PARAMETER OPTIMIZATION IN HIGGS FACTORY DESIGN

C. Zhang, IHEP, CAS, P.O.Box 918, Beijing 100049, China

Abstract

In this paper, parameter optimization of Higgs factories is discussed focusing on the designs of CEPC and FCCee. The total beam current in Higgs factories is limited by synchrotron radiation power, and then the machine size and cost; maximum linear tune shift is limited by beam-beam interaction; reduction of beta-function at interaction point is restricted by the distance of the final focusing quadrupole to the interaction point, bunch length, "hour glass" effect and dynamic aperture. Beamstrahlung effects beam energy spread and lifetime in the colliders, limiting luminosity reach. High luminosity in the Higgs factories requires optimization of parameters.

LUMINOSITY

Discovery of Higgs boson of 125 GeV, shown in Fig.1, not far from the LEPII reached energy, makes it feasible to build circular e^+e^- colliders of 120 GeV and neighbouring energies as Higgs factories with high luminosity.

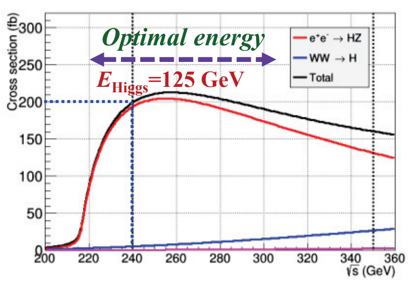

Figure 1: Energy of Higgs boson and Higgs factories.

The luminosity in circular collider is given in Eq. (1) assuming beam aspect ratio $r=\sigma_y/\sigma_x<<1$, σ_x and σ_y being horizontal and vertical beam size at interaction point (IP).

$$L = \frac{1}{2e \cdot r_e E_0} \cdot \xi_y \frac{E \cdot k_b I_b}{\beta_y^*} H_g \qquad (1)$$

Where e is electron charge, r_e is classical radius of electron, E_0 is rest energy of electron, E is colliding beam energy, k_b and I_b are bunch number and current respectively, ξ_y is vertical beam-beam parameter, β_y^* is beta function at IP and H_g is hour glass factor expressed as:

$$H_g = \frac{L}{L_0} = \frac{\beta_y^*}{\sqrt{\pi}\sigma_z} \exp\left(\frac{\beta_y^{*2}}{2\sigma_z^2}\right) K_0\left(\frac{\beta_y^{*2}}{2\sigma_z^2}\right) \qquad (2)$$

Here σ_z is bunch length. The formula is applies to the zero crossing angle case.

It can be seen in Eq. (1) that the luminosity is closely related to total beam current, beta function at IP, maximum beam-beam parameter, hour glass factor which always less than 1. And all parameters are correlated. The correlation of the parameters are illustrated in Figure 2.

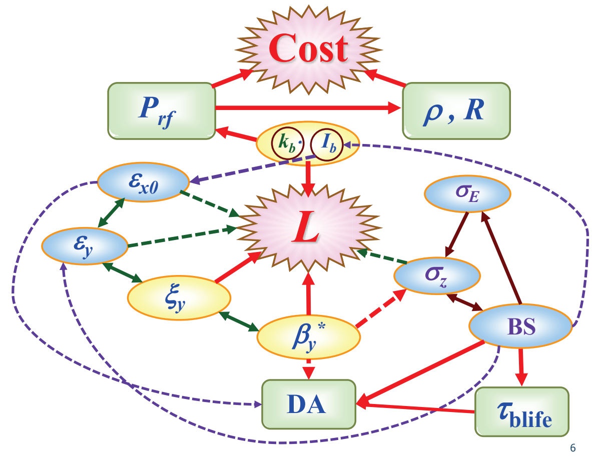

Figure 2: Correlation of parameters in colliders.

As shown in Fig. 2, luminosity is in the centre of the correlation net while the cost of the collider is the top concern especially for such a high energy e^+e^- collider like Higgs factories. In following sections optimization of the parameters related to luminosity and cost are discussed based on the designs of CEPC [1] and FCCee [2].

BEAM-BEAM PARAMETER

Beam-beam parameter, or linear beam-beam tune shift, characterizes the strength of the beam-beam force [3]:

$$\xi_{x,y} = \frac{r_e}{2\pi e} \frac{I_b \beta_{x,y}^*}{\gamma \cdot f_0 \sigma_{x,y}(\sigma_x + \sigma_y)} \qquad (3)$$

here γ is relativistic energy and f_0 is revolution frequency. The larger the beam-beam parameters, the higher the luminosity will be reached. The maximum achievable beam-beam parameter strongly depends on the radiation damping in storage rings. The LEP beam-beam performance gives the following scaling law [4]:

$$\xi_y^{max} = 0.5\tau_E^{-0.4} \qquad (4)$$

Taking advantage of the LEP scaling law of Eq. (3), the maximum vertical beam-beam parameters for CEPC and FCCee are calculated in comparison with their designed values in Table 1 and Fig. 3.

Table 1: Calculated and Designed Maximum Vertical Beam-beam Parameters for CEPC and FCCee

Parameter		CEPC	FCCee			
E (GeV)		120	45.5	80	120	175
τ_E (turns)		39	1320	243	72	23
ξ_y^{max}	Cal.	0.15	0.028	0.056	0.090	0.143
	Des.	0.083	0.03	0.059	0.093	0.092

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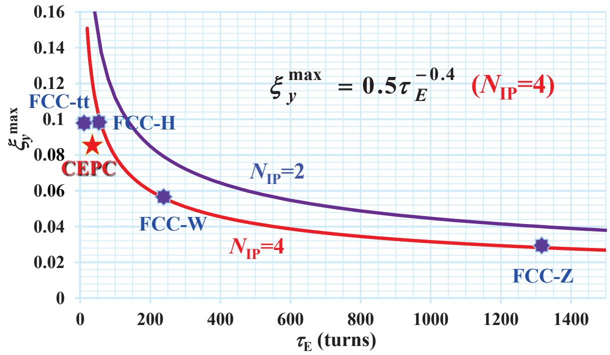

Figure 3: Designed ξ_y^{max} for CEPC and FCCee in comparison with LEP scaling curve.

It can be found in Fig. 3 that the designed ξ_y^{max} values of FCC-H, FCC-W and FCC-Z are well on the LEP scaling curve, while the ξ_y^{max} values of FCC-tt and CEPC are below the curve. The reason for FCC-tt may trace to the limitation of total beam current, while the lower ξ_y^{max} of CEPC design, which is also suggested by beam-beam simulation, may come from other parameters not taken into account in the scaling law especially bunch length.

BEAM INTENSITY

It is well known that the beam intensity in high energy e^+e^- collider is limited by total synchrotron radiation (SR) power:

$$P_{sr} = U_0 \cdot k_b I_b = C_\gamma \frac{E^4}{\rho} k_b I_b \tag{5}$$

Here ρ is bending radius, U_0 is SR loss per revolution and C_γ= 8.846×10^{-5} m/(GeV)^3 is SR constant for electron. As shown in Eq. (5) that the total beam current $k_b \cdot I_b$ is limited by the SR power loss P_{SR}. The luminosity is related to the RF power and bending radius as:

$$L_0 = \frac{3(m_e c^2)^2}{8\pi r_e^2} \frac{\xi_y}{\beta_y^*} \cdot \frac{P_{rf}\rho}{E^3} = 2.45 \times 10^{39} \frac{\xi_y}{\beta_y^*(\mathrm{mm})} \cdot \frac{P_{rf}(\mathrm{MW})\rho(\mathrm{km})}{E^3(\mathrm{GeV}^3)} \tag{6}$$

Figure 4 gives luminosity vs. beam energy for ρ=6.1 km (CEPC), 11km (FCC) and P_{SR}=50 MW (CEPC & FCC).

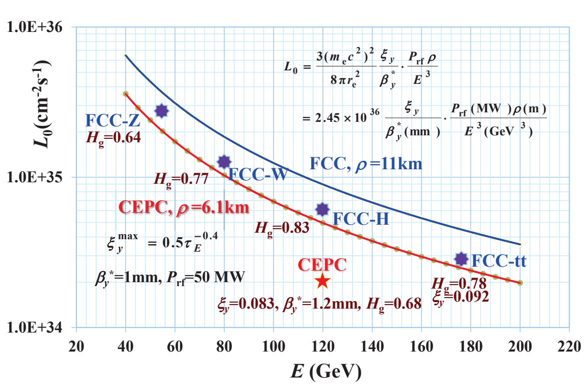

Figure 4: Luminosity vs. beam energy in CEPC & FCC.

It can be seen in Fig.4 that the designed luminosity for both CEPC and FCCee is below the luminosity curve for β_y^* =1mm and P_{SR}=50MW. This is because of the hour glass effect of H_g=L/L_0<1. Another reason for CEPC is the design β_y^*=1.2 mm, 20% larger than 1mm calculated for the luminosity curve.

β_y^* AND BUNCH LENGTH

It is shown in Eq. (3) and Eq. (1) that the smaller the beta function at IP and the smaller the beam-beam tune shift, the higher the initial luminosity L_0 will be. However, the beta-function at the final focusing quadrupole (FFQ) is inversely proportional to β_y^* such that β_y^{FFQ}=L^{*2}/β_y^*, where the L^* is distance from FFQ to IP, which is restricted by the geometry of the detector. Large β_y^{FFQ} requires large aperture of FFQ and also increases chromaticity $\Delta\xi_y$= $(\beta_y \cdot kl)_{FFQ}/4\pi$, causing dynamic aperture problem. This is why the β_y^* is chosen as ~1mm instead of further smaller in Higgs factory design (1.2 mm for CEPC, and 1mm for FFC).

However, the hour glass effect prevents the luminosity reaching to L_0. Figure 5 exhibits the hour glass factor H_g=L/L_0 as a function of σ_z/β_y^*.

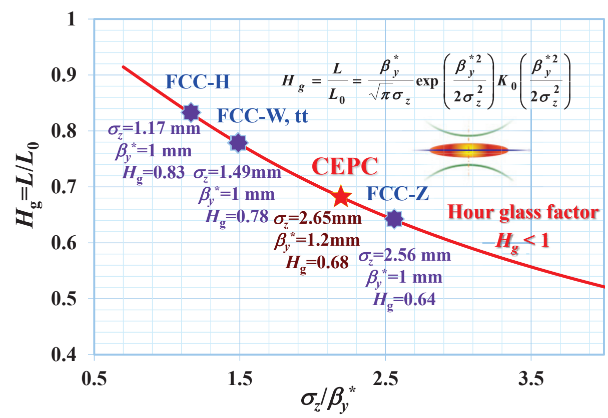

Figure 5: Hour glass factors for CEPC & FCC.

As shown in Fig. 5 that the hour glass factors in Higgs factory designs are 0.83, 0.78, 0.78, 0.68 and 0.64 for FCC-H, FCC-W, FCC-tt, CEPC and FCC-Z respectively. It is realized that the Hg=0.6~0.7 is too small and one needs to further optimize the collider parameters.

The key parameter in the hour glass formula Eq.(2) is the ratio of σ_z/β_y^*. Bunch length σ_z should be short enough comparing to β_y^* in order to restore the initial luminosity L_0 as fully as possible. It is worth to mention that the large σ_z/β_y^* may also cause beam-beam synchro-betatron oscillation and other nonlinear effects.

Bunch length in electron storage rings is determined by synchrotron radiation and collective bunch lengthening effects. The natural bunch length in storage ring σ_{z0} is expressed as:

$$\sigma_{z0} = \frac{R\alpha_p}{\nu_s}\sigma_{E0} = \frac{R\alpha_p}{\nu_s}\gamma\sqrt{\frac{C_q}{J_E\rho}} \tag{7}$$

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Where R is the ring radius, α_p is momentum compaction factor, ν_s is synchrotron tune, σ_{E0} is natural beam energy spread, $J_E\approx2$ is longitudinal damping partition number and C_q=3.832×10^{-13} m is quantum constant.

Small a_p and high V_{rf} (this is a case in Higgs factories) result large ν_s and short bunch, which may help to relax hour class effect. On the other hand, potential and microwave instability, as well as beamstrahlung effects may cause bunch lengthening, and beamstrahlung fractional energy spread $\delta_{bs} \propto \sigma_z^{-2}$. This suggests suitable compromising choice of bunch length. The typical value of σ_{z0} in Higgs factories is 1-2 mm.

BEAMSTRAHLUNG

Beamstrahlung is the radiation from one beam of charged particles in colliders caused by its interaction with the electromagnetic field of an opposite beam.

Beamstrahlung adds energy spread to beam [5]:

$$\Delta\sigma_{\delta,B} = \frac{1}{2}\sqrt{\frac{\tau_E n_{IP}}{T_0}}\sigma_{\delta,B} \quad (8)$$

where n_{IP} is number of IP's in the collider, T_0 is revolution period and $\sigma_{\delta,B}$ is standard deviation of the energy loss expressed as:

$$\sigma_{\delta,B} = 1.24\left(0.333 + \frac{4.583}{n_\gamma}\right)^{1/2}\left[\frac{\alpha\sigma_z}{\bar{\lambda}_c\gamma}\frac{\Upsilon^2}{(1+(1.5\Upsilon)^{2/3})^2}\right] \quad (9)$$

Average emitted photon number per collision is given by:

$$n_\gamma = 2.45\left[\frac{\alpha\sigma_z}{\bar{\lambda}_c\gamma}\frac{\Upsilon}{(1+\Upsilon^{2/3})^{1/2}}\right] \quad (10)$$

Where α=1/137 is fine structure constant, $\bar{\lambda}_c$ is the electron Compton wavelength divided by 2π, $\bar{\lambda}_c = r_e/\alpha$, and Υ is the beamstrahlung strength is the following:

$$\Upsilon = \frac{5}{6}\frac{r_e^2\gamma\cdot N_e}{\alpha\sigma_z(\sigma_x+\sigma_y)} \quad (11)$$

Beamstrahlung effects beam energy spread and lifetime in circular colliders, the beamstrahlung caused lifetime is expressed as [6]:

$$\tau_{BS} = \frac{20\sqrt{6\pi}r_e\gamma\cdot u^{2/3}e^u}{\alpha^2\sigma_e\sigma_z}T_0 \quad (12)$$

with

$$u = \frac{\eta_e E}{E_{cb}} = \frac{\alpha\eta_e\sigma_x\sigma_z}{3\gamma\cdot r_e^2 N_e} \quad (13)$$

Beamstrahlung caused beam energy spread and bunch length are calculated using Eq. (8) – (11) and lifetime is estimated with Eq. (12)-Eq. (13), giving in Table 2.

Table 2: Beamstrahlung Caused Beam Energy Spread and Lifetime Calculated for CEPC and FCCee

Param	Unit	CEPC	FCC			
E	GeV	120	45.5	80	120	175
ξ_y		0.083	0.03	0.059	0.093	0.092
N_e	10^{11}	3.79	1	0.7	0.46	1.4
$r=\varepsilon_y/\varepsilon_x$		0.003	0.002	0.002	0.001	0.001
σ_z	mm	2.65	1.64	1.01	0.81	1.16
$\sigma_{\delta, BS}$	%	0.096	0.044	0.071	0.10	0.12
$\sigma_{\delta, total}$	%	0. 16	0.060	0.10	0.14	0.18
τ_{BS}	min.	47 *	298	73	29	21

* Obtained by beam-beam-simulation

It can be seen from Table 2 that beamstrahlung significantly effects on beam energy spread and lifetime, limiting luminosity reach in CEPC and FCCee. Several measures are taken to mitigate the beamstrahlung effects in Higgs factory design, including to use very flat colliding beams ($r=\sigma_y/\sigma_x$<<1%), to provide large energy acceptance (1.5%-2%) and to optimize bunch length together with hour glass effect. The top-up injection should be applied to keep peak beam current.

BUNCH NUMBER, POPULATION AND EMITTANCE

Total beam current $k_b{\cdot}I_b$ is limited by RF power. The bunch number k_b and population $N_e=I_b/(e{\cdot}f_0)$ need to be optimized to make best use of the expensive current.

As shown in Eq. (11) the beamstrahlung strength Υ is proportional to bunch population N_e, so smaller I_b and larger k_b are preferred for reducing the beamstrahlung effects. In CEPC, the single ring structure prevents a large bunch number, so k_b=50 is chosen and bunch population is 3.79×10^{11}. Taking advantage of double ring structure, more bunches can be operated in FCC. The bunch numbers get to 1360 (FCC-H), 16700 (FCC-Z), 4490 (FCC-W) and 98 (FCC-tt) with populations of 1.0×10^{11}, 0.7×10^{11}, 0.46×10^{11} and 1.4×10^{11} respectively.

Beam emittance is closely related the maximum bunch population allowed in collisions, shown in Eq.(3). In the case of optimum coupling when the coupling coefficient $\kappa=\varepsilon_y/\varepsilon_x=\beta_y^*/\beta_x^*=\sigma_y/\sigma_x$, Eq. (3) can be written as:

$$\xi_x = \xi_y = \frac{r_e}{2\pi\gamma}\frac{N_e}{\varepsilon_{x0}} \quad (14)$$

It is shown in Eq.(14) that horizontal emittance ε_{x0} should be low enough to reach ξ_y^{max} for the small bunch population. The vertical emittance ε_y should be further smaller to reduce the beamstrahlung caused energy spread. The typical value of coupling coefficient κ is 0.001-0.002.

To sum up, the luminosity related parameters of CEPC [1] and FCCee [2] are given in Table 3.

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Table3: Luminosity Related Parameters of CEPC and FCCee

Parameter	CEPC	FCC			
		Z	W	H	tt
E (GeV)	120	45.5	80	120	175
C (km)	54.8	100			
ρ (km)	6.1	11			
N_{IP}	2	4			
k_b	50	16700	4490	1360	98
N_e (10^{11})	3.79	1.0	0.7	0.46	1.4
ε_x (nm·rad)	6.12	29.2	3.3	0.94	2.0
ε_y (pm·rad)	18.4	60	7	1.0	2
$\sigma_{E,SR}$ (%)	0.13	0.04	0.07	0.10	0.14
$\sigma_{E,total}$ (%)	0.16	0.06	0.09	0.14	0.19
$\sigma_{z,SR}$ (mm)	2.14	1.64	1.01	0.81	1.16
$\sigma_{z,total}$ (mm)	2.65	2.56	1.49	1.17	1.49
β_x^* (m)	0.8	0.5	0.5	0.5	1.0
β_y^* (mm)	1.2	1.0	1.0	1.0	1.0
U_0 (GeV)	3.11	0.03	0.33	1.67	7.55
P_{SR} (MW)	50	50			
V_{RF} (GV)	6.9	2.5	4	5.5	11
f_{RF} (MHz)	650	800			
τ_E (turns)	39	1320	243	72	23
A_E (%)	6.0	2.7	7.2	11.2	7.1
f_s	0.18	0.65	0.21	0.096	0.10
H_g	0.68	0.64	0.77	0.83	0.78
L (10^{34}cm^{-2}s^{-1})	2.0	28.0	12.0	6.0	1.8
ξ_x	0.118	0.031	0.06	0.093	0.092
ξ_y	0.83	0.030	0.059	0.093	0.092
τ_{lum} (min)	48	298	73	29	21

CONCLUSION

Low Higgs mass makes it feasible to build circular electron-positron colliders as a Higgs factories with high luminosity. High luminosity in Higgs factories calls for large rings with high RF power, large maximum beam-beam parameters, large dynamic aperture, small beam aspect ratio $r=\sigma_y/\sigma_x$ and optimized beta-function at IP, bunch length, beam emittance and other parameters. ξ_y^{max} is chosen based on the LEP data and beam-beam simulation; 1 mm scale β_y^* can be obtained with optimized L^*, σ_z, dynamic aperture, but remains very challenging; Beamstrahlung limiting beam lifetime prefers smaller bunch population, more bunches and lower beam emittance. Design of proposed Higgs factories is in an active progress, and their parameters have been being optimized.

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