurement of ε_m and calculation of $\varepsilon_m=f(\varepsilon_t)$ (keeping fixed θ and R_t) we find ε_t from the dependence.

At high energies of H^- beam one can obtain analytic expressions for α_t, β_t, ε_t as a functions of the measured parameters by solving the system of linear equations (3).

CONCLUSIONS AND FUTURE PROBLEMS

-It is shown that space charge effect can yield error up to dozens percents in transverse emittance measurements of low energy H^- beam by Allison Scanner

-Method of analysis of emittance data was developed that allows to take into account the error produced by space charge

-At high energies of the beam analytic formulas for the error estimation of measurement can be used

-In future it is desirable to include into the calculations the effect of scattering of ion beam by slit edges

-It is necessary to develop or extend the method of analysis of emittance data for emittance of any shape in phase space

-It is also necessary to confirm the dependences of Fig. 4 experimentally by measuring the emittance as the function of the slit width

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