RESISTIVE WALL INSTABILITY IN CSNS/RCS*

Liangsheng Huang, Sheng Wang#, Y. D. Liu, IHEP, Beijing, P. R. China.

Abstract

Rapid Cycling Synchrotron (RCS) of the China Spallation Neutron Source (CSNS) is a high intensity proton accelerator, with average beam power of 100kW. The collective effects caused by the coupling impedance may be the limit to beam power. The impedance estimation for components on beam line shows that the resistive wall impedance and its instability are more serious than any others. Based on the impedance budget, the instability is theoretically estimated. A simple resistive wall wake field model is used to simulate the bunch oscillation and obtain the growth time. In this simulation model, the continuous resistive wall wake field is concentrated to one position in the ring and the long bunch is sliced into many micro-bunches. By tracking the dynamics of the macro-bunches, the transverse growth time are obtained. The simulation results are also confirmed the restriction to instability by natural chromaticity.

INTRODUCTION

The China Spallation Neutron Source (CSNS) accelerators consists of an H^- Linac and a proton Rapid Cycling Synchrotron (RCS), beam transportations, a target station, spectrometers. RCS is designed to accumulate and accelerate proton beam from 80MeV to 1.6GeV. The extracted 100 KW beam strike the neutron target with repetition rate of 25Hz. Due to the high beam intensity, the ratio of beam loss must be controlled to a very low level. The RCS lattice adopt a triplet cell based 4-fold structure[1]. The time of movement for every pulse is about 20 milliseconds.

Considered the upgrading requirement, the impedance budget needs to be meeting the requirement of higher beam current for 500kW beam. Corresponding to 500kW beam power, the physics parameters should be more flexible on the phase of CSNS design. The impedance and instability need to be restricted seriously. With beam current in upgrades, resistive wall instability will be an important commission limitation for CSNS/RCS. Table 1 listed the main parameters of CSNS/RCS.

Table 1: Main Parameters of CSNS/RCS

Parameters	**Values**
Circumference/m	227.92
Inj. energy/MeV	80
Ext. energy/GeV	1.6
Stainless steel length /m	140
Average beta function(H/V)/m	9.5/10.5
Nominal tunes(H/V)	4.86/4.78
Natural chromaticity(H/V)	-4.64/-8.27
Bunch number	2
Particles per bunch	7.8x1012

Studies on beam instability due to impedance are the vital work in accelerator physical design, so the impedance calculation and control on vacuum components is one of the important challenges in CSNS project. Impedances of RCS mainly come from resistive wall, RF cavities, and collimators. The estimated impedance values were summarised in Table 2. It is clear that resistive wall is much more serious.

The beam loss in RCS caused by resistive wall may be one of the limits to reach the design beam power. Especially, at the low-energy end of each cycle, the resistive wall instability is much more serious. The detailed study on resistive wall instability in RCS is necessary to the design of accelerator.

According to the analytical formula and impedance model on resistive wall instability, the instability is estimated. Them a simple simulation model is introduced and with the tracking method on macro-particles, the bunch oscillation is simulated. Based on the analytical and simulation results, the way for depressing the resistive wall instability is introduced in the last part of the paper.

Table 2: CSNS/RCS Critical Components Impedance Estimated Value, $n=\omega/\omega_0$, ω_0 is Angular Revolution Frequency

Item / **Component**	**Injection longitudinal (Ω/m)**	**Injection transverse(kΩ/m)**	**Extraction longitudinal(Ω/m)**	**Extraction transverse(kΩ/m)**
Space charge	–j811.45	–j16.77k	–j96.62	–j3.30k
Resistive wall	$1.68(1+j)/n^{1/2}$	$36.12(1+j)/n^{1/2}$	$2.61(1+j)/n^{1/2}$	$23.23(1+j)/n^{1/2}$
RF cavity	j0.014	j4.74	j0.033	j4.74
Collimator	j0.17	j0.23	j0.42	j0.23

---

* Supported by National Natural Science Foundation of China (11175193)

wangs@ihep.ac.cn

THEORETICAL ESTIMATION

During the beam moving in the resistive pipe, a following wake field forms. This field can lead to a serious action on its tail bunches or particles. With wake field caused by the resistive wall of the vacuum chamber, the transverse motion of bunches can be expressed as[2,3]

$$\ddot{y}_0(t)+\omega_\beta^2 y_0(t)=-\frac{N_p r_p \upsilon}{\gamma T_0}\sum_{k=0}^{\infty}[W_1(-kC-\frac{C}{2})\times \\ \times y_1(t-kT_0-\frac{T_0}{2})+W_1(-kC)y_0(t-kT_0)] \quad (1)$$

here, the two bunches are specified by indices 0 and 1, ω_β is the frequency of betatron motion, W_1 is transverse wake function, N_p denotes the number of particles in a bunch, γ is relative energy factor, v is velocity of particle, T_0 denotes angular revolution time, and the summation over k sums the wake field over all previous revolutions.

By solving the Eq. 1 in frequency domain, the complex mode frequency shift and growth rate of transverse resistive wall instability are obtained[4]

$$(\Delta\omega_\beta)_{coh} = -\frac{i r_p k_b N_p v Z_{\mathrm{T}}}{2\omega_\beta \gamma T_0^2}, \\ \frac{1}{\tau}=\frac{N_p k_b r_p \omega_0^2}{2\pi\beta\gamma cZ_0}\beta_{av}\,\mathrm{Re}(Z_{\mathrm{T}}). \quad (2)$$

where r_p denotes classical radius of proton, k_b expresses the bunch number, Z_T is transverse impedance. Z_0 is impedance of free space, β_{av} is average beta function, c is the velocity of light and β is the relativistic velocity factor, $\mathrm{Re}(Z_{\mathrm{T}})$ denotes the real party of transverse impedance.

Transverse resistive wall impedances of CSNS/RCS source from stainless steel, Ti vacuum pipe, copper shield, collimator, and RF cavity. For monolayer metal pipe, whose thickness is much thicker than skin depth, transverse impedance of resistive wall can be expressed as[3,5]

$$Z_T = Z_0(\mathrm{sgn}(\omega)+j)\frac{\sqrt{\mu_r}}{b^3}\frac{\delta_{skin}}{\sqrt{n}}\frac{L}{2\pi} \quad (3)$$

Where, μ_r is relative magnetic permeability, μ_r of stainless steel is 1.14 at room temperature, b is the radius of vacuum chamber.

Based on the impedance value listed in Table 2, the instability growth time can be estimated during the phase of injection and extraction. Table 3 shows horizontal and vertical theoretical growth time for injection and extraction. Growth times are far less than particle motion time-20ms, transverse resistive wall instability will occur in this case.

Table 3: Theoretical Estimation of Resistive Wall Instability in CSNS. Inj, Ext, Hor and Ver are Injection, Extraction, Horizontal and Vertical

Inj.Hor.(ms)	Inj.Ver.(ms)	Ext.Hor.(ms)	Ext.Ver.(ms)
10.6	9.5	17.1	15.4

Considering the damping effect from natural chromaticity, the growth time with natural chromaticity becomes

$$\frac{1}{\tau}=\frac{N_p k_b r_p \omega_0^2}{2\pi\beta\gamma cZ_0}\beta_{av}\,\mathrm{Re}(Z_{\mathrm{T}})F_m(\omega_\xi). \quad (4)$$

$F_m(\omega_\xi)$ is form factor. The growth time with different chromaticity is show in Figure 1.

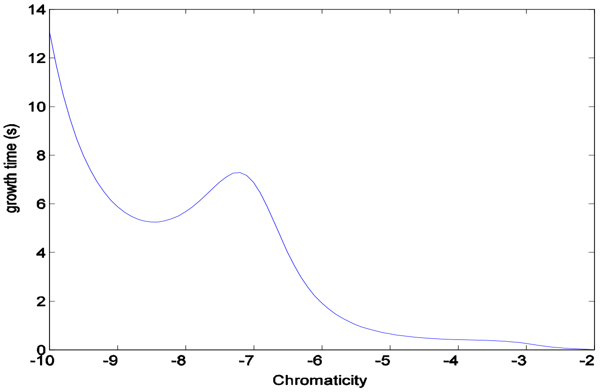

Figure 1: Transverse growth time versus chromaticity.

The above analysis shows the transverse resistive wall instability is serious for CSNS/RCS which may cause beam loss. More detailed simulation studies for the instability will be introduce in following chapter.

SIMATION ON RESISTIVE WALL INSTABILITY

Simulation Model

Resistive wall wake field can be obtained by solving Maxwell equation in general case and wake function is expressed as[6]

$$W_{nj\perp}=-\beta^{3/2}cZ_0\frac{1}{\pi^2 b^3}\sqrt{\frac{\mu_r c}{\sigma}[\frac{1}{|z|^{1/2}}+\frac{3}{8}\frac{b^2-\frac{3}{2}x^2}{\gamma^2|z|^{5/2}}]}. \quad (5)$$

where, b is the radius of chamber, x is the transverse offset, $|z|$ is the distance from beam to test charge, σ is the permittivity, μ_r is the relative permeability of vacuum pipe.

The beam dynamic equation including the effect from wake field in Eq. 1 can be expressed in the form of transfer matrix

ISBN 978-3-95450-118-2

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{i+1} = \begin{pmatrix} \cos(2\pi\nu_{x,y}) & \beta\sin(2\pi\nu_{x,y}) \\ -\frac{1}{\beta}\sin(2\pi\nu_{x,y}) & \cos(2\pi\nu_{x,y}) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_i + \begin{pmatrix} 0 \\ dx' \end{pmatrix}. \quad (6)$$

Where β is average Courant-Snyder parameter of ring, $\nu_{x,y}$ is horizontal or vertical tune, dx' is transverse kick from resistive wall wake field.

In this model, the resistive wall wake field is simplified to one position on RCS and justified to lump resistive wall kick at this point[7]. Beam will experience the kick when it passes the point in each revolution. The wake field can last more than one circle, the multi-turn long range wake will be added up. But in the simulation, the wake must be cut off after some reasonable turns. So the kick seen by beam in a turn is

$$dx' = \sum_n \sum_j \frac{eq<\overline{x_{nj}}>W_{nj\perp}N_{nj}L}{\beta^2 E}. \quad (7)$$

where, L is total length of stainless steel, E is the energy of beam, N_{nj} is number of macros of nth turn jth slice, $<\overline{x_{nj}}>$ is average value of macro-particles, $W_{nj\perp}$ is transverse wake function of test charge, the bunch is considered to be sliced into many micro-bunches longitudinally.

Including the synchrotron motion in the model, the synchrotron motion of bunch can be expressed by transfer matrix form as

$$\begin{pmatrix} \Delta z \\ \delta \end{pmatrix}_{i+1} = \begin{pmatrix} \cos(2\pi\nu_s) & \frac{\eta\beta c}{\nu_s\omega_0}\sin(2\pi\nu_s) \\ -\frac{\nu_s\omega_0}{\eta\beta c}\sin(2\pi\nu_s) & \cos(2\pi\nu_s) \end{pmatrix} \begin{pmatrix} \Delta z \\ \delta \end{pmatrix}_i. \quad (8)$$

where, Δz is longitudinal relative distance, δ is the fractional off-momentum coordinate, ν_s is longitudinal tune, η is slippage factor. The energy ramping progress can be considered in the simulation program, which can be expressed as[8]

$$\begin{aligned} \Delta E_{i+1} &= \Delta E_i + eV(\sin(\phi_i) - \sin(\phi_s)), \\ \phi_{i+1} &= \phi_i + \frac{2\pi h\eta}{\beta^2 E}\Delta E_{i+1}. \end{aligned} \quad (9)$$

where h is harmonic number, ϕ_s is the phase angle for a synchronous particle with respect to the RF wave, ϕ_i is the phase angle for off-momentum particle.

The transverse offset of beam can be attained every turn, and then growth time can be obtained.

Natural chromaticity can damp instabilities. In the simulation model, the effect from the natural chromaticity is included by tune spread in bunch particles. In this case, the betatron tune for each macro-particle expressed as

$$\nu_{x,y} = \nu_{x,y} + \xi_{x,y}\delta_{x,y}. \quad (10)$$

where $\nu_{x,y}$ is particle real tune, $\nu_{x,y}$ is designed tune of particle, $\xi_{x,y}$ is natural chromaticity.

Simulation Result

According to comparison, a Gaussian bunch is sliced into 50 pieces and each bunch is represented by 3000 macro particles. The horizontal and vertical average beta functions are 9.5 and 10.5 meters respectively. The radius of vacuum pipe is 14 centimetres. The bunch is tracked turn by turn and all resistive wall nonultrarelativistic wake forces in ring lumped at one point where alpha is zero for simplification on transfer matrix. The simulation results, on the transverse growth time for injection and extraction energy respectively are in Table 4 by tracking macro-particles oscillation after ten thousands turns. Comparing the simulation and analyses result in table 4 and table 3, the growth times are also consistent.

Table 4: Transverse Simulated Growth Time of Resistive Wall Instability in CSNS

	Growth time/ms
Injection Horizontal	9.3
Injection Vertical	9.2
Extraction Horizontal	18.9
Extraction Vertical	19.1
Ramping Horizontal	11.5
Ramping Vertical	12.3

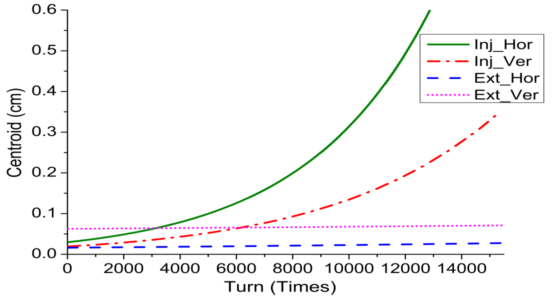

Figure 2: 100kW injection and extraction transverse bunch centre variety versus turn in CSNS/RCS.

The simulated bunch centroid oscillation in injection and extraction is shown in the Figure 2, beam energy is fixed during this tracking. Considering the actual ramping process, the case of ramping program is simulated, and energy increasing comes from RF cavity though synchrotron motion. The horizontal and vertical simulation results are shown in Figure 3. Growth time of horizontal and vertical are 11.5ms and 12.3ms respectively. One should note that the growth curve at that time is not exponential. The growth times are

ISBN 978-3-95450-118-2

obtained by calculating the times when transverse offsets increased *e*(2.71828) times of initial offsets.

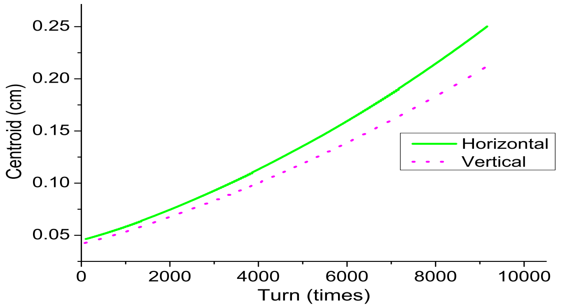

Figure 3: Transverse growth envelope of CSNS in ramping process.

Chromaticity leading the tune spread can suppress resistive wall instability in CSNS/RCS. The maximum deviations of tunes caused by natural chromaticity in RCS are about 0.04 and 0.06 respectively. The horizontal and vertical instabilities are depressed entirely. The chromaticity can be corrected by sextupoles in operation. Figure 4 shows the change of growth time versus different chromaticity.

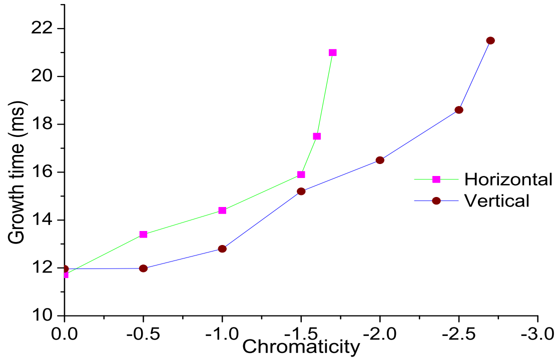

Figure 4: Transverse growth time vs chromaticity.

Conclusion

According to transverse betatron movement, resistive wall instability model is found and simulated code is obtained according to this model. The 100kW simulation results of transverse two directions during the phase of injection and extraction, 9.3ms, 9.2ms and 18.9ms, 19.1ms respectively, agree well with theoretical calculation results-10.6ms, 9.5ms, 17.1ms, and 15.4ms. Including the synchrotron oscillation in tracking simulation, the growth times are about 11.5ms in horizontal plane and 12.3ms in vertical plane. The resistive wall instability is not an ignored effect. The transverse resistive wall instability can be depressed while the chromaticity effect is considered.

REFERENCES

[1] S. Wang, S.Y. Xu et al., An Overview of Design for CSNS Accelerators, SCIENCE CHINA physics, Mechanics&Astronomy.
[2] A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerator, Wiley, New York (1993).
[3] King-Yuan Ng, Fermilab-FN-0713 (2002).
[4] Francesco Ruggiero, Single-beam Collective Effects in the LHC, CERN SL/95-09, 1995.
[5] CSNS design report, IHEP-0107 (2011).
[6] Frank Zimmermann, Resistive-wall Wake and Impedance Nonultrarelativistic beams, Phys. Rev., 2004.
[7] K.A.Thompson and R.D.Ruth, Transverse Coupled bunch Instabilities in Damping Rings of High-Energy Linear Colliders, Phys. Rev., D43:3049-3062,1991.
[8] S. Y. Lee, Accelerator Physics, (World Scientific, Singapors, 2004).