

# SIMULATION AND MEASUREMENT OF HALF INTEGER RESONANCE IN COASTING BEAMS IN THE ISIS RING

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## Abstract

ISIS is the spallation neutron source at the Rutherford Appleton Laboratory in the UK. Operation centres on a high intensity proton synchrotron, accelerating  $3 \times 10^{13}$  ppp from 70-800 MeV, at a repetition rate of 50 Hz. Present studies are looking at key aspects of high intensity behaviour with a view to increasing operational intensity, identifying optimal upgrade routes and understanding loss mechanisms. Of particular interest is the space charge limit imposed by half integer resonance: we present results from coasting beam experiments with the ISIS ring in storage ring mode, along with detailed 3D (ORBIT) simulations to help interpret observations. The methods for experimentally approaching resonance, and the implications on beam behaviour, measurement and interpretation, are discussed. In addition, results from simpler 2D simulations and analytical models are reviewed to help interpret expected beam loss and halo evolution. Plans and challenges for the measurement and understanding of this important beam loss mechanism are summarised, as are some closely related areas of high intensity work on ISIS.

## INTRODUCTION

### *The ISIS Synchrotron*

The ISIS synchrotron accelerates  $3 \times 10^{13}$  protons per pulse (ppp) from 70-800 MeV on the 10 ms rising edge of the sinusoidal main magnet field. At the repetition rate of 50 Hz this corresponds to a beam power of 0.2 MW. Charge-exchange injection takes place over 130 turns as the high intensity beam is accumulated, with painting in both transverse planes over the collimated acceptances of  $\sim 300 \pi$  mm mr. The ring has a circumference of 163 m, with a revolution time of 1.48  $\mu$ s at injection. Nominal betatron tunes are  $(Q_x, Q_y) = (4.31, 3.83)$ , but these are varied using two families of 10 trim quadrupoles. A dual harmonic RF system captures and accelerates the unbunched injected beam, and allows enhanced bunching factors. Peak incoherent tune shifts exceed -0.4 at about 80 MeV during the bunching process. Single turn extraction makes use of a fast vertical kicker at 800 MeV. Main loss mechanisms are associated with non-adiabatic trapping and transverse space charge. The loss associated with half integer resonance is highly relevant for present ISIS operations and proposed upgrades [1].

### *Half Integer Loss and Aims of Study*

The ‘‘space charge limit’’ of a high intensity proton ring is generally expected to be that imposed by the half integer resonance. The inclusion of self-consistency leads to the coherent resonance condition as a basic guideline

for designs [2]. For realistic beams however, behaviour leading to loss is more complicated than this simplified model suggests. These studies look at this loss mechanism in more detail.

For a rapid cycling synchrotron (RCS) such as ISIS, half integer resonance is further complicated by addition of longitudinal motion (i.e. 3D dynamics) and fast changes of parameters. To reliably predict the high intensity limit of such machines a deeper understanding is desirable. This paper is part of a programme of study to address these topics, presently working on the simpler case of 2D coasting beams, with plans to study non-accelerated bunched beams next, and finally the full RCS case. A central objective is to perform detailed measurements of the loss processes, allowing benchmarking of codes and theory. This in turn has the potential for more detailed beam optimisations, with perhaps increased beam intensities.

### *ISIS Studies and Present Experiments*

Earlier work has looked in detail at various aspects of half integer resonance in 2D on ISIS: solution of the envelope equation and coherent modes for the large tune-split case [3,4]; comparison with ORBIT simulations, evolution of halo and beam loss near coherent resonance [4,5]; and more recently development of new experiments and more sophisticated simulations to explain them [6,7]. This paper continues the later work where the essential aim is to measure and simulate half integer resonance, and compare with relevant models.

In [7] measurements and simulations of half integer resonance in the ISIS ring were shown to be in agreement with predictions from coherent resonance theory. This gives the following relations for the envelope frequencies (large tune split case, equal transverse emittances):

$$\begin{aligned} \omega_x^2 &= 4Q_{0x}^2 - 5Q_{0x}\Delta Q_{inc} \\ \omega_y^2 &= 4Q_{0y}^2 - 5Q_{0x}\Delta Q_{inc} \end{aligned} \quad \text{with } \Delta Q_{inc} = \frac{r_p N}{2\pi\beta^2\gamma^3\epsilon} \frac{1}{B} \quad (1)$$

where:  $\omega_{x,y}$  envelope frequencies,  $Q_{0x,y}$  zero intensity tunes,  $\Delta Q_{inc}$ , incoherent tune shift of RMS equivalent KV beam,  $r_p$  proton radius,  $N$  intensity,  $\epsilon = 4\epsilon_{rms}$ ,  $\epsilon_{rms}$  RMS emittance,  $B$  bunching factor and  $\beta, \gamma$  relativistic parameters. From (1) the coherent frequencies as a function of intensity can be calculated, and predict resonance and loss. An example, appropriate for these experiments, is shown in Figure 1: the vertical coherent tune is approaching 7, equivalent to the  $2Q_y = 7$  line.

In [6,7] work concentrated on the experimental observation of loss as a function of tune and intensity. We now take this a stage further, looking at evolution of

transverse beam distributions as coherent resonance is approached.

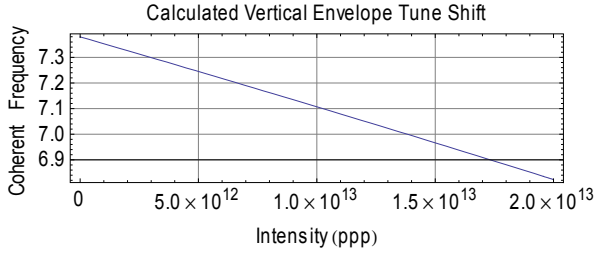


Figure 1: Coherent frequency as a function of intensity for typical beam parameters in the experiment.

## EXPERIMENTS

### Machine Configuration, Experimental Method

A basic aim of these experiments is to provide the simplest practical beam configuration that allows the essentials of the loss mechanisms to be studied. For this reason the ISIS synchrotron is reconfigured in storage ring mode (SRM), with RF off and main magnet fields set at a constant level appropriate for the 70 MeV injection energy. The result is coasting, unbunched 2D beams, where adjustment of injection painting facilitates independent control of transverse emittances. To reach appropriate incoherent tune shifts, and allow observation of beam tails or “halo” generated by half integer effects, beam is painted over a small fraction ( $\epsilon_{100\%} \sim \leq 100 \pi \text{ mm mr}$ ) of the machine acceptances ( $\sim 300 \pi \text{ mm mr}$ ). The two sets of 10 trim quadrupoles in the ISIS lattice allow adjustment of tunes and application of harmonic driving terms.

There are a number of ways in which half integer coherent resonance may be approached, e.g. ramping tune with quadrupoles or ramping intensity. The latter has been selected for these studies. Injection painting was configured to provide a beam of selected, constant emittance,  $\epsilon_{rmsx} \approx \epsilon_{rmsy} \approx 20 \pi \text{ mm mr}$  through the injected pulse. Tunes were set at constant (zero intensity) values  $(Q_x, Q_y) = (4.38, 3.63)$ , such that as the intensity ramps (linearly  $0.0\text{--}1.3 \times 10^{13}$  ppp) over  $\sim 130 \mu\text{s}$  of injection, the beam crosses the vertical half integer resonance ( $2Q_y = 7$ ). These values are selected to avoid other resonances and loss mechanisms. Monitoring of beam longitudinal and transverse beam spectra, along with experience from earlier studies, ensured instabilities were not encountered [6]. As the intensity ramps through the injection process, circulating beam currents and losses were recorded with toroids and loss monitors, which indicate when resonance is approached. The evolution of transverse beam distributions was also simultaneously recorded on residual gas ionisation (RGI) profile monitors.

### Changing the Phase of the Driving Term

These measurements were repeated as a function of strength and phase of the  $2Q_y = 7$  harmonic driving terms applied to the trim quadrupoles. These put a harmonic driving term around the machine azimuth,  $\theta$ , of the form

$\Delta k(\theta) = k_0 \cos(2Q_y \theta + \phi)$ , with  $2Q_y = 7$ . Three measurements were taken, one with zero driving term,  $k_0 = 0$  ( $p_0$ ), and two with  $k_0 \sim 0.02 \text{ m}^{-2}$  at equivalent phases of  $\phi = 0$  ( $p_1$ ) and  $\phi = \pi$  ( $p_2$ ). When the beam encounters a driven half integer resonance, the corresponding resonance structure is locked to the machine azimuth and corresponding phase of the driving harmonic: the two-fold symmetrical phase space structure rotates  $Q$  times around the machine. When observing the beam profile at one azimuth, i.e. a monitor, it is possible that key features will not be visible. This is because the profile is the projection of the distribution on the spatial axis and depends on the beam orientation in phase space. Taking measurements at a number of driving phases ensures observation of expected features, and *importantly* demonstrates behaviour expected specifically for half integer resonance. This is illustrated below.

### Key Diagnostics

Essential to these measurements is the use of non-destructive RGI profile monitors [8]. Previous studies [9] have described correction schemes to compensate for errors from drift field non-linearities and beam space charge. These corrections have been applied here and are expected to give beam widths to within about  $\pm 6 \text{ mm}$ . The corrected profiles give key information on the evolution of the beam distribution: ongoing work to further improve monitor accuracy is outlined below.

Use of special diagnostic chopped beams at injection [10] have allowed the accurate measurement of zero intensity  $(Q_x, Q_y)$  and injected betatron amplitudes defining the painting process. These are used in the ORBIT model.

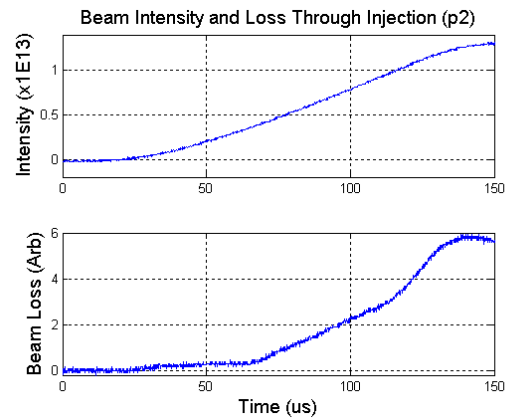


Figure 2: Beam current and loss measurements through injection (for driving term  $p_2$ ).

### Results

As beam is accumulated through injection, the intensity ramps and the coherent frequency depresses as indicated in (1) and Figure 1. This pushes beam toward resonance, resulting in emittance growth and beam loss. An example of intensity and beam loss as a function of time (for the  $p_2$  case) is shown in Figure 2. As with previous observations, this shows very large loss as beam pushes onto resonance (peak loss  $\sim 10\%$  of total). What is also noticeable is the

gradual increase of loss, before the “brick wall” of resonance is reached. Measurements at all three driving term settings showed increased losses as beam approached resonance, after  $\sim 100 \mu\text{s}$ . Loss levels were about the same for both phases with driving terms on ( $p_1$ ,  $p_2$ ), but significantly reduced with no driving term ( $p_0$ ).

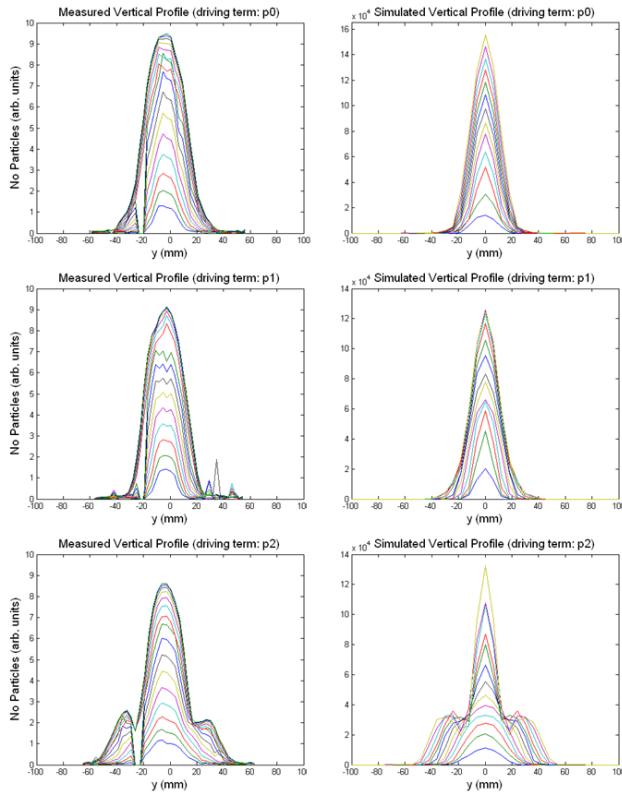


Figure 3: Vertical beam profiles: measured (left) and simulated (right) for three driving term settings:  $p_0$ ,  $p_1$ ,  $p_2$  at the top, middle and bottom respectively.

The measured vertical beam profiles over the injection interval, for each of the three driving term settings, are shown on the left column of Figure 3. These show beam profiles at intervals of  $10 \mu\text{s}$  for the first  $150 \mu\text{s}$  of injection. Corrections for drift field and space charge have been applied, as noted above. Horizontal profiles were also measured, but these showed no notable changes. Similarly, monitoring of transverse and longitudinal beam spectra indicated no action of instabilities. Figure 4 shows the detailed development of the vertical profile over  $400 \mu\text{s}$  (in this plot space charge corrections include approximations, see comments on development work). Some anomalous readings are visible on the measurements due to spurious detector calibrations.

Of greatest interest is the appearance of distinctive “hips” on the beam profiles in Figure 3, which appear just for one phase setting of the driving term ( $p_2$ ), not being visible when the phase is “reversed” ( $p_1$ ), or with driving term removed ( $p_0$ ). Observation of this feature, with the appropriate driving term, was highly repeatable.

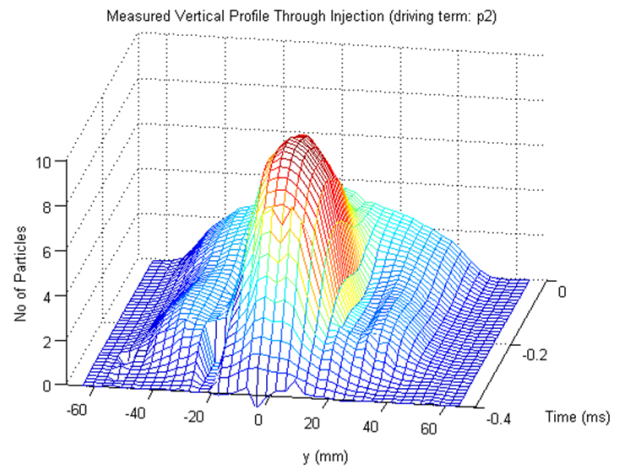


Figure 4: Evolution of the measured vertical profile over  $400 \mu\text{s}$  with the  $p_2$  driving term.

## SIMULATIONS

### ORBIT Model of the Ring and Experiment

A detailed ORBIT [11] model of the ISIS ring has been established as described in [12,7]. This includes: the detailed AG lattice, 3D beam dynamics with space charge, injection painting, the foil, apertures and collimation. This was adapted to model the machine in SRM, including constant injection painting, suitable  $Q$  values, and harmonic driving terms. This is valuable in two ways: (i) it gives checks that the simulation and experimental behaviour agree (benchmarking), and (ii) allows study of detailed simulation data (not available experimentally) to help understand behaviour.

The measurements with low intensity beams provided accurate values for  $Q$  and injection painting amplitudes. However, limited experimental time required nominal values for some parameters to be assumed for the ORBIT model, with reasonable variations made to explore behaviour. Essential parameters were adjusted to be close to the experimental values above, with  $\epsilon_{rms,x} \approx \epsilon_{rms,y} \approx 20 \pi \text{ mm mrad}$  and  $(2Q_y=7)$  driving terms applied to the lattice trim quadrupoles as they are in hardware. ORBIT simulations were run for  $\sim 200 \mu\text{s}$ , including injection painting and accumulation, with a total of  $\sim 10^6$  macro-particles. Simulation runs were repeated for the three driving term settings, with intensity ramping to  $1.3 \times 10^{13}$  ppp over injection. Beam distributions at the lattice locations of profile monitors were produced and analysed. Beam tunes were also determined as required.

### Results

Vertical beam profiles at the lattice location of the profile monitor, through the injection pulse at the three different phase settings, are shown on the right hand side of Figure 3. The intervals between plots are  $7.5 \mu\text{s}$  (5 turns). It can be seen the general form of profiles agrees well with experiment, most notably with the appearance of hips just at the one expected driving term setting ( $p_2$ ).

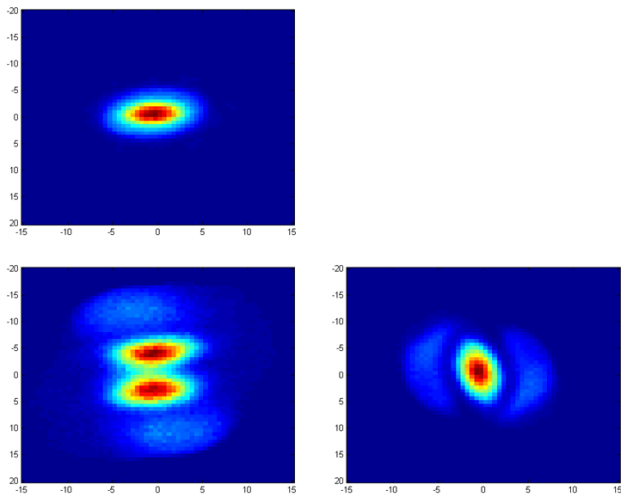


Figure 5: Normalised phase space distributions  $(Y, Y')$   $\sim 85 \mu\text{s}$  through injection with driving terms  $p_0, p_1, p_2$ , at top, left, right respectively (corresponding to Figure 3).

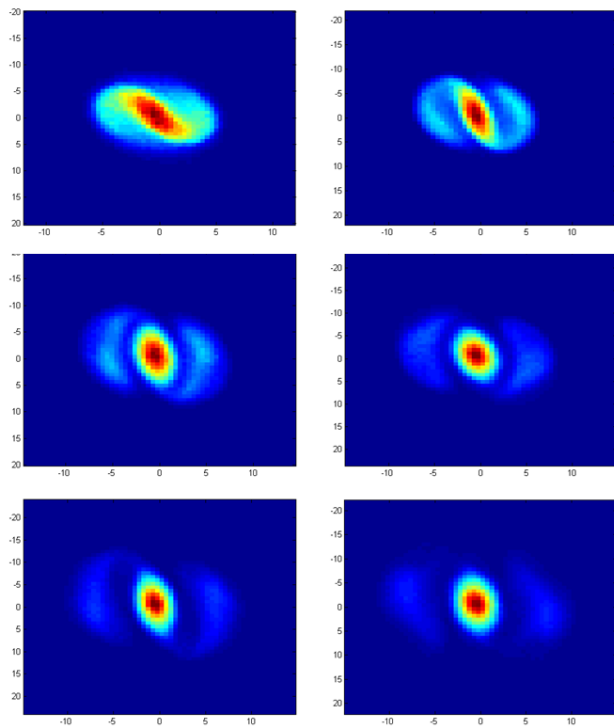


Figure 6: Evolution of phase space from 40-120  $\mu\text{s}$  through injection, at 15  $\mu\text{s}$  (10 turn) intervals for  $p_2$  phase driving term (time increases left-right, top-bottom).

The beam distributions in normalised  $(Y, Y')$  phase space at the monitor location, corresponding to the three driving term settings of Figure 3, are shown in Figure 5. The distinctive half integer structure, with two-fold symmetry, is evident when the driving terms are applied. The orientation of the resonant “lobes” in phase space at different driving phases shows why hips are only visible at one phase setting. The measured profile is the projection of this distribution on the  $Y$  (horizontal) axis: the orientation is appropriate only for the  $p_2$  case.

Beam behaviour in simulations clearly indicates half integer resonance, with excitation of coherent moments at expected frequencies as described in [7]. The evolution of the  $(Y, Y')$  beam distributions through the injected pulse over  $\sim 80 \mu\text{s}$ , from  $\sim 0.2-1.2 \times 10^{13}$  ppp for the  $p_2$  case, is shown in Figure 6. The development of the half integer structure is clear as intensity ramps and resonance approached. Monitoring of incoherent betatron frequencies clearly shows much of the beam to be locked into the  $Q_y=3.5$  resonance once the two-fold structure appears. A typical example of the tune footprint is shown in Figure 7.

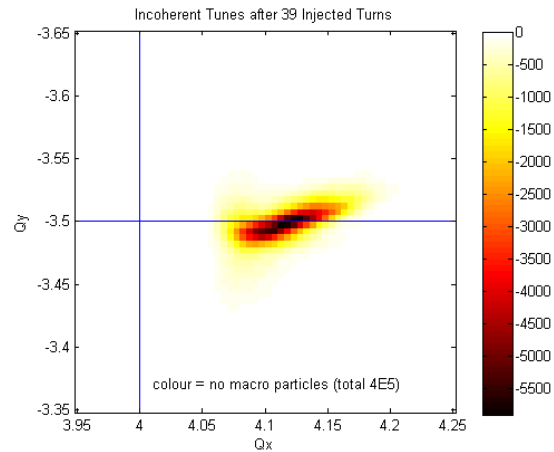


Figure 7: Incoherent tune footprint  $(Q_x, Q_y)$  in simulations corresponding to appearance of resonant structure in Figure 6, showing particle tunes at  $Q_y=3.5$ .

## INTERPRETATION AND FUTURE WORK

### *Appearance of Half Integer Halo*

The measurement of hips on profiles that can be manipulated predictably with the magnitude and phase of quadrupole driving terms, in agreement with detailed simulations, indicates that these features are indeed halo associated with half integer resonance. These features are also coincident with the onset of coherent resonance. The close correspondence between codes, experiment and basic coherent resonance theory is a useful confirmation of expectations. This opens up new possibilities for detailed study of the loss mechanism.

### *Details of Beam Behaviour*

The evolution of simulated beam distributions in Figure 6 shows interesting behaviour, with lobes moving outward as intensity increases. Although appearance of hips in measurements of Figure 4 is clear, and on the right time scale, discerning their detailed evolution requires more work. To understand these observations, more detailed study of profile monitor and ORBIT simulation models is required: see below.

### *Beam Models*

The most basic model of the experiment assumes a constant emittance beam of ramping intensity (as

accumulated beam is painted at constant amplitudes) which approaches resonance as given by (1). This gives useful predictions of beam loss, but is obviously limited once emittance changes and the beam redistributes.

Perhaps the most relevant model in the literature is [13], where halo structure is predicted for a self-consistent model of a KV beam driven by harmonic errors. Previous comparisons of this model with ISIS simulations [5] have shown similar behaviour. The behaviour in Figure 6 would seem to be compatible with that predicted in [13], but differences in the model should be accounted for (e.g. large tune splits and non-KV distributions).

The confirmed appearance of parametric structure suggests that a relatively simple (smooth focusing) analytical or simulation model may have some predictive power over short time scales. With this in mind, a (non self-consistent, particle-core) 1D model based on a waterbag beam and driving term is currently being studied.

### *Developments of Measurements and Simulations*

Ongoing developments of diagnostics and experimental methods have made these new half integer halo measurements possible. However, this work is far from complete. The installation and refinement of new hardware for the multi-channel profile monitors [8], including fast calibration systems, higher operating voltages, and benchmarking experiments with “harp” monitors is currently under way. These will improve performance and reduce errors in key parameters. More detailed simulation studies of the profile monitors [9] are also planned, to check performance with non-standard beam distributions and probe limits to resolution. Development of new instrumentation that allows fast measurement of essential parameters is making more detailed parameterisation of the machine possible, thus reducing uncertainties in the models. In addition, more experimental time will allow the refinement and development of the methods used. Collectively, this will reduce error margins and increase value for benchmarking. It is hoped this will provide experimental confirmation of the rich behaviour seen in the simulations.

### *Related Work*

Studies for upgrades to the ISIS ring [1], looking at intensities into the 0.5 MW regime, are addressing numerous high intensity topics related to those here. Closely related study [14] extends the present work to include other resonances and image effects. Longitudinal effects and intensity limitations are discussed in [15]. Development of a new 3D ISIS code (Set 3Di) will provide a valuable tool for these studies.

## SUMMARY

Development of diagnostics and experimental methods is now allowing detailed study of half integer generated halo. Experimental observation shows good agreement

with ORBIT simulations, and in combination these are giving valuable information on this important loss mechanism. Developments and refinements of the methods above will allow more detailed study and accurate benchmarking. It is hoped that some simplified analytical or numerical models will provide additional insight into beam behaviour. Extension to the RCS case in due course should improve understanding of half integer high intensity limits on proton rapid cycling synchrotrons.

## ACKNOWLEDGMENTS

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## REFERENCES

- [1] C M Warsop et al., Status of Injection Upgrade Studies for the ISIS Synchrotron, Proc. IPAC11, p2760.
- [2] R Baartman, Betatron Resonances with Space Charge, AIP Conf. Proc. CP448, 1998.
- [3] C M Warsop et al., Transverse Space Charge Studies for the ISIS Synchrotron, Proc. EPAC06, p312.
- [4] C M Warsop et al., Studies of Space Charge Loss Mechanisms on the ISIS RCS, Proc. PAC09, p4408.
- [5] C M Warsop et al., Space Charge Loss Mechanisms Associated with Half Integer Resonance on the ISIS Synchrotron, Proc. EPAC08, p373.
- [6] C M Warsop et al., High Intensity Studies on the ISIS Synchrotron, Including Key Factors for Upgrades and the Effects of Half Integer Resonance, Proc. HB2010, p619.
- [7] C M Warsop et al., Simulation and Measurement of Half Integer Resonance in Coasting Beams on the ISIS Ring, Proc. IPAC11, p637.
- [8] S J Payne et al., A Self Calibrating Real Time Multi-Channel Profile Monitor for the ISIS Proton Synchrotron., Proc. DIPAC 2007, p364.
- [9] R E Williamson et al., Analysis of Measurement Errors in Residual Gas Ionisation Profile Monitors in a High Intensity Proton Beam, Proc. EPAC08, p1317.
- [10] C M Warsop, Low Intensity and Injection Studies on the ISIS Synchrotron, Proc. EPAC94, p1722.
- [11] J Galambos, ORBIT – A Ring Injection Code with Space Charge, Proc. PAC99, p.3143.
- [12] D J Adams et al., Beam Loss Studies of the ISIS Synchrotron Using ORBIT, Proc. IPAC12, p3942.
- [13] M Venturini, R L Gluckstern, Resonance Analysis for a Space Charge Dominated Beam in a Circular Lattice, PRST-AB, V3, 034203, 2000.
- [14] B G Pine et al., Space Charge Limits on the ISIS Synchrotron Including the Effects of Images, Proc. this conference.
- [15] R E Williamson, High Intensity Longitudinal Dynamics Studies for an ISIS Injection Upgrade, Proc. this conference.