NON-SCALING FFAG AND THEIR APPLICATIONS*

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Abstract

Examples of the Non-Scaling Fixed Field Alternating Gradient (NS-FFAG) in applications are shown. NS-FFAG designs, with beam passing either once of just a few turns, are shown: the medical gantries in the cancer therapy, Recirculating Linac Accelerators (RLA) with droplets or race track, and a muon accelerating ring with distributed RF cavities. A small permanent magnet proton cancer therapy machine and one GeV racetrack with superconducting magnets are presented as examples of NS-FFAG with larger numbers of turns. Consideration of the possible use of the NS-FFAG for storage rings for protons or pions for longitudinal manipulations is assessed.

INTRODUCTION

This report describes possible applications of the Non-Scaling Fixed Field Alternating Gradient (NS-FFAG) accelerators. In recent years there has been a clear revival of the previous concept of the FFAG's, developed mostly in the 1950's [1, 2, 3] mostly with a Midwestern Universities Research Association (MURA). More recent information about Scaling FFAG (S-FFAG) concept can be found elsewhere [4]. In the second chapter, a concept of the NS-FFAG is described. It includes a description of the linear properties of the structures, explains the rationale of the large momentum acceptance and small aperture requirements. In the basic concept description of the tune, time of flight, dispersion and amplitude functions variations with a momentum are shown. Applications of NS-FFAG's are divided into two categories: where a beam passes through the structure one or a few times, or where the beam has multiple - hundreds of turns. This distinction comes as a consequence of the tune variations with momentum. Although the NS-FFAG is a linear machine with a large dynamical aperture and large momentum acceptance the integer crossings of the tunes represents a problem. This comes from misalignment of more than 20-40 μm or from the magnetic field error larger than $\delta B/B > 10^{-3}$. The following chapter describes NS-FFAG examples with a single or a few beam passes: the carbon/proton medical gantries for the cancer therapy, the RLA either racetrack or droplets, an RLA for electron ion collider acceleration, thirteen turns NS-FFAG muon accelerator made of triplet combined function magnets, and finally a storage ring to capture muons from pion decays, which is fully described in the program Phase Rotated Intense Slow Muon (PRISM) beam in Japan [5]. The higher momentum pions, of few hundred MeV/c (idea of C. Ankenbrandt -Muon Inc.), could be stored in the same fashion as in the PRISM and from a decay of relativistic pions obtain muons with a smaller momentum spread. The spectrum that ranges from about 100% to about 50% of the pion momentum. So a NS-FFAG ring with a momentum acceptance of 50% and a high RF voltage would capture most of the decay muons from an injected pion beam in no more than 6 turns. The third chapter describes NS-FFAG for non-relativistic beams where a larger number of turns is assumed. A proton accelerator for the cancer therapy made of NS-FFAG made with permanent Halbach magnets, and a 1 GeV superconducting race track are described. Additional heavy-ion acceleration replacing long and very expensive superconducting linac was just noted (more information is available in the previous publication [6]). From the particle tracking of all the examples at the central momentum, where usually the momentum compaction is equal to zero becomes a clear possibility for using NS-FFAG as a storage ring for longitudinal beam manipulations as the momentum aperture is $\delta p/p = \pm 50\%$.

BASIC CONCEPT OF THE NS-FFAG

In S-FFAGs the tunes ν_x and ν_y are constant with a zero chromaticity for all particle energies as the orbits radii scale with energy but with the field index $|n| \sim 500$. This necessarily makes the field nonlinear. The benefit of the NS-FFAGs is due to the relationship $\Delta x = D_x \Delta p/p$, where Δx is the radial beam offset, D_x is the lattice dispersion function and $\Delta p/p$ is the fractional momentum deviation. The value for Δx may be kept less than ±50 mm for a $\Delta p/p = \pm\ 60\ \%$, if the D_x is <0.08 m. The dispersion function or the dispersion action H is well controlled, similar to the request for the minimum of the H in the light source lattice. The NS-FFAG has a very strong focusing structure to obtain small values of D and β, and hence small magnet sizes. Linear magnetic field dependence with respect to the radial axis $B \propto r$ is an additional simplification. The strongest focusing, smallest dispersion, and best circumference is achieved by the combined function magnets in the triplet FDF where in the middle is a larger defocusing main bending element surrounded by two smaller focusing combined function magnets with opposite bend [8], as shown in Fig. 1.

In the non-scaling case the magnetic fields are linear even though they lead to a substantial range of tunes; that is acceptable in an accelerator for muons because the acceleration has to take place very rapidly, and therefore betatron resonances are traversed so fast that they have very little

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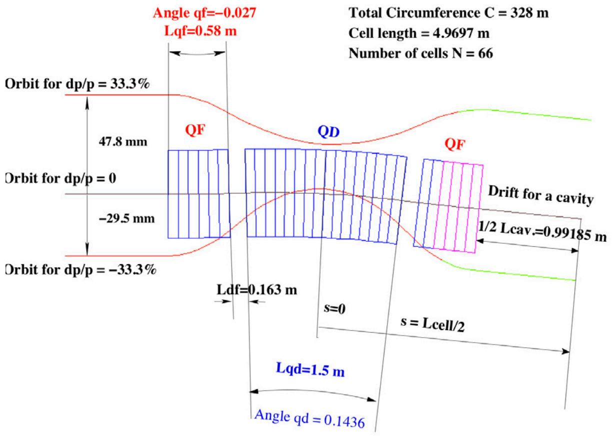

Figure 1: Basic half cell. The cell is symmetric with respect to $s = 0$. The total length of the cell equals $L_{cell} = L_{cavity} + 2 * L_{QF} + L_{QD} + 2 * L_{df}$. There are two types of magnets, QD (normal bend, field 4.95 T) and QF (negative bend, field -2.29T). Each has constant focusing strength, K_f and K_d.

effect. The first NS-FFAG: Electron Model for Many Applications - EMMA [7] has been built and presently is being commissioned in Daresbury Laboratory, England. This is a proof of principle of the NS-FFAG concept for the fixed frequency acceleration of relativistic electrons, muons, and other ions.

NS-FFAG WITH A FEW BEAM PASSES

The NS-FFAG concept arose during a study of future Muon Collider or Neutrino Factory [9]. The NS-FFAG reduced the cost by using more circulating turns than in the re-circulating linacs they replaced the high linac cost by using the same linac with multiple turns. A simplest NS-FFAG application due to the large momentum acceptance is a use of it as the beam line for example for the isocentric gantries as shown in the next.

Medical Carbon/Proton Gantries

The highest cost in the proton or carbon facilities is that of the delivery. A present world-class facility for the carbon and other ion cancer therapy at Heidelberg is already operating with the 630 tons isocentric gantry where the transport elements weight is 135 tons [10]. The NS-FFAG concept provides a reduction of the transport elements for the carbon ions to about 1.5 tons. In addition to the reduction in size, the magnetic field is fixed for all energies required for treatment, additional simplification for the operation. Ions pass through the gantry oscillating around the central orbit with very small orbit offsets.

Two examples of NS-FFAG isocentric gantry designs are presented [11]; one with the superconducting combined function magnets, to allow carbon and proton ion transport and delivery, and the second one with separated functions permanent Halbach magnets. The gantry, made of the NS-FFAG cells, accepts and propagates different energy ions with very small variation of the orbit in a kinetic energy range 149.7-400 MeV/u or in momentum -25 $\% \leq \delta p/p \leq$ 30 $\%$ (as shown in Fig. 2). The dispersion function and the slope are set to zero at the beginning of the gantry in the middle of the required momentum range. The largest orbit offsets are at the focusing quadrupoles, and have similar values at the lowest and highest momentum-energy by optimization of the bending angles. Carbon/proton ions of different energies reach the end of the gantry within ±6 mm. The ion position at the patient is adjusted with the scanning and triplet focusing magnets for each energy separately.

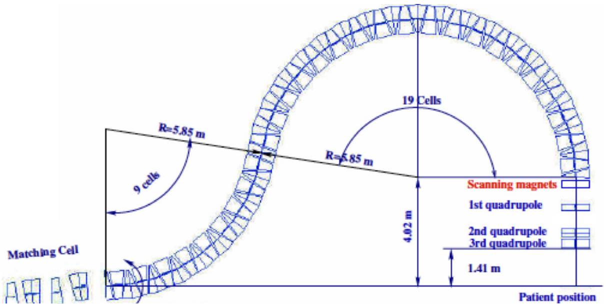

Figure 2: The carbon/proton isocentric gantry with superconducting magnets.

For the proton cancer facilities a permanent Halbach separate function magnet design is presented in Fig. 3. The maximum magnetic field in the center of the Halbach dipole magnet is $B_g = B_r \ln(\mathrm{OD/ID})$, where B_r is the material permanent magnetic field value, while OD and ID are outside and inside diameters of the material modules. The range of proton energies under the fixed magnetic field is between 68-250 MeV. Magnets are made of Neodymium-Iron-Boron compounds (Nd-Fe-B) assuming the maximum operating temperature of 70^o C and with the magnetic field of B_r=1.35 T. The NS-FFAG isocentric gantries provide simple solutions: they are easy to operate as the field is fixed for all treatment energies, the cost is reduced, they are made of light elements (a weight of the whole gantry is $\sim$500 kg), and there is small power consumption. The scanning and focusing system is above the patient at a distance of $\sim$3 m.

RLA with Racetrack or Droplets

A combination of the Recirculating Linear Accelerators (RLA) with NS-FFAG represent a very competitive way to achieve the rapid acceleration of short-lived muons to multi-GeV energies, required for Neutrino Factories and TeV energies required for Muon Colliders. The competitiveness is due to better "compaction" with respect the NS-

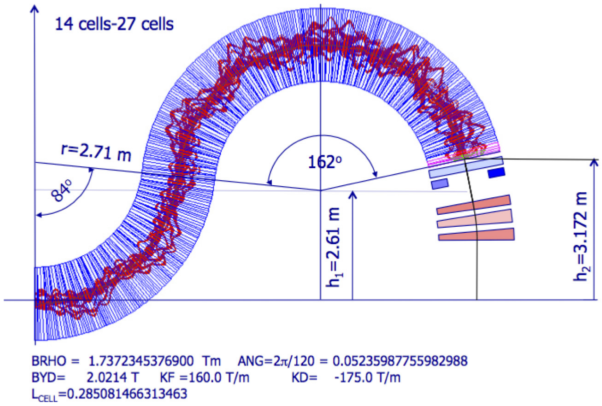

Figure 3: Proton isocentric gantry designed with the permanent Halbach magnets [12].

FFAG rings. The ”droplet” is constructed by a 60^o outward bend, a 300^o inward bend and another 60^o outward bend so that the net bend is 180^o. This arc has the advantage that if the outward and inward bends are made up of the same kind of cells, the geometry automatically closes without the need for any additional straight sections, thus making it simpler. An additional advantage is a possibility to transport different energy muons of both charges μ^+ and μ^- through the same arc structure. Difficulties arise from dependence on momentum: of the orbit offsets (they need to be removed before arriving to linac), of the time of flight through the arcs and of the beam size and amplitude functions. Two types of designs the racetrack and the ”droplet” are shown. The racetrack layout with betatron function and dispersion at the central energy is shown in Fig. 4 . The muon orbits at momentum in a range of $\delta p/p = \pm 60\%$, are shown in Fig. 5. Two linacs are placed opposite to each other.

The second ”droplet” solution, shown in Fig. 6 without linac and matching cells, allows use of both charges μ^+ and μ^-.

RLA for Electron Ion-Colliders

The future relativistic electron hadron collider requires acceleration of electrons with multiple passes through the linac. The same principle is used in the Jefferson Laboratory accelerator. There the multiplearcs are connected to the linacs in the racetrack. For the electron ion colliders in Relativistic Heavy Ion Collider (eRHIC) the linacs could be placed in the two RHIC tunnel straight sections, while arcs could follow the existing RHIC superconducting heavy ion accelerator. Multiple arcs can be replaced with a two NS-FFAG arcs allowing momentum range of $\Delta p/p = \pm 60\%$ or energy range 7.5-30 GeV and 1.875 - 7.5 GeV. More details of this design were presented elsewhere [15].

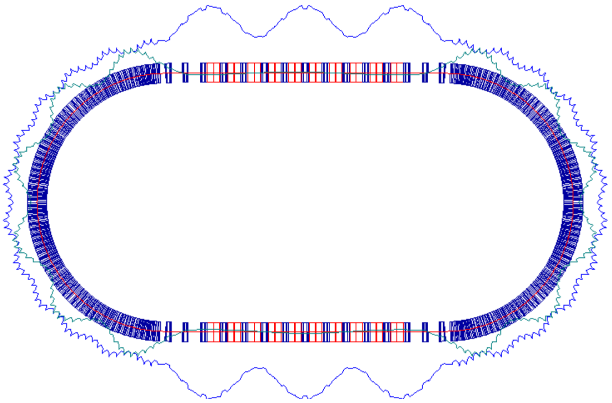

Figure 4: Matching at central muon energy of 6.8 GeV and maximum energy of 10 GeV for $\Delta p/p = \pm$ 60 % with minimum energy of 2.5 GeV, the β_x -(blue color) and dispersion (green color) are shown with magnifications of 5 and 15 respectively.

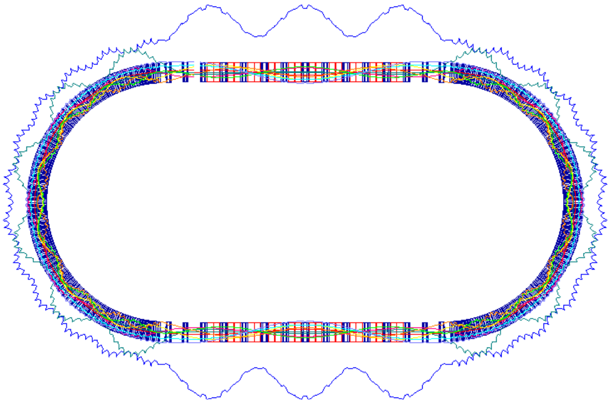

Figure 5: Orbits magnified 100 times at $\Delta p/p = \pm$ 60 % with β_x and dispersion.

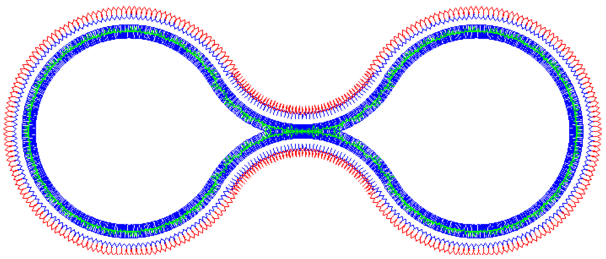

Figure 6: Betatron functions magnified 10 times at -40% $\leq \delta p/p \leq$60 % with β_x and dispersion. Total length of 612.5 m, maximum energy of 10 GeV.

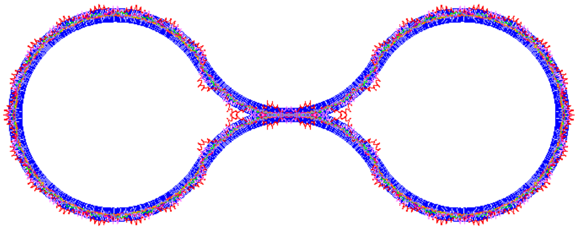

Figure 7: Orbits magnified 100 times at -40% $\leq \delta p/p \leq$60%.

Muon Acceleration with Triplet NS-FFAG

The basic cell of the muon accelerating ring is shown in Fig. 1. The acceleration in thirteen turns has been simulated and previously presented [8].

Pion Storage Rings for the Phase Rotation

A recent modification of the project PRISM is a racetrack solution with the S-FFAG [13]. The same problem could be solved with the NS-FFAG [14]. A very large aperture for the magnets is required and the time of pions spending during the phase rotation is very limited $\sim$2 μs. The longer straight section is required for extraction and injection and this is the main reason to apply the racetrack solution like the one shown Fig. 4. The dispersion function is much larger than in examples shown above as the periodic cell number is much smaller.

NS-FFAG WITH A LARGE NUMBER OF BEAM PASSES

Other possible applications of the NS-FFAG is accelerating of the non-relativistic ions: proton/carbon cancer therapy accelerators, a proton driver for muon production or an Accelerator Driven Subcritical nuclear Reactor (ADSR), and acceleration of radioactive heavy ions [6]. Tune and chromaticity variation with energy present a very serious problem as hundreds of turns are required due to the limits of how fast the RF frequency or phase can change during a short time. The non- relativistic acceleration of the proton or light ions with the NS-FFAG has to be fast enough to avoid emittance or beam amplitude blowup due to crossing integer resonances. The amplitude growth during the resonance crossings depends on errors in the magnetic field and on the speed of the resonance crossing. For the magnetic field and alignment errors smaller than $dB/B \leq 10^{-3}$ or for a ring of a circumference 26 m $\Delta x, y \leq 20 - 40 \mu m$ amplitude growth [16] is tolerable. It is very clear that the fast acceleration is a must, not only to avoid the emittance growth due to resonance crossings, but also to make faster repetition rate producing more particles for the same time. This is especially important for the ADSR application as it would be a major advantage with respect to synchrotrons where presently maximum repetition rate is of the order of 60 Hz compared to the 1 kHz. Novel methods for very fast acceleration are necessary. Few solutions have been found like phase jump [16] or the harmonic jump method [17] but research in this field is still necessary.

NS-FFAG for the Proton Cancer Treatment

The proton cancer treatment facilities today typically use cyclotrons as they are compact and reasonable priced. The application of the NS-FFAG for the 250 MeV proton acceleration with a single turn extraction for the required energy could provide many advantages with respect to the cyclotrons or synchrotrons. The cyclotrons have a fixed proton energy requiring to use degraders to obtain the required energy for a treatment. This produces radiation and unavoidable emittance blow up of the beam. Both synchrotrons and NS-FFAG have advantage with respect to the cyclotrons as the final energy is adjustable. The NS-FFAG would have an advantage with respect to the fast cycling synchrotron due to a higher repetition rate of 1 kHz as the magnetic field is fixed. Two examples for the proton cancer therapy machine are presented: one with a combined function magnets with 26.88 meter circumference [18] and the other with permanent Halbach separated function magnets. The acceleration is assumed to be with the phase jump and fixed frequency 374 MHz. The ring with accelerating cavities (12 green lines), injection and extraction kickers (red boxes), and doublet combined function magnets (blue trapezoids) is shown in Fig. 8.

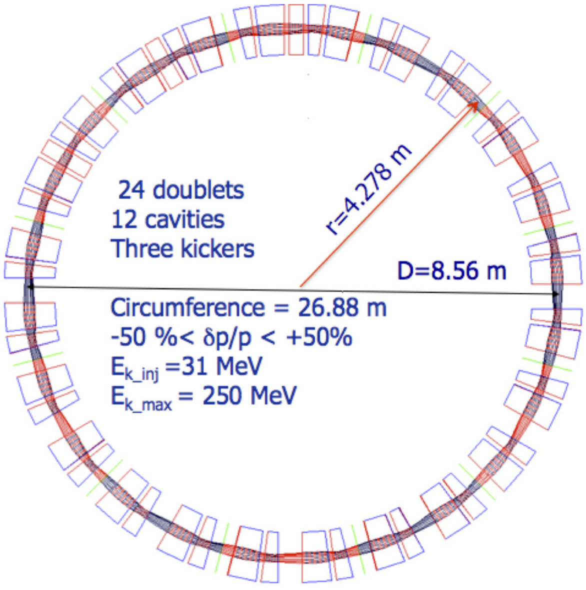

Figure 8: NS-FFAG for proton cancer therapy.

The second example is a racetrack NS-FFAG made of Halbach magnets. There are sixty cells in each arc. The large number of cells produces very strong focusing and reduces the maximum orbit offset to 16.8 mm. This allows use of very small magnets [12]. The racetrack is shown in Fig. 9.

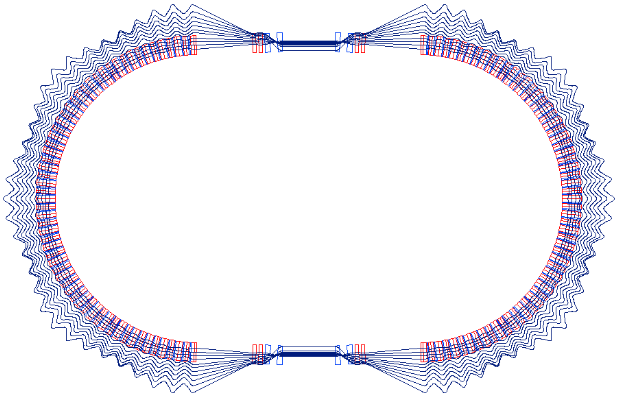

Figure 9: NS-FFAG racetrack made of Halbach separated function magnets. The radius of each arc is 2.85 m, the length with the two straight sections is 26.8 m.

NS-FFAG for 1 GeV Proton Acceleration

A principle of proton therapy accelerator described in the previous chapter is applied for the 1 Gev NS-FFAG accelerator but using combine function superconducting magnets with both magnets bending in the same direction for the central momentum. The defocusing magnet is L_d=16 cm long with the bending field of B_d= 2.4 T and gradient of G_d=130 T/m. The maximum orbit offsets at the middle of the defocusing magnet are 25 mm. The focusing magnet is 18 cm long with a bending field of B_d= 0.85 T and a focusing gradient of G_f=120 T/m. A radius of the arc is 3 m while a length of the racetrack is 26 m.

SUMMARY

Applications of the NS-FFAG are divided in two categories: first where only one or few passes are needed and the second where few hundred turns are necessary mostly for the case of accelerating non-relativistic particles. Examples from the first category like the carbon/proton cancer therapy gantries, the racetrack and droplet solutions for muon acceleration with RLA's are shown. In the second category the ring and racetrack examples of the NS-FFAG for the proton cancer therapy are shown. Other examples are just briefly discussed due to a limited space.

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