# IBS FOR NON-GAUSSIAN DISTRIBUTIONS*

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*Abstract*

In many situations distribution can significantly deviate from Gaussian which requires accurate treatment of IBS. Our original interest in this problem was motivated by the need to have an accurate description of beam evolution due to IBS while distribution is strongly affected by the external electron cooling force [1]. A variety of models with various degrees of approximation were developed and implemented in BETACOOL in the past to address this topic [2]. A more complete treatment based on the friction coefficient and full 3-D diffusion tensor was introduced in BETACOOL at the end of 2007 under the name "local IBS model" [3]. Such a model allowed us calculation of IBS for an arbitrary beam distribution. The numerical benchmarking of this local IBS algorithm and its comparison with other models was reported before. In this paper, after briefly describing the model and its limitations, we present its comparison with available experimental data.

## INTRODUCTION

Typically, in the absence of beam loss and external amplitude-dependent force, time evolution of beam profiles due to the Intrabeam Scattering (IBS) can be described by Gaussian distribution. Thus, analytic models of IBS developed for Gaussian distribution are very useful and provide good agreement with experimental measurements (see Ref. [4], for example). When longitudinal distribution starts to deviate from Gaussian for example due to the losses from the RF bucket, assumption of Gaussian distribution may already result in inaccurate prediction of intensity loss. To address this issue 1-D Fokker-Planck approach was effectively used before [5-6]. A more dramatic situation occurs when there is an externally applied force, like electron cooling. Since electron cooling force depends on the amplitudes of individual particles, the distribution under such force very quickly deviates from Gaussian. This effect is especially magnified when electron cooling is "magnetized" [7]. The problem of how to accurately account for IBS for such distributions became of special interest with a proposal to use electron cooling directly in a collider. For realistic prediction of luminosity gain from electron cooling an accurate treatment of IBS is required. Several approximate models were developed in the past to address this issue [1, 8-9]. However, a more general description requires full treatment of kinetic problem. Such a treatment was introduced in the BETACOOL code under the name "local IBS model" [3].

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## LOCAL IBS MODEL

The process of change of distribution function as a result of many small-angle scatterings can be described by Fokker-Planck equation, which offers self-consistent description of the system in diffusion approximation. The diffusion approximation reduces the problem of determining the effect of the fluctuations in the interaction force to the calculation of the dynamical friction $F$ and diffusion coefficient $D$, which are related to the first and second velocity jump moments, respectively. For the case of Coulomb interaction, expressions for the friction force and diffusion tensor are well known from plasma physics and are given by:

$$\vec{F} = \frac{\langle \Delta\vec{p} \rangle}{\Delta t} = -\frac{4\pi n e^4 Z_t^2 Z_f^2}{\left(\dfrac{m_f m_t}{m_f + m_t}\right)} \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{\vec{U}}{U^3} f(\vec{v}) d^3 v \quad (1)$$

$$D_{\alpha,\beta} = \frac{\langle \Delta p_\alpha \Delta p_\beta \rangle}{\Delta t} = = 4\pi n e^4 Z_t^2 Z_f^2 \int \ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right) \frac{U^2 \delta_{\alpha,\beta} - U_\alpha U_\beta}{U^3} f(\vec{v}) d^3 v \quad (2)$$

Here $\alpha$, $\beta$ = $x$, $y$, $z$, the angular brackets indicate averaging over the field particles, $Z_t$, $Z_f$ are the charge numbers of the test and field particle, $\vec{U} = \vec{V} - \vec{v}$ is the relative velocity of the test and field particle, $n$ is the mean density and $f(v)$ is the distribution function in the velocity space of field particles, respectively. The logarithm under the integrals is also called Coulomb logarithm, and is a measure of the relative contribution of (weak) remote interactions compared to (strong) near interactions. A validity of the diffusion approximation requires Coulomb logarithm >> 1, which is valid for IBS.

In standard treatment of IBS one usually assumes Gaussian distribution function and also averages over beam distribution to produce expressions for the growth rate of beam emittances. For present problem we need to keep dependence of the friction and diffusion coefficients on particle amplitudes. The beam distribution is represented by an array of particles. For each of the particles a smaller array of local particles is chosen, and local density and rms parameters of the particle distribution in each local array are calculated. The local parameters are used for calculation of the friction and diffusion coefficients. Since evaluation of the friction and diffusion coefficients is done numerically, the algorithm applies for an arbitrary distribution, although a faster option is available for Gaussian distribution with analytic evaluation of the integrals. Details of numerical implementation can be found in Refs. [2-3].

The "local" algorithm of calculating amplitude-dependent friction and diffusion coefficients in BETACOOL is common both for IBS and electron cooling simulation based on arbitrary distribution. It was first benchmarked in simulations of electron cooling with an arbitrary distribution function of electrons with the results reported in Refs. [10-11].

A full numerical solution (on a grid) of Fokker-Planck equation with more than two degrees of freedom is complicated. However, it is possible to transform a Fokker-Planck equation into an equivalent system of Langevin equations [12], which is adopted in the BETACOOL code. For simulation of the beam distribution function evolution in time, the Model Beam algorithm is used in the code. In the framework of this algorithm the ion beam is represented by an array of model particles, and all the effects changing the distribution function lead to the variation of the particle momentum components. The program then solves Langevin equation for each model particle from the particle array. The particle momentum during simulations is changed regularly by an action of the friction force and randomly by diffusion. In the three dimensional case each component of the particle momentum is changed according to the step of integration over time $\Delta t$ as:

$$P_i(t+\Delta t) = P_i(t) + F_i \Delta t + \sqrt{\Delta t}\sum_{j=1}^{3} C_{i,j}\xi_j \quad , \qquad (3)$$

where $F_i$ are the components of the friction vector, $\xi_j$ are independent random numbers, the coefficients $C_{i,j}$ are calculated from the diffusion tensor components according to:

$$\sum_{k=1}^{3} C_{i,k} C_{j,k} = D_{i,j} \, , \qquad (4)$$

which is a system of 6 non-linear algebraic equations [3]. The algorithm of finding solutions of these equations follows closely the one presented in Ref. [13], but with more general expressions for the friction and diffusion coefficients. The model developed in Ref. [13] was also implemented in BETACOOL under the name "kinetic model" and was used for benchmarking purposes [3].

For IBS simulations, the friction and diffusion coefficients have to be calculated at each optics element of the ring. To keep the calculation time reasonable for simulation run on a PC, the total number of optics elements should be reduced to just a few. This procedure should be done without sufficient distortion of the optics structure with respect to its IBS properties. For the lattice like the one used in RHIC, we found that such reduction is possible with an accuracy of IBS rates calculation within 10% compared to a full lattice with thousands of optics elements. Thus, for simulations with "local" IBS model presented here, we used simplified optics structure with only 15-30 optics elements.

## BENCHMARKING FOR GAUSSIAN DISTRIBUTION

A series of benchmarking tests were done first for a Gaussian distribution to make sure that "local" IBS model produces the same results as an analytic formalism available for Gaussian distributions.

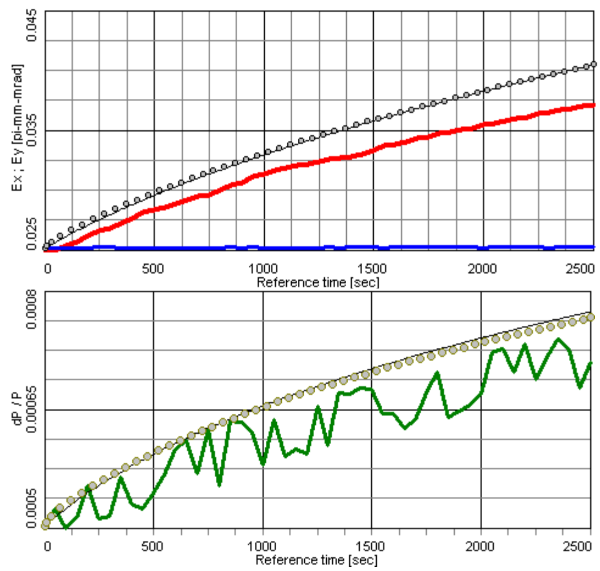


Figure 1: Simulations of beam emittance (upper plot) and momentum spread (lower plot). Circles and thin black top lines – analytic model for Gaussian distribution for full and reduced lattice, respectively. Color curves (red and green) – "local" IBS model with only diagonal elements of the diffusion tensor included.

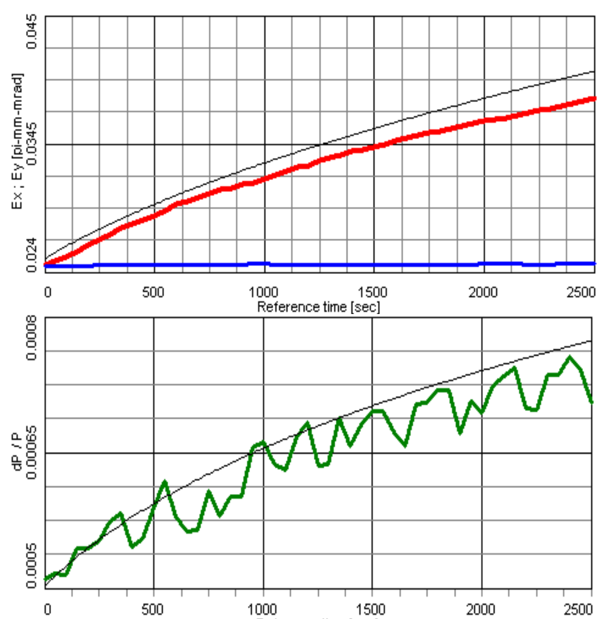


Figure 2: Simulation of beam emittance (upper plot) and momentum spread (lower plot). Thin top black lines – analytic model for Gaussian distribution. Color curves (red and green) – "local" IBS model including off-diagonal elements of the diffusion tensor.

For RHIC operation at top energy, which is much higher than transition energy, IBS diffusion is dominated by the longitudinal component $D_{zz}$ of the diffusion tensor. In such a case, the account of the off-diagonal tensor elements has little effect on the results of the simulations, as shown in Figs. 1-2. Also, for a Gaussian distribution, the evolution of beam emittances is predicted with very good accuracy even with constant diffusion tensor components (independent of particle coordinates and velocities), which was confirmed by using "kinetic" model of IBS. For the case of distributions which significantly deviate from Gaussian, the use of amplitude-dependent diffusion coefficients becomes important.

## HOLLOW LONGITUDINAL DISTRIBUTION

The "local" IBS model was benchmarked vs. dedicated IBS measurements with "hollow" longitudinal distribution done in RHIC for Au ions at 100 GeV/n in 2004. In that experiment, the RF synchrotron phase in one of the RHIC rings was jumped by about 90 degrees to create hollow longitudinal particle distribution. In both the horizontal and vertical directions, the particle distribution remained Gaussian and fully coupled transversely. As in a typical dedicated IBS measurement [4], several bunches of different bunch intensity were injected. Time evolution of emittance for each individual bunch was recorded with the ionization profile monitor. The longitudinal beam profiles and de-bunching beam loss were recorded using wall current monitor.

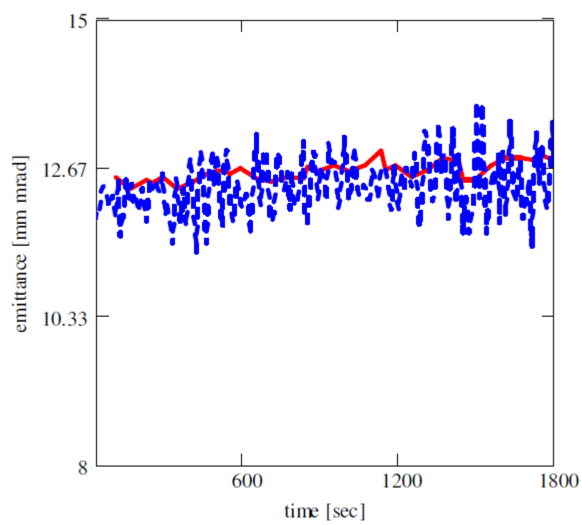


Figure 3: Time evolution of transverse emittance (95% normalized): red solid curve – measurements, blue dash curve – simulations with local IBS model.

It was already reported before, that to simulate correctly evolution of such longitudinal profile and achieve good agreement with the measured de-bunching loss, the use of 1-D longitudinal Fokker-Planck is required [5]. Here we present comparison of simulations using "local" IBS model with the same experimental data as reported in Ref. [5]. In present simulations with the local IBS, no approximations were used either for the longitudinal or transverse IBS, with off-diagonal elements of the diffusion tensor included in calculations. Although we should note that for this specific example, when transverse distributions stay approximately Gaussian and transverse IBS rates are very weak, even the use of an approximate analytic expression for the transverse growth rates gives reasonably good agreement with the measurements [5].

As an example, comparison of simulations using "local" IBS model (assuming fully coupled transverse motion) with measured data for a single bunch of medium intensity is shown in Figs. 3-6. Similar agreement between the measurements and simulations was observed for other bunches with different intensities and emittances as well.

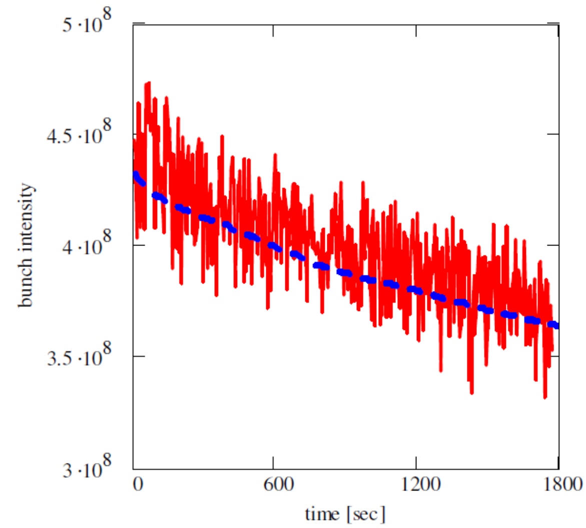


Figure 4: Bunch intensity evolution: red curve – measurements, blue dash curve – simulations with local IBS model.

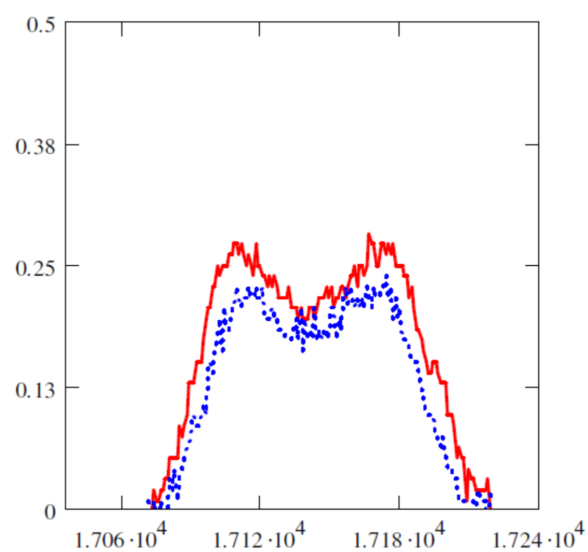


Figure 5: Measured longitudinal bunch profile: red (top) curve – initial, blue (lower curve) – after 1800s.

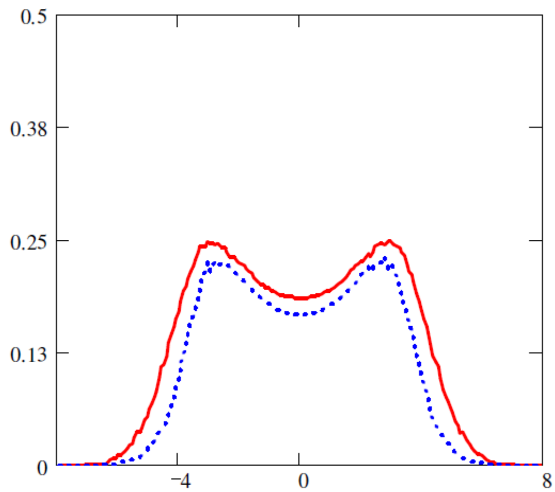


Figure 6: Simulated longitudinal bunch profile with local IBS model: red (top) curve – initial, blue (lower curve) – after 1800s.

## DISTRIBUTION WITH LOSSES FROM RF BUCKET

For another benchmarking of the model with experimental data we have chosen a data set for a different ring lattice with the reduced transverse IBS [14-15]. Due to large losses from the RF bucket during that experiment, longitudinal distribution quickly deviated from Gaussian. As an example, in Figs. 7-9 comparison between experimental data and simulation with "local" IBS model is shown for a single bunch from the 2007 IBS measurements during dedicated Accelerator Physics Experiments (APEX) in RHIC [15].

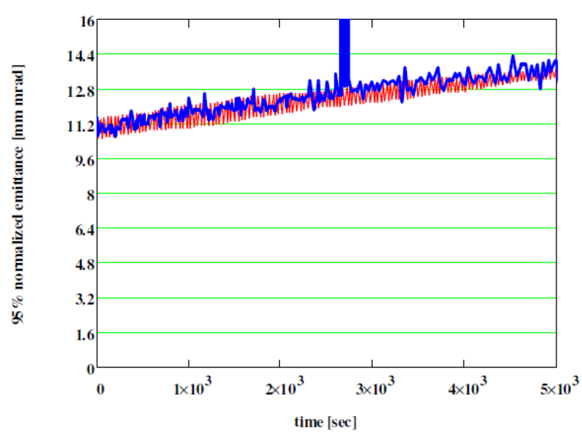


Figure 7: Time evolution of transverse emittance (fully coupled): blue curve – measurements using ionization profile monitor, red curve – simulations using local IBS model.

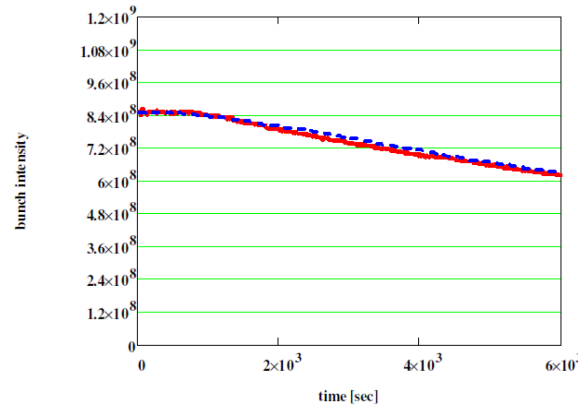


Figure 8: Bunch intensity: red solid line – measurements, blue dash curve – simulations with local IBS model.

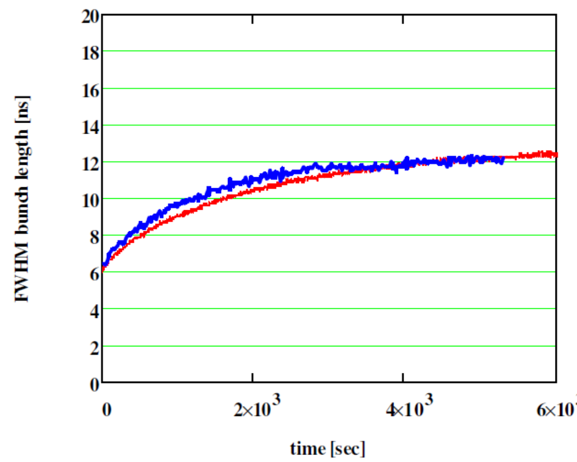


Figure 9: Bunch length evolution: red – measurements, blue – simulations with local IBS model.

If Gaussian approximation is used instead of "local" IBS, then evolution of the longitudinal profile and de-bunching loss become inaccurate (Figs. 10-11).

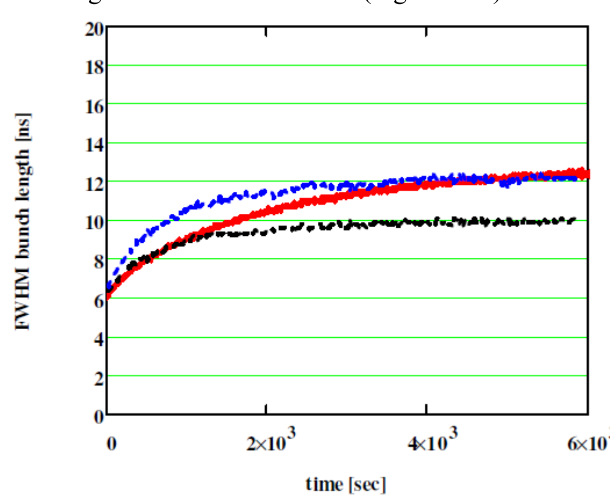


Figure 10: Bunch length: red – measurements, blue (upper curve) – simulations using Gaussian approximation with rms calculated from all particles, black (lower curve) – simulations using Gaussian approximation with calculation of FWHM of the distribution.

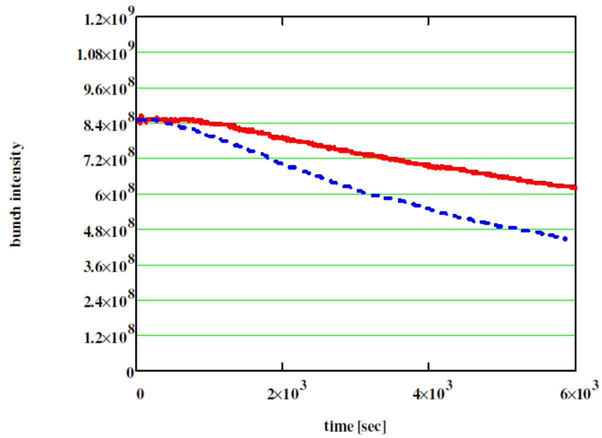


Figure 11: Bunch intensity: red (top curve) – measurements, blue dash curve – simulations using Gaussian approximation.

## SUMMARY

A model of IBS based on numerical evaluation of amplitude-dependent diffusion and friction coefficients was developed and implemented in the BETACOOL code. This model is suitable for IBS calculation for an arbitrary distribution function. In addition to the numerical benchmarking which was reported before, in this paper we presented comparison of simulations based on this model with available experimental data.

In many cases when distribution does not deviate significantly from Gaussian, the use of analytic IBS models available for Gaussian distribution seems to be well justified, especially since they do not require significant computational resources, as needed for local IBS model. However, in some cases, as shown in this paper, an amplitude-dependent treatment of IBS may be desired.

Of a special interest is accurate prediction of ion beam distribution evolution in time under combined effects of both IBS and electron cooling, as proposed for several collider projects, since resulting luminosity directly depends on the details of the distribution. The local IBS model developed allows verification of previous simulation results based on the approximate models. Such comparison will be reported elsewhere.

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## REFERENCES

[1] A.V. Fedotov et al., "IBS for distributions under electron cooling", Proc. of PAC05 (Knoxville, TN, 2005), p. 4263.

[2] BETACOOL code : http://betacool.jinr.ru; A. Sidorin et al., NIM A 558, p. 325 (2006).

[3] BNL BETACOOL report, December 2007: http://betacool.jinr.ru/reports/

[4] A.V. Fedotov et al., "Experimental studies of IBS in RHIC and comparison with theories", Proc. of HB2006 Workshop (Tsukuba, Japan, 2008), p. 259.

[5] J. Wei et al., "IBS theory and RHIC experiments", AIP Conf. Proc. 773 (HB2004), p. 389.

[6] V. Lebedev, "Single and multiple IBS in hadron colliders", AIP Conf. Proc. 773 (HB2004), p. 440.

[7] A.V. Fedotov et al., "High-energy electron cooling in a collider", New Journal of Phys. 8, p. 283 (2006).

[8] A. Burov, "Electron cooling against IBS for high-energy collider", FERMILAB-TM-2213 (2003).

[9] G. Parzen. BNL Tech Note C-AD/AP 144, 150 (2004).

[10] A.V. Fedotov et al., "High-energy electron cooling based on realistic six dimensional distribution of electrons", Proc. of PAC07 (Albuquerque, NM, 2007), p. 3699.

[11] A. Sidorin et al., "Electron cooling simulation for arbitrary distribution of electrons", Proc. of COOL07 (Bad Kreuznach, Germany, 2007), p. 159.

[12] H. Risken, "The Fokker-Planck equation", (Springer, Berlin, 1984).

[13] P. Zenkevich, O. Boine-Frankenheim, A. Bolshakov, Nucl. Instr. Meth. A 561, p. 284 (2006).

[14] V. N. Litvinenko et al., "Experience with IBS-suppression lattice in RHIC", Proc. of EPAC'08 (Genoa, Italy, 2008), p. 2557.

[15] A.V. Fedotov et al., "IBS suppression lattice in RHIC: theory and experimental verification", Proc. of HB2008 Workshop (Nashville, TN, 2008), p. 148.