HIGH INTENSITY PROTON FFAG RING WITH SERPENTINE ACCELERATION FOR ADS

E. Yamakawa*
Graduate school of Engineering, Kyoto University, Japan
J.-B. Lagrange, T. Uesugi, Y. Kuriyama, Y. Ishi, Y. Mori
Kyoto University Research Reactor Institute, Kumatori, Japan

Abstract

In order to produce high intensity proton beam for ADS, a new type of fixed rf acceleration scheme, so-called serpentine acceleration, is examined in scaling FFAG. Longitudinal hamiltonian for scaling FFAG is first derived analytically. Then the features of serpentine acceleration in longitudinal phase space are studied. Ring design for ADS is finally shown.

INTRODUCTION

High beam power accelerator to produce intense secondary particle beams are desired for Accelerator Driven System (ADS) [1]. Linear accelerators have been considered as a proper candidate so far. An alternative candidate is Fixed-field alternating gradient (FFAG) accelerator [2]. There are two types of FFAG; non-scaling type and scaling type. Scaling FFAG ring is composed of non-linear magnetic fields so that the betatron tune is constant for every particle momentum, contrary to the non-scaling FFAG accelerator.

In order to obtain large current beam, in FFAG accelerators, the acceleration scheme with fixed rf frequency has been proposed. In scaling FFAG, the stationary bucket acceleration [3, 4] has been considered. In this scheme, however, the total acceleration energy gain is limited by the bucket height. In order to make a large bucket height, the acceleration in the relativistic energy region is preferable. On the other hand, in non-scaling FFAG, to minimize orbit shifts during acceleration, the parabolic variation in orbit length with energy is created by the appropriate selection of parameters. At the bottom of the parabola, the momentum compaction approaches zero. Furthermore, for relativistic particles with the parabolic variation in orbit length, time of flight is also approximately parabolic. Therefore, the decreased variation in orbital period allows to operate a fixed rf frequency acceleration scheme with an appropriate selection of rf frequency and a high enough rf voltage. This new type of fixed rf frequency acceleration scheme is called serpentine acceleration [5, 6].

For both types of FFAG accelerators, only relativistic energy particles are suitable for a fixed rf frequency acceleration. However, if serpentine acceleration is applied to scaling FFAG, high-power beam can be also obtained even in the non-relativistic energy region. In this paper, the longitudinal hamiltonian in scaling FFAG with fixed rf frequency is derived analytically first [7]. Then the preliminary ring design of proton driver for ADS based on serpentine acceleration is also presented.

* yamakawa@post3.rri.kyoto-u.ac.jp

LONGITUDINAL HAMILTONIAN IN SCALING FFAG WITH FIXED RF FREQUENCY

In cylindrical coordinates, the magnetic field in scaling FFAG in the mid-plane has the form:

$$B_z(r, z=0) = B_0\left(\frac{r}{r_0}\right)^k, \tag{1}$$

where r is the radial coordinate with respect to the center of the ring, B_0 is the magnetic field at r_0, k is the geometrical field index, and z is the vertical coordinate. The closed orbits for different momenta P are given by

$$r = r_0\left(\frac{P}{P_0}\right)^{\frac{1}{k+1}}, \tag{2}$$

where r_0 is the radius of the closed orbit at the momentum P_0.

In longitudinal particle dynamics with constant rf frequency acceleration in the scaling FFAG, the phase discrepancy $\Delta\phi$ per revolution is written by

$$\Delta\phi = 2\pi(f_{rf}\cdot T - h), \tag{3}$$

where h is the harmonic number, f_{rf} is the rf frequency and T is the revolution period of a non-synchronous particle. Then Eq. 3 becomes

$$\frac{\Delta\phi}{2\pi} = \frac{hT}{T_s} - h, \tag{4}$$

where T_s is the revolution period of a synchronous particle. Equation 4 is also expressed with another description based on Eq. 2 as follows;

$$\begin{aligned}\frac{T}{T_s} &= \frac{r}{r_s}\Big/\frac{P/E}{P_s/E_s} \\ &= P_s^{1-\alpha}\frac{E}{E_s}P^{\alpha-1},\end{aligned} \tag{5}$$

where r_s is the reference radius, α is the momentum compaction factor and E_s is the reference energy at the reference radius r_s. Combining Eq. 4 and Eq. 5, the phase difference $\Delta\phi$ becomes

$$\Delta\phi = 2\pi h\left[\frac{P_s^{1-\alpha}}{E_s}EP^{\alpha-1} - 1\right]. \tag{6}$$

ISBN 978-3-95450-118-2

Assuming that the energy gain is uniformly distributed around the ring, we can approximate $\Delta\phi/2\pi$ by $d\phi/d\Theta$ to derive the phase and energy equations of motion,

$$\begin{aligned} \frac{d\phi}{d\Theta} &= h\left[\frac{(E_s^2-m^2)^{\frac{1-\alpha}{2}}}{E_s}E(E^2-m^2)^{\frac{\alpha-1}{2}}-1\right] \\ \frac{dE}{d\Theta} &= \frac{eV_0}{2\pi}\sin\phi, \end{aligned} \quad (7)$$

where m is the rest mass, V_0 is the rf peak voltage per turn and Θ is the azimuthal angle in the machine. We introduce the energy variable E canonically conjugate to the coordinate variable ϕ. Then Eq. 7 derive the longitudinal hamiltonian:

$$H(E,\phi;\Theta) = h\left[\frac{1}{\alpha+1}\frac{(E^2-m^2)^{\frac{\alpha+1}{2}}}{E_s(E_s^2-m^2)^{\frac{\alpha-1}{2}}}-E\right]+\frac{eV_0}{2\pi}. \quad (8)$$

Longitudinal Phase Space in Non-relativistic Energy Region

When the transition energy is fixed in non-relativistic energy region and rf frequency is fixed near the transition energy, serpentine channel between the two stationary buckets appears in non-relativistic energy region as shown in Fig. 1.

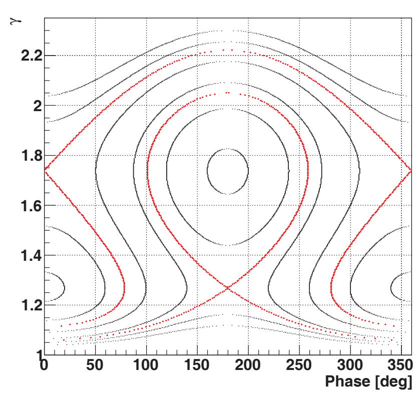

Figure 1: Longitudinal phase space in non-relativistic energy region. Red lines are the separatrixes, and γ is the Lorentz factor.

PROTON RACETRACK RING DESIGN FOR ADS

The accelerator for ADS is required to make proton beam up to several tens of MW power with over 1 GeV beam energy. Furthermore, a low-injection energy around 200 MeV is desired. From the study of serpentine acceleration in longitudinal motion so far, in order to make injection energy lower, small k-value is required. Small harmonic number is also desired for decreasing rf voltage per turn. Since the circumference of the ring should be small to satisfy these requirements, long drift spaces to put many rf cavities are considered. In this case, focusing elements have to be installed in a long straight section to keep the beam stable [8]. Since the field laws in circular and straight sections are different, discontinuity of reference trajectories can occur at the border between these two sections. In order to combine the reference trajectories at the border, dispersion functions in circular and straight sections must be matched at one momentum P_0 [9].

In order to satisfy the zero-chromaticity in race track ring, ring tune needs to be constant for different momenta. Once geometrical configuration is given in straight section, horizontal and vertical tunes in straight section are independent of momentum. Then if horizontal and vertical phase-advance are satisfied with $n\times\pi$ (n is integer) in the circular part, ring tunes of horizontal and vertical are also constant.

In this section, preliminary proton race track ring design for ADS is presented analytically in longitudinal, and then in transverse with linear transfer matrix method.

Longitudinal Design

The phase equation with long straight sections can be derived as

$$\frac{d\phi}{d\Theta} = h\left[\left(\frac{1}{1+\frac{L}{2\pi R_s}}\right)\frac{EP_s}{E_sP}\left(\frac{P^\alpha}{P_s^\alpha}+\frac{L}{2\pi R_s}\right)-1\right], \quad (9)$$

where L is the total length of straight sections. From Eqs. 3 and 9, the longitudinal Hamiltonian with long straight sections can be derived as

$$\begin{aligned} &H(\phi,E,\Theta) = \\ &h\left[\frac{1}{(1+\frac{L}{2\pi R_s})}\left(\frac{1}{1+\alpha}\frac{\sqrt{E^2-m^2}^{1+\alpha}}{E_s\sqrt{E_s^2-m^2}^{\alpha-1}}\right.\right. \\ &\left.\left.+\frac{L}{2\pi R_s}\frac{\sqrt{E_s^2-m^2}}{E_s}\sqrt{E^2-m^2}\right)-E\right]+\frac{eV}{2\pi}\cos\phi. \end{aligned} \quad (10)$$

The longitudinal parameters of proton race track ring for ADS are presented in Tab. 1. The resulting longitudinal phase space is shown in Fig. 2. The initial energy is 200 MeV, and the final energy is over 1 GeV.

Table 1: Longitudinal Parameters for the Proton Race Track Ring

k-value at circular section	0.01
Mean radius of the arc part (200 MeV) [m]	1.6
Long straight section [m]	12
Stationary energy below transition[MeV]	820
rf voltage [MV/turn]	20 (h=1)
rf frequency [MHz]	5.45 (h=1)

Transverse Design

The first step is to determine the characteristics of the closed orbit for given geometrical parameters of the cell.

ISBN 978-3-95450-118-2

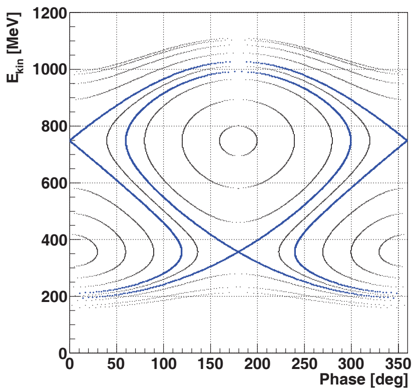

Figure 2: Longitudinal phase space with hamiltonian contours. Blue lines indicate separatrixes.

For simplicity reasons, we assume that the curvature of the closed orbit for the same momentum is constant in each magnet, and null in the drift spaces between magnets. The denomination of the closed orbit parameters in the FDF arc section and the DFD straight section are drawn in Fig. 3 and Fig. 4, respectively. A schematic view of half of the ring is also drawn in Fig. 5.

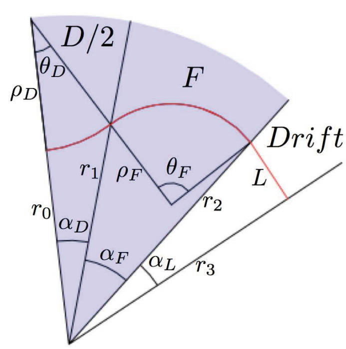

Figure 3: Schematic view of half FDF cell in the arc section.

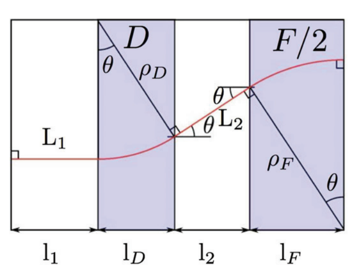

Figure 4: Schematic view of half DFD cell in the straight section.

The relative strength x between F and D magnets is defined as $x = \theta_F/\theta_D$ in the arc section. From the geometrical consideration in Fig. 3, the unknown parameters: θ_F, θ_D, ρ_D, ρ_F, and L can be expressed with x, α_F, α_D, α_L,

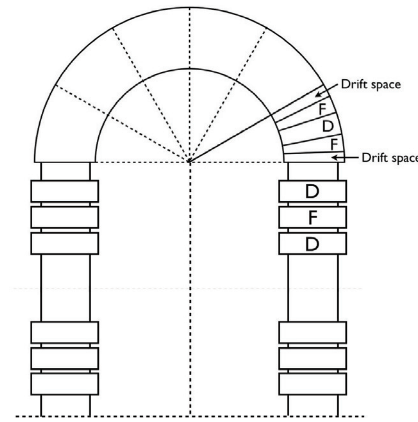

Figure 5: Schematic view of half of the ring. The ring is composed of circular sections FDF cells, and long straight sections DFD cells.

and r_1. In the straight section, the unique solutions ρ_F, ρ_D, and θ can be obtained from known parameters: l_F, l_D, l_1 defined in Fig. 4.

In the circular section with linear approximation, the field index n can be written with the geometrical filed index k as $n = \pm\frac{\rho}{r}k$ where the $+$ and $-$ signs corresponding to the case of D and F magnets, respectively. The normalized field gradient m in the straight section is defined as $m = \frac{1}{B}\frac{dB}{d\chi}$ where χ is average abscissa and B is the magnetic field. In the straight section with linear approximation, the field index n can be written with the normalized field gradient m as $n = \pm m\rho$ where the $+$ and $-$ signs corresponding to the case of D and F magnets, respectively [9].

Horizontal and vertical beta-functions are obtained from linear transfer matrix. Matrix elements are calculated with the closed orbit geometrical parameters and the field index. The resulting horizontal and vertical beta-functions are shown in Fig. 6. Tune diagram is presented in Fig. 7, and parameters for the proton race track ring are expressed in Tab. 2.

SUMMARY

In order to obtain high power proton beam for ADS, serpentine acceleration has been proposed for the scaling FFAG. First, longitudinal hamiltonian in scaling FFAG with fixed rf frequency has been derived analytically. Then, we have confirmed that serpentine acceleration in scaling FFAG can be worked in non-relativistic energy region. Preliminary ring design of proton race track ring for ADS has been done with linear transfer matrix method. With this

ISBN 978-3-95450-118-2

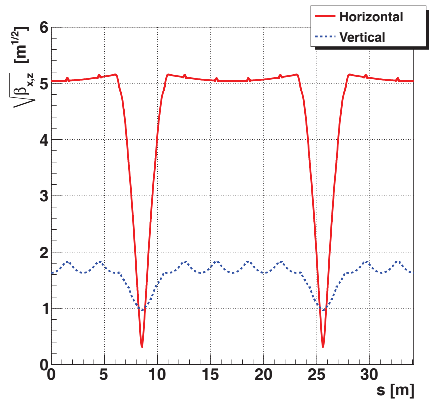

Figure 6: Horizontal (red) and vertical (blue) beta functions for the proton race track ring.

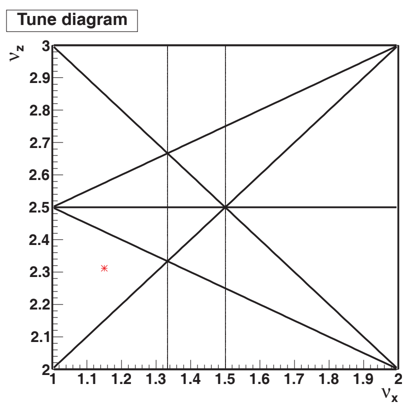

Figure 7: Tune diagram for the proton race track ring.

Table 2: Geometrical Parameters for the Proton Race Track Ring

Circular section:FDF cells	
k	0.01
Number of cells per arc section	6
α_F and α_D [deg]	12.2, 1.8
α_L [deg]	1.0
θ_F and θ_D [deg]	17.7, 2.73
Mean radius (200 MeV) [m]	1.6
ρ_F and ρ_D (at 200 MeV) [m]	1.11, 1.05
Horizontal and vertical phase advance per cell [deg]	30.2, 30.0
Straight section:DFD cells	
Number of cells per straight section	4
l_F and l_D [m]	0.10, 0.10
l_1 and l_2 [m]	1.25, 0.05
θ [deg]	5.22
ρ_F and ρ_D [m]	1.10, 1.10
Horizontal and vertical phase advance per cell [deg]	3.05, 65.5
Circumference (at 200 MeV) [m]	34.1
Horizontal and vertical ring tune	1.15, 2.31

machine, maximum length to put rf cavities is 2.5 m in the long straight section. Further studies are needed to evaluate more precisely serpentine acceleration scheme in scaling FFAG. Not only transverse dynamics but also detailed technical design of rf cavity to produce big rf voltage is necessary. Furthermore, to make a small ring size, superconducting magnet design in scaling FFAG is also important issue to make realize a proton driver for ADS with serpentine acceleration in scaling FFAG.

REFERENCES

[1] K. Mishima et al, *Journal of Nuclear Science and Technology*, vol.44(2007), NO3 Special Issue on GLOBAL 2005 p.499-503.

[2] C. Ohkawa, in *Proc. of JPS*, (1953)

[3] Y. Mori, *International Workshop on FFAG Accelerators(FFAG2006)*, FNAL, Chicago, USA(2006)

[4] T. Planche, E. Yamakawa, T. Uesugi, J.-B. Lagrange, Y. Kuriyama, K. Okabe, Y. Ishi and Y. Mori, *Nuclear Inst. and Methods in Physics Research, A*, vol.622, No.1, p.21-27 (2010)

[5] R. Barlow et al, *Nuclear Inst. and Methods in Physics Research, A*, vol.624, No1, p.1-19 (2010)

[6] S. Machida et al, *Nature Physics*, 8, pp. 243-247, (2012)

[7] E. Yamakawa , T.Uesugi and Y.Mori, *International Journal of Modern Physics A*, Vol.26, p1685-1872, (2011).

[8] J.-B. Lagrange, T. Planche, E. Yamakawa, T. Uesugi, Y. Ishi, Y. Kuriyama, B. Qin, K. Okabe and Y. Mori, *Nuclear Inst. and Methods in Physics Research A*, vol. 691, pp. 55-63 (2011)

[9] J.-B. Lagrange, T. Planche, and Y. Mori, *International Journal of Modern Physics A*, vol. 26, pp. 1785-1793 (2011)

ISBN 978-3-95450-118-2