# THE STUDY ON MEASURING BETA FUNCTIONS AND PHASE ADVANCES IN THE CSNS/RCS

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*Abstract*

As a key component of the China Spallation Neutron Source (CSNS) Project, the Rapid Cycling Synchrotron (RCS) will accumulate and accelerate the proton beams from 80MeV to 1.6GeV for extracting and striking the target with a repletion rate of 25Hz. To check linear optics and locate the quadruple errors, beta function plays an important role in beam diagnostics of a particle accelerate system. The Independent Component Analysis (ICA) is a robust beam diagnosis method by decomposing the samples recorded by turn by turn BPMs (beam position monitors) into the independent components which represent the inherent motion of the beam. The beta functions and phase advances can be derived from the corresponding independent components. Because the linear part of the space charge gives a defocusing effect to the beam, beta function variation will be induced. We find that the ICA method can measure beta functions with a reasonable tolerance under the conditions of strong space charge effects.

## INTRODUCTION

The CSNS accelerator consists a low energy H- Linac and high energy RCS. A 4-fold symmetry structure with 16 triplet cells is adopted for the lattice design of RCS. In one super-period, there are 8 turn-by-turn beam position monitors (BPMs) located near quadruples for better understanding the beta functions of the RCS Lattice, and the main parameters of RCS are shown in Table 1 [1].

Table 1: Main Parameters of RCS

| Parameters | Units | Values |
|---|---|---|
| Circumference | m | 227.92 |
| Inj. Energy | MeV | 80 |
| Ext. Energy | GeV | 1.6 |
| Repetition Rate | Hz | 25 |
| Average current | µA | 62.5 |
| Quadruples | | 48 |
| Dipoles | | 24 |
| Nominal Tunes(H/V) | | 4.86/4.78 |
| Maximum β | m | 12/26 |
| BPM | | 32 |

Measuring beta functions is a critical issue for many synchrotrons to understand the linear Optics of the Lattice. With turn-by-turn BPMs installing in lots of synchrotrons, getting the beta functions from the samples recorded by turn-by-turn BPMs is becoming more and more popular. Independent components analysis (ICA) is a data mining method that can provide the independent components (also called ICs) which represent the inherent property of the samples data. ICA is first introduced in the accelerator by the team of Professor S.Y. Lee [2] and then applied to many accelerators.

## DATA ACQUISITION

In order to get the samples from turn-by-turn BPMs, RF kicker or pinger is often used to excite betatron oscillation. The beam emmittance will be diluted in the case of pinger, because pinger is a pulsed magnet and the field variation is non-adiabatically. However, the field of RF kicker can be ramped up or down with a sine curve slowly [3], and the beam emmitance can be preserved.

Let's define $\omega_0$ is the orbital angular frequency, and $\omega_m$ is the RF kicker frequency, and $\nu_m = \omega_m / \omega_0$ as the modulation tune. When the fractional part of the modulation tune equals that of tune of the transverse motion, or the sum of two fractional parts equals one, the coherent betatron oscillation can be excited. Figure 1 shows the driven oscillation excited by RF Kicker with the amplitude of 0.2mrad. The RF kicker is switched off when the beam oscillation reaches proper amplitude, and the data of the free oscillations are stored for ICA processing. The first 200 turns is the rise time, and the latter 800 turns is free oscillation after excitation. The RF kicker works at a repetition rate of 449 kHz, which means $\nu_m$=0.14, corresponding to the beam angular frequency of 3.212 MHz in the beam energy of 80 MeV.

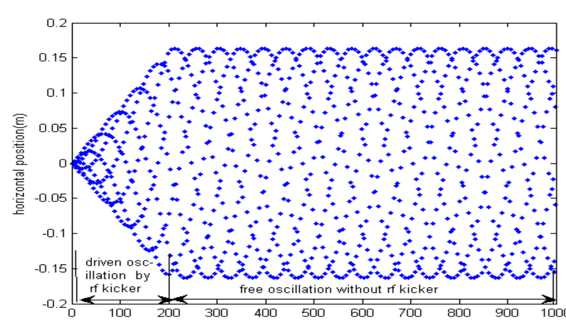


Figure 1: Driven betatron oscillation with RF kicker and free betatron oscillation without RF kicker.

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## SAMPLES DECOMPOSITION

The samples obtained from turn by turn BPMs can be recorded in the vector $\mathbf{x}(t)=[x_1(t),\cdots,x_m(t)]^T$, and assumed to be generated by

$$\mathbf{x}(t)=\mathbf{A}\mathbf{s}(t)+\mathbf{N}(t)\,, \qquad (1)$$

where $\mathbf{A}\in\Re^{m\times n}$ is the mixing matrix with $m\geq n$ (n is the row index of the estimated matrix) , and **N**(t) is the noise matrix with the same size of samples **x**(t).

Many data technologies can be used to extract the ICs from the samples according the character of the source signal (or called ICs). Second order blind source separation (SOBI) [4] algorithm is more robust when applying to the samples recorded by turn-by-turn BPMs. The source signals are assumed to be mutually independent and temporally correlated. Consequently the covariance matrix is diagonal. The samples are the linear combination of the independent components, so the covariance matrix of the samples is not diagonal. In order to get the relationship between the independent components and the samples, Singular Value Decomposition (SVD) method can be used to extract the independent components from samples because the covariance matrix of the samples can be decomposed into the product of the unitary matrix and the diagonal matrix, and the diagonal matrix is just the covariance matrix of the independent components. Because the unitary matrix is not always unique in the process of the SVD, a proper unitary matrix needs to be figured to be multiplied to the former unitary matrix to make the covariance matrix of the extracted independent components more diagonal. A time-lagged factor is always used in the process of estimating the mixing matrix and the independent components matrix, and the samples covariance matrix can be decomposed, as

$$\mathbf{C_X(\tau)}=\mathbf{A}\mathbf{C_s(\tau)}\mathbf{A}'\,, \qquad (2)$$

where $C_X(\tau)\equiv\langle X(t)X(t+\tau)'\rangle$ and $C_s(\tau)\equiv\langle s(t)s(t+\tau)'\rangle$, represent the covariance matrix of the samples and independent signals with time-lagged factor τ respectively, and prime means transposition. Finally, the independent components **s** and mixing matrix **A** can be written as [2]

$$\mathbf{s}=\mathbf{W}'\mathbf{\Lambda}_1^{-1/2}\mathbf{U}_1'\mathbf{X}\,, \qquad (3)$$

$$\mathbf{A}=(\mathbf{\Lambda}_1^{-1/2}\mathbf{U}_1')^{-1}\mathbf{W}\,, \qquad (4)$$

where $\mathbf{\Lambda_1}$ and $\mathbf{U}_1$ represent the singular value matrix and unitary matrix in the SVD process of $\mathbf{C_x(0)}$ with proper truncation respectively.

## CALCULATION OF BETA FUNCTIONS AND PHASE ADANCE IN CSNS/RCS

The samples recorded by turn by turn BPM include betatron motion, synchrotron motion, and other sources that can effects the beam oscillation. There are 32 horizontal BPMs and 32 vertical BPMs accommodated in the RCS. To get coherent betatron oscillation and synchrotron oscillation, 0.015m (H)/0.015m (V) are set as initial amplitude of beam oscillation, while the noise level is set to 0.1mm. The RF cavity is also included in the simulation. By using the Accelerator Toolbox (AT) [5], the turn by turn BPM tracking data were obtained.

In order to get the beta functions and phase advance, the transverse mode should be focused. Because the independent source signals reflect the coherent motion of the beam, the frequencies of the independent source signals are the fractional part of the transverse tunes. Figure 2 shows the frequency spectrum of the independent transverse source signals in the transverse plane. S1 and S2 correspond to the horizontal motion, while S3 and S4 correspond to the vertical motion.

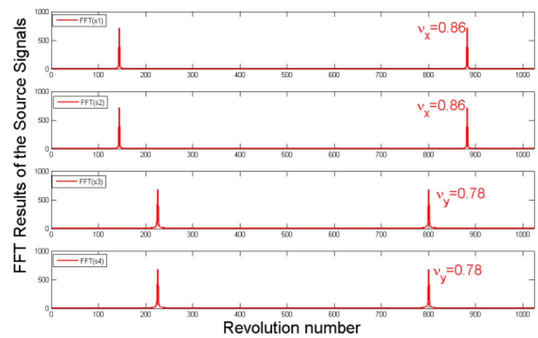


Figure 2: The frequency spectrum of the independent source signals in the transverse plane.

The beta function and phase advance can be derived from the mixing matrix of the transverse function from Eq.4.

$$\beta_x=a(A_{s1}^2+A_{s2}^2),\qquad \varphi_x=\tan^{-1}(A_{s1}/A_{s2})\,, \qquad (5)$$

where a is a constant scale number should be fixed. The constant $a$ equals $2*J$ ($J$ is the action of the beam in Hamiltonian system) in a linear conservative system, and can be scaled from the model

$$\sum_{i=1}^{M}\frac{1}{\beta_{i,ICA}}=\sum_{i=1}^{M}\frac{1}{\beta_{i,Modle}}. \qquad (6)$$

Figure 3 represents beta functions and phase advance from ICA calculation and that provided by model. The value calculated from ICA technology agrees well with model.

We also note that the accuracy between ICA and model decreased with the noise level growth. However, the goodness of fit is still in the range of 1% even the noise level reached 2mm, and that was a satisfied result for CSNS/RCS.

## SPACE CHARGE EFFECTS TO THE BETA FUNCTIONS

When a charged particle travels around an accelerator, it feels the force of the neighbouring particles that travel in the same direction, which is called space charge effects, and the actual motion of the particles will be modified.



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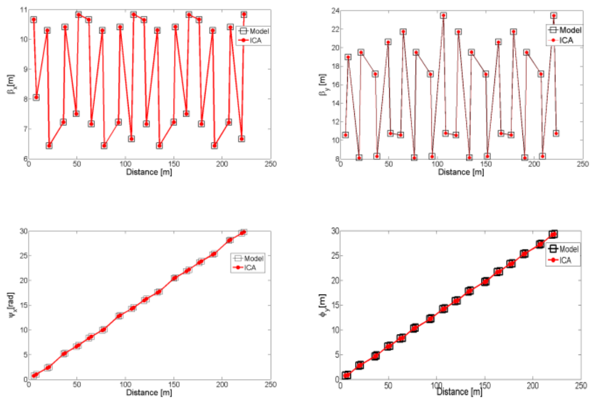


Figure 3: Top: The beta functions (left belongs to horizontal and right belongs to vertical) from ICA calculation and provided by Model. Bottom: The phase advance (left belongs to horizontal plane and right belongs to vertical plane) from ICA calculation and provided by model.

The dominant region of the space charge is the linear part, and that can be equalled by an element that has the defocusing effects in both of the transverse plane, horizontal and vertical. In our simulation, 4 space charge elements were added to the lattice of CSNS/RCS, and the beta functions were modified. Figure 4 shows the location of the space charge elements which are accommodated uniformly along the ring.

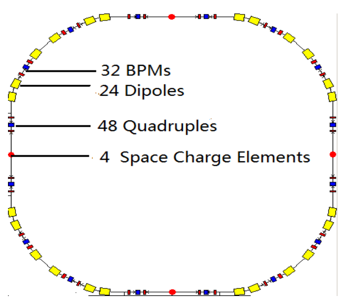


Figure 4: The location (red dot) of the space charge elements in the lattice of CSNS/RCS.

The phase advance between two neighbored BPMs can be calculated by ICA accurately even in the circumstance of strong space charge, because the phase advance between two neighboring BPMs is just related to the ratio of the corresponding vector of the mixing matrix. However, the beta functions scale factor will be changed because the action of the beam is different with that without space charge. In our simulation, the length of the four space charge elements equals 0.01m, and the strength of that element varies uniformly from 0.01 to 1 m$^{-2}$ in the step of 0.01, and the tune decreased from 4.86/4.78 to 4.66/4.58. From Eq.5 and Eq.6, the beta-beat from ICA calculation and model can be obtained. Figure 5 shows the relationship of the beta-beat between ICA calculation and model.

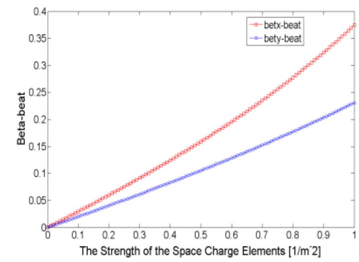


Figure 5: Beta-beat between ICA calculation and model. The red square and blue dot represent horizontal and vertical beta-beat respectively.

From Figure 5, we note that the beta-beat growth as the strength of the space charge elements increased. Because the phase advance can be fixed by ICA technology accurately, we refer an equation empirically to fix the scale constant [6]

$$\beta_1 = \beta_1^m \sqrt{\frac{\beta_2 / \beta_1}{\beta_2^m / \beta_1^m} \frac{\sin \phi_{12}^m}{\sin \phi_{12}}}, \qquad (7)$$

where the superscript ‘m’ represents the model. And the beta functions obtained from ICA are shown as Figure 6.

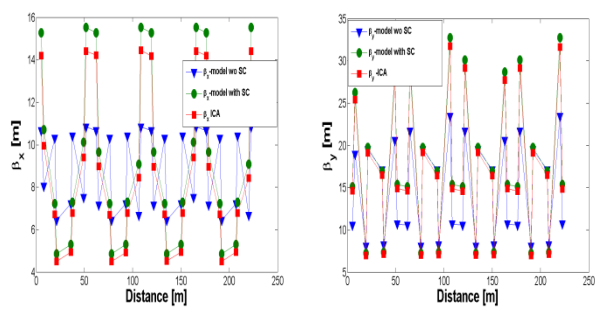


Figure 6: The beta functions comparison between model and ICA calculation.

## CONCLUSION

ICA technology is a robust beam diagnosis method when applying at CSNS/RCS. In order to get rid of the beam emmittance dilution, the RF kicker was introduced to excite the coherent betatron motion. The beta functions can be obtained precisely even if the random noise level reached 2mm. However, when the space charge is strong enough, the beta function calculated from ICA varied much from the model. Fortunately, we found an empirical equation to fix the constant scale factor of the beta functions, and the results seemed satisfied.


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