TRANSVERSE DECOHERENCE IN BUNCHES WITH SPACE CHARGE

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Abstract

Transverse bunch offsets typically occur after bunch-to-bucket transfer between synchrotrons. Decoherence of the oscillations can cause emittance growth and beam loss, which should be avoided in high-intensity synchrotrons, like the projected SIS-100 synchrotron of the FAIR project. In this contribution we investigate how space charge modifies the bunch decoherence and associated diagnostics methods as turn-by-turn chromaticity measurements.

DECOHERENCE DUE TO CHROMATICITY

As a result of an initial transverse displacement A_0, a bunch oscillates in the corresponding plane (here x). In the case of the Gaussian momentum distribution, the amplitude of the bunch offset evolves with the turn number N as [1]

$$A(N) = A_0 \exp\left\{ -2\Big(\frac{\xi Q_0 \delta_p}{Q_{\rm s}} \sin(\pi Q_{\rm s} N)\Big)^2 \right\}, \qquad (1)$$

here Q_0 is the bare betatron tune, ξ is the chromaticity: $\Delta Q_\xi / Q = \xi \Delta p/p$, $\delta_p = \sigma_p/p$ is the normalized rms momentum spread and $Q_{\rm s}$ is the synchrotron tune. Here, the linear synchrotron motion is assumed, the only source of the tune spread is the chromaticity with the momentum spread. Figure 1 shows an example for bunch decoherence after the kick $\overline{x} = \sigma_{x0}$, where σ_{x0} is the horizontal rms beam width, ε_0 is the initial transverse rms emittance. The effect of the chromaticity is usually quantified by the betatron phase shift over the bunch length,

$$\chi_{\rm b} = \frac{Q_0 \xi}{\eta} \tau_{\rm b}\,, \qquad (2)$$

where η is the slip factor and $\tau_{\rm b}$ is the bunch length in radian, calculated accordingly to the longitudinal truncation at 2σ, which was taken for simulations. Figure 1 demonstrates that a higher chromaticity provides a faster decoherence, and that after the synchrotron period $N_{\rm s} = 1/Q_{\rm s}$ the initial offset amplitude appears again, which is called recoherence.

The usual rms emittance, to which we refer here as the "global" rms emittance, is given by

$$\varepsilon = \Big[\langle (x-\overline{x}_{\rm b})^2 \rangle \langle (x'-\overline{x'}_{\rm b})^2 \rangle - \langle (x-\overline{x}_{\rm b})(x'-\overline{x'}_{\rm b}) \rangle^2 \Big]^{1/2}, \qquad (3)$$

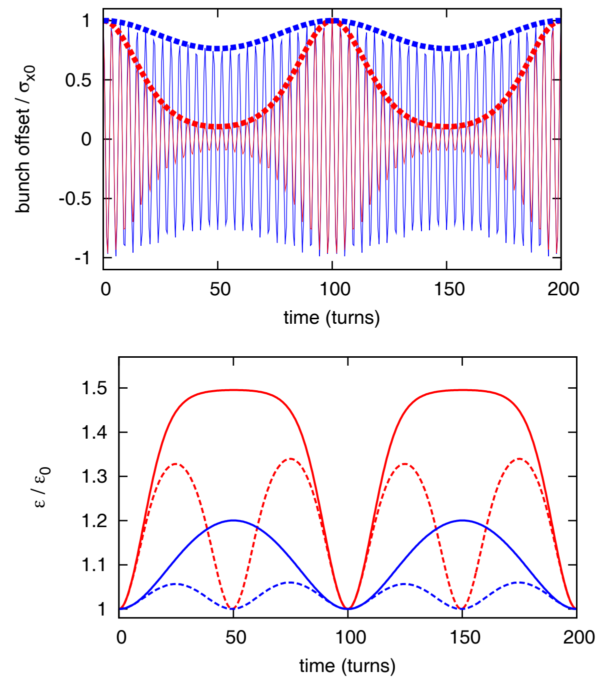

Figure 1: A particle tracking simulation for a Gaussian bunch after an offset kick $\overline{x}(\tau) = \text{const}$. Top plot: time evolution of the bunch offset for the head-tail phase shift $\chi_{\rm b}$=1.5 (blue) and $\chi_{\rm b}$=4.3 (red), the dashed lines are given by Eq. (1). Bottom plot: time evolution of the global rms emittance [full lines, Eq. (3)] and of the local rms emittance [dashed lines, Eq. (4)]. Line colors correspond to the top plot. The synchrotron period is $N_{\rm s}$=100 turns.

where $\langle ... \rangle$ denotes averaging over bunch particles; $\overline{x}_{\rm b}$ and $\overline{x'}_{\rm b}$ are the coordinate offset and the momentum offset of the whole bunch. Additionally, the so-called "local" rms emittance can be defined,

$$\varepsilon_{\rm local} = \Big\{ \langle [x-\overline{x(\tau)}]^2 \rangle \langle [x'-\overline{x'(\tau)}]^2 \rangle - \langle (x-\overline{x(\tau)})(x'-\overline{x'(\tau)}) \rangle^2 \Big\}^{1/2}, \qquad (4)$$

where $\overline{x(\tau)}$ and $\overline{x'(\tau)}$ are the bunch offsets at the longitudinal particle position τ in the bunch. Figure 1 (bottom) demonstrates the difference between these two values for the bunch decoherence and recoherence due to the chromaticity.

For the decoherence due to the chromaticity, all the bunch parameters behave periodically. The opposite case is an oscillation damping due to e.g. a transverse nonlinearity. Figure 2 shows an example for an octupole nonlinearity, where the bunch decoherence is irreversible. Damping of the coherent oscillations is accompanied by an increase in the transverse emittance [3],

$$\frac{\varepsilon_{\max}}{\varepsilon_0} = 1 + \frac{1}{2}\Big(\frac{A_0}{\sigma_{x0}}\Big)^2 , \tag{5}$$

what we can observe in Fig. 2, where $\varepsilon_{\max} = 1.5\varepsilon_0$. The total and the local emittances are nearly identical in this case. We should point out that in the case of bunch decoherence due to chromaticity the largest rms emittance is also given by this $\varepsilon_{\max}$, and it can be reached if the betatron phase shift $\chi_{\rm b}$ is high enough, see Fig. 1.

Transverse oscillations excited by a bunch kick can be also used to measure the chromaticity [2]. After an offset kick $\overline{x}(\tau) = \text{const}$ the phase difference in coherent betatron oscillations between head and tail evolves as

$$\Delta\psi_{\rm ht} = \Delta\chi[1 - \cos(2\pi N Q_{\rm s})] , \tag{6}$$

where $\Delta\chi = Q_0\xi\Delta\tau/\eta$ is the head-tail phase shift between two bunch positions. The turn-by-turn chromaticity can then be determined as

$$\xi(N) = \frac{\eta}{Q_0\Delta\tau}\frac{\Delta\psi_{\rm ht}(N)}{1 - \cos(2\pi N Q_{\rm s})} . \tag{7}$$

This method can be also useful for numerical simulations, as we demonstrate in the present work. Figure 3 shows the head-tail phase difference between two bunch positions and the resulting from Eq. (7) chromaticity, both in the turn-by-turn mode. The phase difference was obtained using a harmonic analysis for each single turn signal, for the bunch head and for the bunch tail. In Fig. 3 (right) we see that this method clearly reproduces the chromaticity.

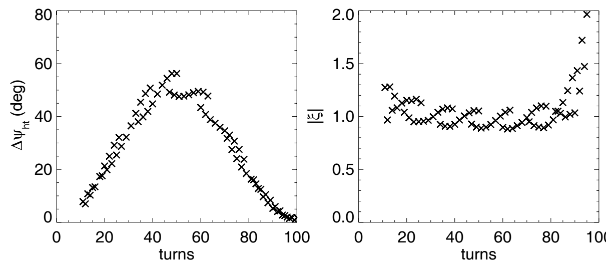

Figure 3: Time evolution of the turn-by-turn head-tail phase difference (left) and the corresponding chromaticity [right, obtained using Eq. (7)] after an offset kick. The particle tracking simulation was made for a Gaussian bunch, chromaticity $\xi = -1$ and synchrotron period $N_{\rm s}$=100 turns.

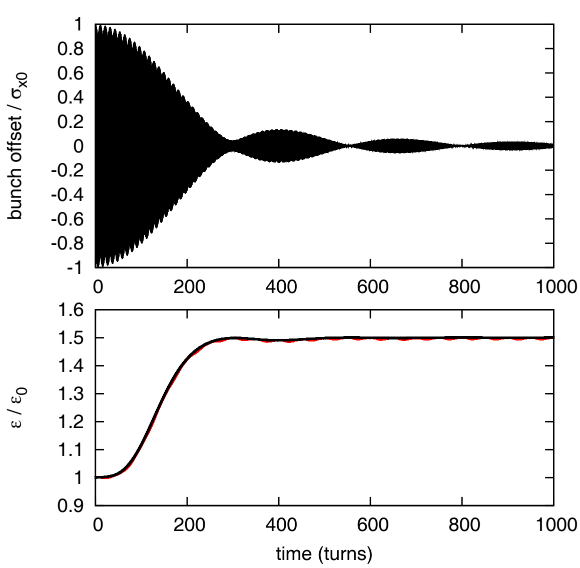

Figure 2: Bunch decoherence with an octupole nonlinearity: time evolution of the bunch offset (top plot), rms emittance (bottom plot). The global (black) and the local (red) rms emittance is shown. Simulation parameters correspond to Fig. 1.

DECOHERENCE WITH SPACE CHARGE

Decoherence in bunches with self-field space-charge can be understood using the effect of Landau damping [4, 5, 6]. The space-charge tune spread due to the longitudinal density profile provides Landau damping. This damping is induced exclusively by the space-charge effect. In order to characterize the space-charge strength, we introduce the space-charge parameter $q = \Delta Q_{\rm sc}/Q_{\rm s}$, where $\Delta Q_{\rm sc}$ is the peak space-charge tune shift (i.e. in the bunch middle). In this work we assume a Gaussian longitudinal profile.

As we see in Ref. [6] (Fig. 2 there), at moderate space charge $\Delta Q_{\rm sc} \sim Q_{\rm s}$, the head-tail mode $k = 1$ is effectively damped by the space-charge effect, and higher-order modes are damped much faster, while the $k = 0$ mode is not affected by space charge. If we consider the initial offset kick $\overline{x}(\tau) = \text{const}$ as a superposition of the head-tail eigenmodes, it is clear that the modes $k \geq 1$ will be damped and the eigenmode $k = 0$ will continue to oscillate if there are no other damping mechanisms. For stronger space charge, Landau damping for the lowest-order eigenmodes becomes much weaker (see Fig. 2 in [6]), thus a combination of the $k = 0$ mode with higher-order modes will continue to oscillate, depending on q.

In Ref. [6] we also discuss that the transverse eigenfunctions in a Gaussian bunch with space charge are very close to the airbag [7] eigenmodes, $\overline{x}_k(\tau) = A_0\exp(-i\xi Q_0\tau/\eta)\cos(k\pi\tau/\tau_{\rm b})$. Hence, in the further discussion here we use the airbag [7] eigenmodes as a reasonable approximation.

Particle tracking simulations presented in this work were done using PIC codes PATRIC [8] and HEADTAIL [9]. For the transverse space charge force, the "frozen" electric field model was used, i.e. a fixed potential configuration which follows the mass center for each single slice. A round trans-

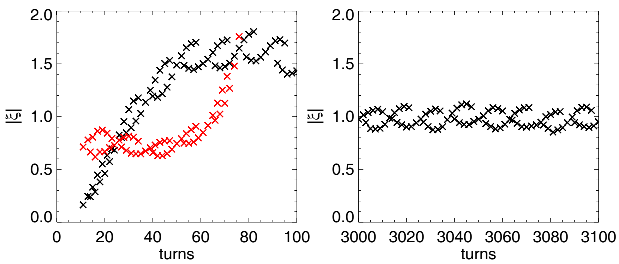

Figure 4: Turn-by-turn chromaticity after an offset kick for a bunch with space charge $q = \Delta Q_{\rm sc}/Q_{\rm s} = 1$, directly after the kick (left) and 3000 turns later (right). Black symbols: obtained using Eq. (9), red symbols: obtained using Eq. (7).

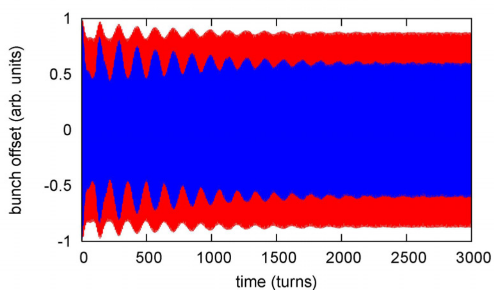

Figure 5: Time evolution of the bunch offset after an offset kick for a Gaussian bunch with space charge $q = 1$; $\chi_{\rm b} = 1.5$ (the red line) and $\chi_{\rm b} = 2.9$ (the blue line).

verse cross-section and a homogeneous transverse beam profile were used in the simulations in this work. The code validation, especially for long-time simulations with space charge, was presented in [10].

In order to demonstrate the decoherence with space charge, we use the method of the head-tail phase difference. Figure 4 shows results of the chromaticity evaluation in a simulation for a Gaussian bunch with space charge $q = 1$. Red symbols in the left plot of Fig. 4 demonstrate that it is not possible to use Eq. (7) for the chromaticity in the case of a bunch with space charge. However, after some time the bunch oscillations saturate, as we show in Fig. 5. If we now consider a bunch with the $k = 0$ head-tail mode,

$$\overline{x}(\tau) = A_0 \cos\Big(\frac{\chi_{\rm b}\tau}{\tau_{\rm b}}\Big) \tag{8}$$

we see that the head-tail phase difference $\Delta\psi_{\rm ht} = \Delta\chi$ is constant and the turn-by-turn chromaticity is given by

$$\xi(N) = \frac{\eta}{Q_0 \Delta\tau} \Delta\psi_{\rm ht} \, . \tag{9}$$

The black symbols in Fig. 4 are calculated using this expression. The right plot confirms that only the $k = 0$ mode remained in the bunch oscillations. Directly after the kick (the left plot in Fig. 4), the head-tail phase difference can not be described by Eqs. (7), (9), because the oscillation is

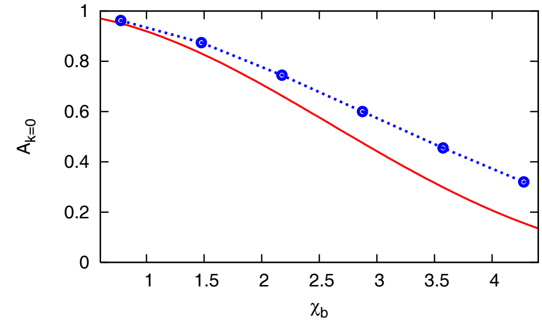

Figure 6: Relative oscillation amplitude of the $k = 0$ mode in a $\overline{x}(0) = \text{const}$ kick. The blue circles: results of decoherence simulations for a Gaussian bunch with space charge $q = 1$. The red line: analytical expression Eq. (11).

a complex mixture of eigenmodes, each of them affected by space charge differently.

Important bunch parameters, as the oscillation amplitude and the beam size, which stay after the active phase of bunch decoherence with space charge, depend on the space-charge strength q and on the head-tail phase shift $\chi_{\rm b}$. As discussed above, at $q = 1$ only the mode $k = 0$ continue to oscillate. The final oscillation amplitude can be readily estimated. The $\overline{x}(\tau) = \text{const}$ kick can be decomposed into a superposition of head-tail eigenmodes,

$$1 = \sum_k a_k \exp\Big(-i\frac{\chi_{\rm b}\tau}{\tau_{\rm b}} + i\phi_k\Big) \cos\Big(\frac{k\pi\tau}{\tau_{\rm b}}\Big) \, . \tag{10}$$

The relative oscillation amplitude of an eigenmode can be calculated as $A_k = a_k d_{\max}$, where $d_{\max}$ is the maximum bunch offset produced by the mode k. For the mode $k = 0$ we obtain

$$A_{k=0} = \frac{4}{\chi_{\rm b}^2} \sin^2\Big(\frac{\chi_{\rm b}}{2}\Big) \, . \tag{11}$$

In order to obtain $A_{k=0}(\chi_{\rm b})$ for a realistic Gaussian bunch with space charge, we perform simulation runs for $q = 1$ with an offset kick $\overline{x} = \sigma_{x0}$ for different $\chi_{\rm b}$, see an example in Fig. 5. The resulting amplitudes of the $k = 0$ mode are presented in Fig. 6, together with the estimation Eq. (11). The agreement is not perfect, since the airbag [7] eigenmodes differ from the eigenmodes of a Gaussian bunch, but the $\chi_{\rm b}$-dependencies are similar.

The transverse emittance blow-up after the bunch decoherence is also of high importance. Here we would like to discuss not only the global rms emittance, but also the local rms emittance from Eq. (4), because it gives a more complete picture, especially in the case of persistent bunch oscillations. Figure 7 shows an example for global [Eq. (3)] and local [Eq. (4)] rms emittances in a decoherence simulations with $\chi_{\rm b} = 2.9$, $q = 1$. With space charge, the global rms emittance (the blue line) increases irreversibly by 32%. But this does not describe the decrease of the phase-space density correctly, because the bunch continues to oscillate.

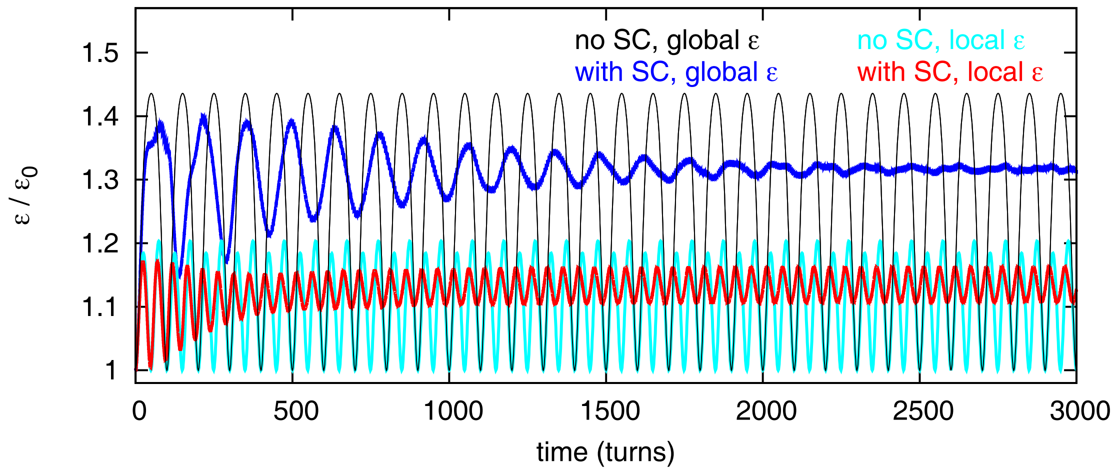

Figure 7: A comparison between the global rms emittance [Eq. (3)] and the local rms emittance [Eq. (4)] in a decoherence simulation after an offset kick $\overline{x} = \sigma_{x0}$ for a Gaussian bunch with space charge $q = 1$ and without space charge.

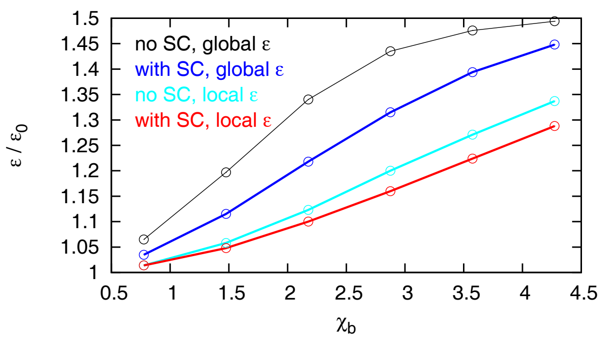

Figure 8: Head-tail phase shift dependence of the emittance blow-up after the bunch decoherence with an offset kick $\overline{x} = \sigma_{x0}$ for a Gaussian bunch with space charge $q = 1$ and without space charge. For the oscillating emittances, the maximum values are given.

The maximum emittance blow-up for $\varepsilon_{\rm local}$ (the red line) is only 16%.

Results for the emittance blow-up in decoherence with space charge are summarized in Fig. 8. For oscillating emittances, the maximum values are given. The increase of the global emittance for decoherence with space charge (the blue line) is always smaller than the maximum emittance blow-up for the case without space charge (the black line). The local emittance blow-up, which is relevant in terms of the phase space density, is roughly a half of the increase in the global rms emittance for the cases considered. In a comparison with the damping due to nonlinearities, which causes $\varepsilon_{\rm final} = 1.5\varepsilon_0$ for our parameters here, the decoherence with space charge provides a smaller emittance blow-up at moderate $\chi_{\rm b}$. Only at large head-tail phase shifts the emittance increase reaches the limit of the nonlinearity damping Eq. (5).

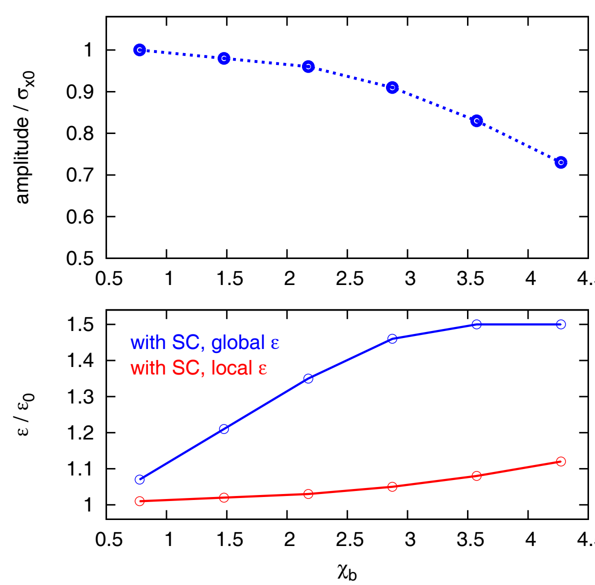

Figure 9: Summary of the bunch decoherence simulations for a Gaussian bunch with space charge $q = 6$: the oscillation amplitude (top plot), the emittance blow-up (bottom plot). For the oscillating emittances, the maximum values are given.

As discussed above, at stronger space charge the mode $k = 1$ is nearly not damped, and after the decoherence phase the oscillation does not correspond to the $k = 0$ mode, but it is a mixture of modes. The simulation results for space charge $q = 6$ are presented in Fig. 9. The oscillation amplitude does not saturate in these decoherence runs, as it is the case in Fig. 5, instead it "beats" periodically. The same is true for the rms emittance. Thus, the corresponding maximum values are plotted in Fig. 9. In a comparison to the $q = 1$ case (Figs. 6 and 8), for $q = 6$ the amplitude reduction with growing $\chi_{\rm b}$ is weaker, but the global rms emittance blow-up is stronger. Instead, the increase in the local rms emittance is small. This difference is related to the large oscillation amplitudes and small reductions in the phase space density.

CONCLUSIONS

Decoherence in bunches with space charge has been studied using particle tracking simulations. The initial kick $\overline{x}(\tau) = \mathrm{const}$ can be considered as a superposition of head-tail eigenmodes. Landau damping [4, 5, 6], induced in bunches by the space-charge tune spread due to the longitudinal density profile, effectively suppresses the head-tail modes. The relative amplitude of an eigenmode in the offset kick depends on the head-tail phase shift χ_{b}, while the Landau damping decrement for this mode depends on the space-charge parameter $q = \Delta Q_{\mathrm{sc}}/Q_{\mathrm{s}}$. Thus, the oscillation after the decoherence can be a complex mixture of eigenmodes, each of them affected by space charge differently.

The case $q = 1$ has been considered in detail. Landau damping suppresses strongly all the head-tail modes $k \geq 1$ for this space charge strength. The oscillation after the decoherence phase corresponds to the eigenmode $k = 0$. The oscillation amplitude reduction with growing χ_{b} can be easily understood in this case as the relative amplitude of the $k = 0$ mode in the offset kick.

The method to determine the chromaticity [2] using the head-tail phase difference in coherent oscillations has been used. Generally, for bunches with space charge it is not possible to use the usual model Eq. (7) for the turn-by-turn chromaticity. It has been demonstrated, that, if Landau damping due to space charge suppresses the $k \geq 1$ modes, which is the case for e.g. $q = 1$, the chromaticity can be obtained using Eq. (9).

The emittance blow-up due to bunch decoherence has been examined. Without space charge, the rms emittance increases periodically, recovering to the initial value each $1/Q_{\mathrm{s}}$ turns. The emittance blow-up depends on χ_{b}, but the maximum emittance increase corresponds to that of the damping due to nonlinearities, see Eq. (5). For bunch decoherence with space charge, the increase of the rms emittance is normally smaller than the maximum emittance blow-up for the case without space charge. In a comparison with the damping due to nonlinearities, the decoherence with space charge provides a smaller emittance blow-up at moderate χ_{b}. Only at large head-tail phase shifts the emittance increase reaches the limit of the nonlinearity damping Eq. (5). In order to describe the emittance blow-up more accurately, the concept of the local rms emittance Eq. (4) has been used. It has been demonstrated, that in the case of an oscillation with a large amplitude, the increase in the global emittance can be large. However, this is not always true for the phase space density. This can be seen using the local rms emittance which can have a small increase.

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