DYNAMIC APERTURE OPTIMIZATION IN SuperKEKB

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Abstract

Colliders squeeze a beam spot at an interaction point(IP) to obtain higher luminosity. Large natural chromaticity generated in a final focus system should be corrected by strong sextupole magnets. Nonlinear effects in the sextupole field and the final focusing magnets decreases the dynamic aperture significantly. Optimization of the dynamic aperture is based on a numerical particle-tracking simulations since aberrations of particle motions due to nonlinear and higher-order effects are treated. In particular, low emittance and low beta functions at IP in SuperKEKB, the dynamic aperture is one of the important issues for both Touschek lifetime and injection efficiency. We present an optimization procedure of the dynamic aperture in SuperKEKB.

INTRODUCTION

The target luminosity of SuperKEKB is 8×10^{35} cm^{-2}s^{-1} which is 40 times higher than the peak luminosity of KEKB. In order to achieve the target luminosity, the vertical beta function at the interaction point (IP) is necessary to be squeezed down to about 300 μm and the beam current needs to be increased 3.6 A in LER with keeping the same beam-beam parameter in the vertical direction, $\sim$0.09 as KEKB.

A bunch length is 5$\sim$6 mm which is much longer than the vertical beta function to suppress coherent synchrotron radiation (CSR). "Nano-beam scheme" proposed by P. Raimondi[1] is adopted to avoid a luminosity degradation due to an hourglass effect. A large Piwinski angle is introduced in the nano-beam scheme. The crossing-angle is 83 mrad in the horizontal direction between a positron low energy ring (4 GeV, LER) and an electron high energy ring (7 GeV, HER). The horizontal emittance is reduced to 3.2$\sim$4.6 nm and the horizontal beta function is also squeezed to 25$\sim$32 mm to realized the nano-beam scheme. A small vertical emittance is necessary to obtain a higher luminosity in the nano-beam scheme. The ratio of the vertical emittance to the horizontal emittance is required to be less than $\sim$0.27 % under an influence of the beam-beam interaction as well as including machine error. The machine parameters will be found in elsewhere[2].

Touschek lifetime will be expected to be very short and the linac injector will need to be improved to provide enough injection beams to compensate short lifetime. A dynamic aperture is one of important issues at SuperKEKB because the dynamic aperture will affect both of the lifetime and the injection efficiency.

LATTICE DESIGN

The linear chromaticity of focusing magnets in a ring is written by

$$\xi_{x,y} = \frac{\partial \nu_{x,y}}{\partial \delta} = -\frac{1}{4\pi}\int K(s)\beta_{x,y}(s)ds, \qquad (1)$$

where $\nu_{x,y}$ is betatron tune and $\delta = \Delta p/p_0$ the momentum deviation from the design momentum p_0, $K(s)$ the focusing strength, $\beta_{x,y}(s)$ the beta function as a function of location s. The linear chromaticity is $\xi_x = -105$ and $\xi_y = -776$ in the LER and $\xi_x = -171$ and $\xi_y = -1081$ in the HER, respectively. The linear chromaticity is corrected with noninterleaved sextupoles at SuperKEKB. The noninterleaved chromaticity corrections in the arc section are shown in Figures 1 and 2.There are 50 pairs of sextupole magnets in the arc section and 4 pairs in the interaction region (IR). The transfer matrix between two identical sextuple magnets is $-I'$ to compensate a nonlinear kick due

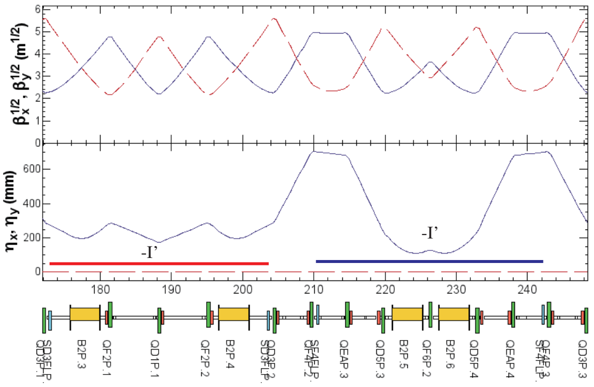

Figure 1: Arc cell in LER.

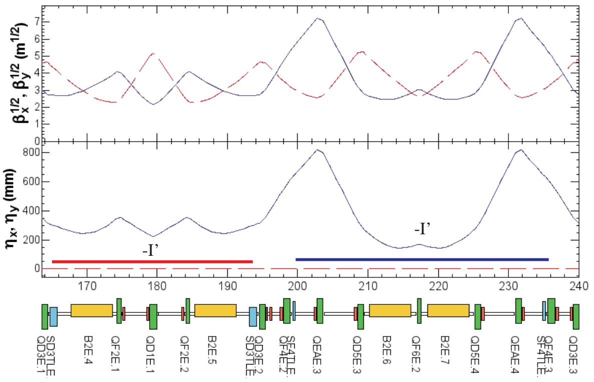

Figure 2: Arc cell in HER.

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to strong field of the sextupoles. The phase advance between two sextupoles is π in the horizontal and the vertical direction. The configuration of the focusing and the defocusing sextupoles are noninterleaved. A nonlinear kick due to one of the sextupole pair can be canceled by the other sextupole magnet for an on-momentum particle, however, the chromaticity can be corrected properly.

The final focus (FF) is designed to achieve extremely low beta functions at IP. In order to squeeze the beta functions, doublets of a vertical focusing (QC1) and a horizontal focusing quadrupole magnet (QC2) are adopted. The magnet system consists of superconducting magnets to make strong focusing strength. The magnitude of $\xi_{x,y}$ in Eq. (1) is determined by a linear optics and becomes large as increasing the focusing strength. Since approximately 80 % of the linear chromaticity in the vertical direction is induced in the FF, a local chromaticity correction (LCC) is adopted to correct the large chromaticity near the FF. There are 2 pairs for the vertical direction (Y-LCC) and another 2 pairs of sextupoles for the horizontal direction (X-LCC) in the IR. The phase advance between QC1 and the Y-LCC is π in the vertical direction and between QC2 and X-LCC is 2π in the horizontal direction for each side of IP. Horizontal dispersions are created at the LCC by using several dipole magnets. Figures 3 and 4 show the lattice design of the LCC region.

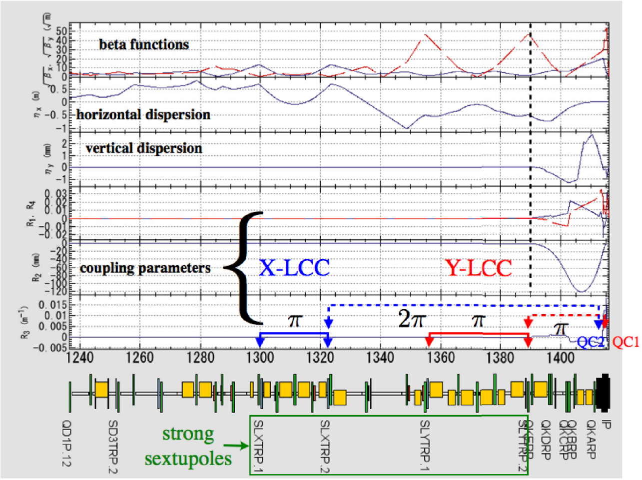

Figure 3: Local chromaticity correction in LER.

The nonlinear effect in the final focus system decreases the dynamic aperture significantly. In addition to the nonlinear magnetic field, the drift space is not linear system as shown in a Hamiltonian. Especially, when the beta function is squeezed in the vicinity of IP and decreased with distance from IP, the effect cannot be ignored. The aperture of the motion can be described by a simple one-dimensional Hamiltonian[3]. The aperture for the initial action, J_0 is expressed by

$$J_{y0} = \frac{\beta_y^{*2}}{(1+\frac{2}{3} \mid K \mid L^{*2})\, L^*} A(\mu_y), \qquad (2)$$

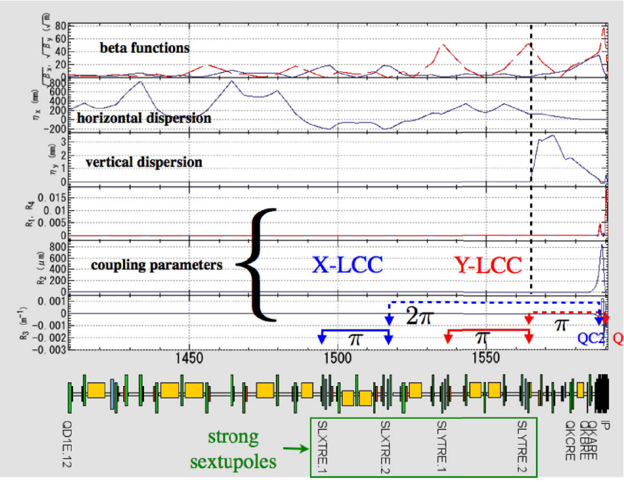

Figure 4: Local chromaticity correction in HER.

where $K = B'/B\rho$ is the focusing strength of the final focusing quadrupole in the vertical direction, L^* the distance from IP to the final focusing quadrupole, $A(\mu_y)$ a universal function determined by the one-dimensional model. Table 1 shows the parameters to evaluate the dynamic aperture in SuperKEKB.

Table 1: Estimation of the Dynamic Aperture

	LER	HER	Unit
β_y^*	270	300	μm
K	-5.1	-3.1	1/m^2
L^*	0.76	1.22	m
$J_{y0}/A(\mu_y)$	0.032	0.018	μm

OPTIMIZATION OF DYNAMIC APERTURE

It is difficult to apply either an analytic approach or a perturbative method to an evaluation of the dynamic aperture since there is the final focus system as described above and the sextupole magnets to correct the large chromaticity causes strong nonlinear effects. Therefore, the dynamic aperture is estimated by using SAD[4]. Six canonical variables, x, p_x, y, p_y, z, and δ are used to describe the motion of a particle, while p_x and p_y are transverse canonical momenta normalized by the design momentum, and δ is the relative momentum deviation from p_0. A synchrotron oscillation is included while a synchrotron radiation and a quantum excitation are turned off during tracking simulations. The FF region within ± 4 m from IP, the magnetic field of Belle II and the anti-solenoids and QCS(QC1 and QC2) along the longitudinal direction on the beam line is sliced by thickness of 10 mm of constant B_z or $K_1 = B'L/B\rho$ to make the lattice model. Higher order multipole fields up to 44-poles for normal and skew fields are included in the slices[5]. The three-dimensional solenoid field is calculated by using ANSYS[6] which is a electromagnetic field simulation code. The behavior of the solenoid field is also implemented by slices in the model[7]. The fringe field of the solenoid field and higher order multipole fields of the final focusing magnets affect the dynamic aperture significantly.

The beam lifetime should be long enough to store the beam currents stably. Touschek lifetime contributes the total lifetime significantly because of the low emittance in SuperKEKB. The target of Touschek lifetime is 600 sec for the nominal machine parameters in SuperKEKB. Touschek lifetime depends on the dynamic aperture and the density of particles in a bunch. The larger dynamic aperture is obtained by optimizing 54 families of sextupoles and 12 (LER) or 10 (HER) families of skew sextupoles in both and the LCC, and 3 (LER) or 2 (HER) octupole coils in QCs. The optimization utilizes an off-momentum matching and a down-hill simplex method as a function of an area of the dynamic aperture. The octupole magnets make the transverse dynamic aperture larger by deforming the phase space at the large amplitude to fit the physical aperture. The skew sextupole magnets correct chromatic X-Y coupling. The skew sextupole filed is induced by rotating the normal sextupole in the LER. In the HER, the skew sextupole magnets are placed in the vicinity of the sextupole magnets in the arc cell.

The dynamic aperture is estimated by a particle tracking in the LER and the HER, respectively. The particle tracking is performed for 1000 turns to define a stable region with synchrotron oscillations. The dynamic aperture is important for keeping enough Touschek lifetime as well as an injection aperture. Figures 5 and 6 show the dynamic aperture in the LER and the HER, respectively. The area of the dynamic aperture is fitted by an ellipse to estimate Touschek lifetime. Two initial betatron phases of $(0, 0)$ and $(\pi/2, \pi/2)$ in the horizontal and vertical plane are calculated in the dynamic aperture survey. Touschek lifetime is defined by average of two cases since the larger betatron amplitude becomes a nonlinear region and a Poincare plot differs from a circle. The requirement of Touschek lifetime is almost satisfied in the ideal lattice and the optimization has been still continued. The transverse dynamic aperture in the plane of betatron tunes for the case that the initial vertical amplitude is zero with the on-momentum particle is shown in Figure 7.The nominal betatron tune is $(\nu_x, \nu_y) = (44.53, 46.57)$ in the LER and $(\nu_x, \nu_y) = (45.53, 43.57)$ in the HER, respectively.

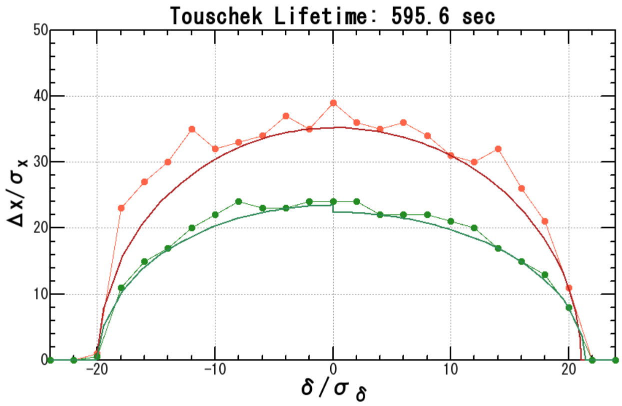

Figure 5: Dynamic aperture and Touschek lifetime in LER.

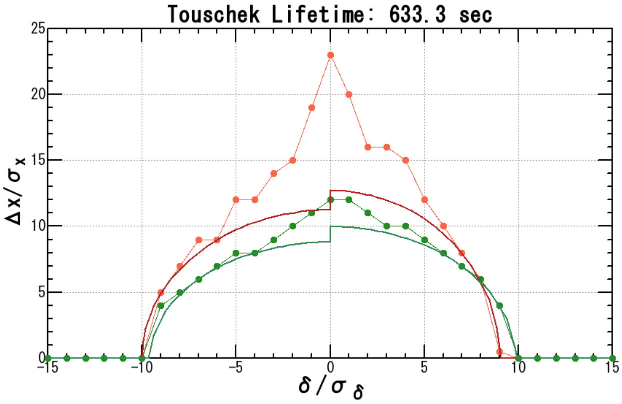

Figure 6: Dynamic aperture and Touschek lifetime in HER.

DYNAMIC APERTURE UNDER INFLUENCE OF BEAM-BEAM EFFECT

The dynamic aperture will be reduced under the influence of beam-beam effects in the nano-beam scheme. A particle with a horizontal amplitude collides at a location different from IP in the longitudinal direction due to the crossing angle of 83 mrad between two beam lines in the horizontal plane. The deviation along the beam axis is written by

$$\Delta z = \frac{\Delta x}{2\phi_x}, \tag{3}$$

where Δx is the horizontal amplitude and ϕ_x the half crossing angle. The beta function is written by a function of the

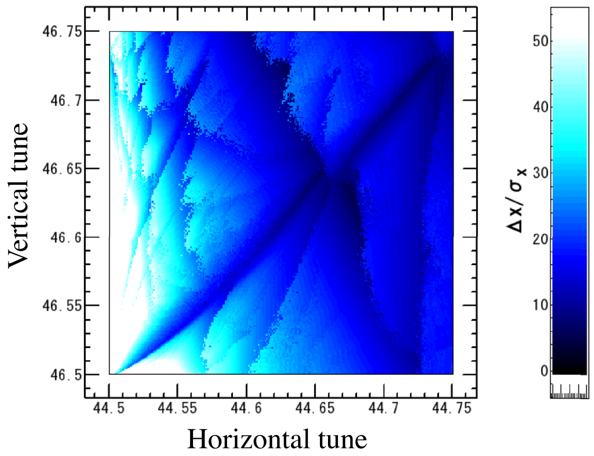

Figure 7: Tune scan for transverse aperture in LER.

distance from IP:

$$\beta_y(\Delta z) = \beta_y^* + \frac{\Delta z^2}{\beta_y^*}. \tag{4}$$

Therefore, the particle with a horizontal amplitude is kicked at a large vertical beta function and the vertical amplitude will increase due to the beam-beam interactions for an initial non-zero vertical amplitude. This behavior is a kind of an hourglass effect. The vertical amplitude given by the beam-beam kick is

$$\Delta y \propto \theta_{bb,y}\sqrt{\beta_y(\Delta z)}. \tag{5}$$

The particle is lost if the vertical amplitude increases and is out of a stable region. In the case of a particle with the horizontal amplitude of $30\sigma_x$ in the LER, the deviation from IP becomes 3.6 mm in the longitudinal direction where the vertical beta function is 48 mm. The vertical beta function becomes 180 times of the nominal beta function at IP.

Touschek lifetime in the HER reduces about 10 % due to the beam-beam effect, however, the impact in the LER is much larger than the HER. Figure 8 shows the dynamic aperture in the vertical and the horizontal plane under the influence of the beam-beam effect. The initial momentum deviation is zero in the figure. The transverse aperture is reduced significantly compared with the dynamic aperture without the beam-beam effect. The particle with the horizontal amplitude larger than $10\sigma_x$ is lost due to the vertical oscillation even though the initial amplitude is zero in the vertical direction because of the vertical amplitude is induced by nonlinearities such as X-Y coupling originate from the IR.

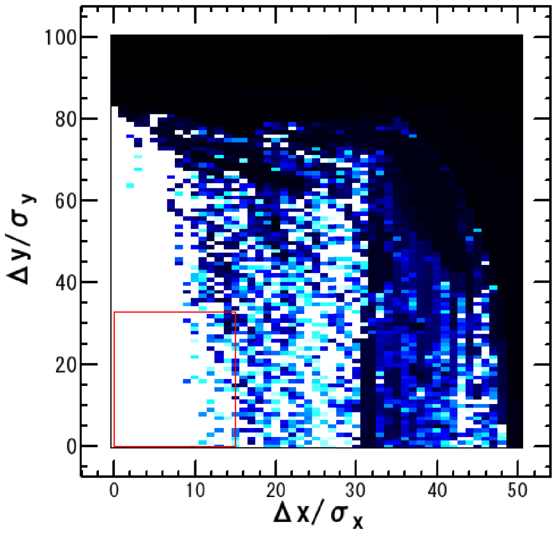

Figure 8: Dynamic aperture in the transverse plane when the initial momentum deviation is zero in LER. White color indicates a stable and black color indicates an unstable region. Rectangle (red) shows the necessary injection acceptance.

Touschek lifetime in the LER will be about 100 sec under the influence of the beam-beam effect without any optimization procedures. If we change the working point from the nominal betatron tunes to (ν_x,ν_y)=(44.53,46.54), Touschek lifetime will be improved to be 230 sec with re-optimization of the sextupoles and so on.

CRAB-WAIST SCHEME

Another approach to compensate the beam-beam effect for the large horizontal amplitude is "Crab-waist scheme"[8]. Hamiltonian of the crab-waist term is

$$H_{cw} = \frac{\lambda}{2} x p_y^2, \tag{6}$$

where

$$\lambda = \frac{1}{\tan 2\phi_x}. \tag{7}$$

If we consider the ideal case of the crab-waist scheme, the map of the beam-beam interaction is replaced with

$$f_{BB} \to f_{cw}(+\lambda)\cdot f_{BB}\cdot f_{cw}(-\lambda), \tag{8}$$

where the map of the crab-waist is

$$\begin{aligned} f_{cw}(\lambda): p_x &\to p_x + \frac{\lambda}{2}p_y^2 && (9)\\ y &\to y - \lambda x p_y. && (10) \end{aligned}$$

A feasibility of the ideal crab-waist scheme has been studied by using tracking simulations. The dynamic aperture is almost recovered by the ideal crab-waist which is compared with Figure 8. In order to accomplish the crab-waist in the realistic lattice, two sextupole magnets are utilized. In the case of the realistic lattice, the x^3 term comes from the sextupole is added to the crab-waist term in the Hamiltonian. However, it can be ignored by choosing a large ratio of the vertical beta function to the horizontal at the sextupole magnet.

Two sextupole magnets are placed for each side of IP and shift a waist of colliding particles having a horizontal amplitude to cancel the deviation from the waist. The betatron phase advance between a crab-waist sextupole and IP is adjusted to be $m\pi$ in the horizontal direction and $(n+1/2)\pi$ in the vertical direction, where m and n are arbitrary integers. The strength of the crab-waist sextupoles are

$$\mid K_2 \mid = \frac{1}{\tan 2\phi_x \beta_y^* \beta_{y,s}}\sqrt{\frac{\beta_x^*}{\beta_{x,s}}}, \tag{11}$$

where $\beta_{x,s}$ and $\beta_{y,s}$ are the horizontal and the vertical beta function at the sextupoles, respectively. The sign of K_2 is chosen so as to shift the waist at IP properly and cancel a nonlinear kick between a pair of the crab-waist sextupoles. The lattice design for the crab-waist scheme in the LER is shown in Figure 9.The crab-waist sextupole is assumed to be a thin lens to make the model simple in this report. The

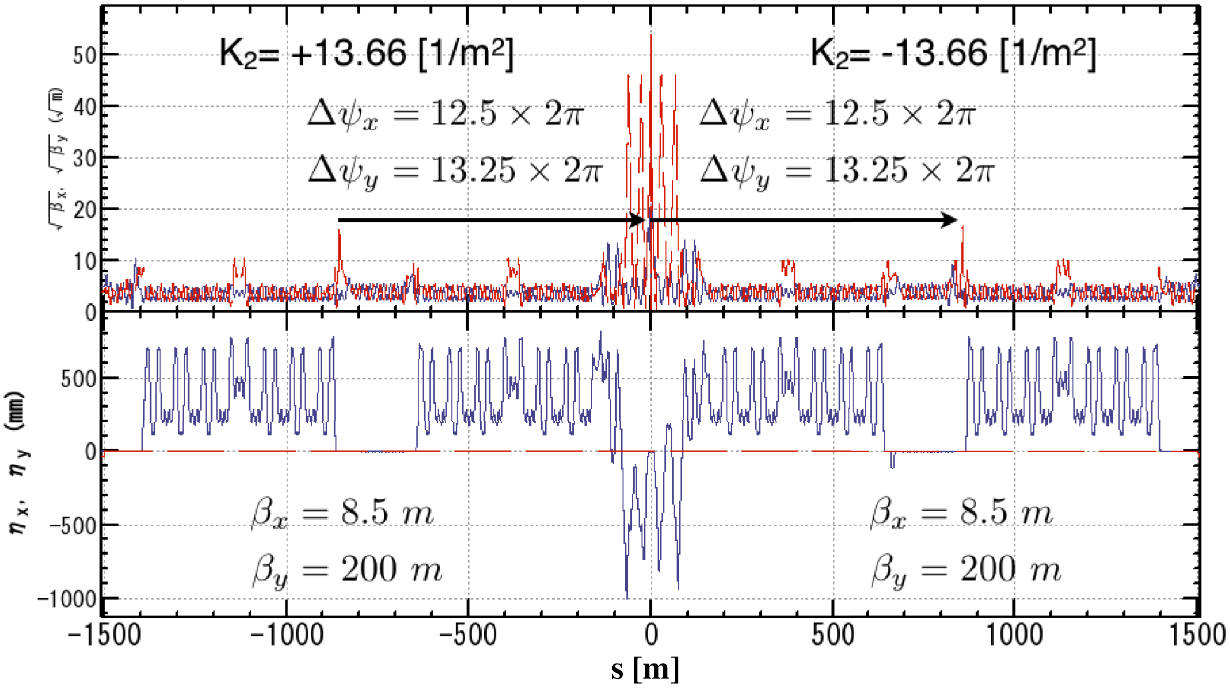

Figure 9: Lattice candidate for the crab-waist scheme at SuperKEKB.

Table 2: Machine Parameters for the Crab-waist Scheme in the LER

	Symbol	LER	Unit
Horizontal beta at IP	β_x^*	32	mm
Vertical beta at IP	β_y^*	270	μm
Half crossing angle	ϕ_x	41.5	mrad
Horizontal beta at sextupole	$\beta_{x,s}$	8.5	m
Vertical beta at sextupole	$\beta_{y,s}$	200	m
Horizontal phase advance	$\Delta\psi_x$	25π	rad
Vertical phase advance	$\Delta\psi_y$	26.5π	rad
Nominal field of sextupole	$\mid K_2 \mid$	13.66	1/m^2

machine parameters for the crab-waist scheme in the LER are shown in Table 2.

Figure 10 shows transverse dynamic aperture in the LER as a function of K_2 for the crab-waist sextupoles. The initial momentum deviation and the vertical amplitude are zero in the simulations. The dynamic aperture decreases as increasing the strength of the sextupoles. The beam-beam effect is turned off in the simulation. The nonlinear kick due to the crab-waist sextupole can be canceled by another sextupole for the reference particle, however, it cannot be canceled for a particle with a large initial amplitude. The transfer map between two sextupoles which includes the IR with the final focus is no longer the linear map. The term of $\Delta p_y = K_2 xy$ will increase the vertical amplitude, then the particle will be lost and the dynamic aperture will be reduced.

Figure 11 shows the transverse dynamic aperture in the LER which is similar to Figure 10, but the beam-beam effect is turned on. The dynamic aperture is indeed recovered by the crab-waist sextupoles as increasing the field strength until the nonlinear kick from the sextupoles restricts the dynamic aperture. Therefore, it implies the difficulty comes from the cancellation of the nonlinear kick by a pair of crab-waist sextupoles for the large horizontal amplitude of a particle without the beam-beam effect.

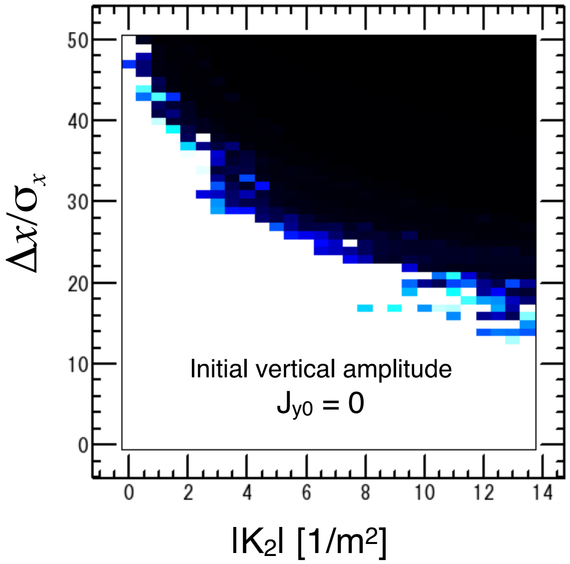

Figure 10: Dynamic aperture in the transverse plan for the crab-waist lattice without beam-beam effects.

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Fourier analysis for the crab-waist on and off is performed to investigate what frequency is a dominant source for the amplitude growth. The frequency spectrum for the vertical action is shown in Figure 12.The initial horizontal amplitude increases from 1 to 15 sigmas with keeping the initial vertical amplitude zero and the vertical amplitudes for the frequency are plotted. In the case that the crab-waist turned on, the strength of the crab-waist sextupole is K_2 =11 m^{-2} which is 80 % of the nominal value. When the spectrum is compared for each other, the spectrum for the crab-waist on is almost similar to that of turned off except for the component of the vertical tune. The vertical amplitude corresponds to the vertical tune increases as increasing the initial horizontal amplitude significantly for the crab-waist on.

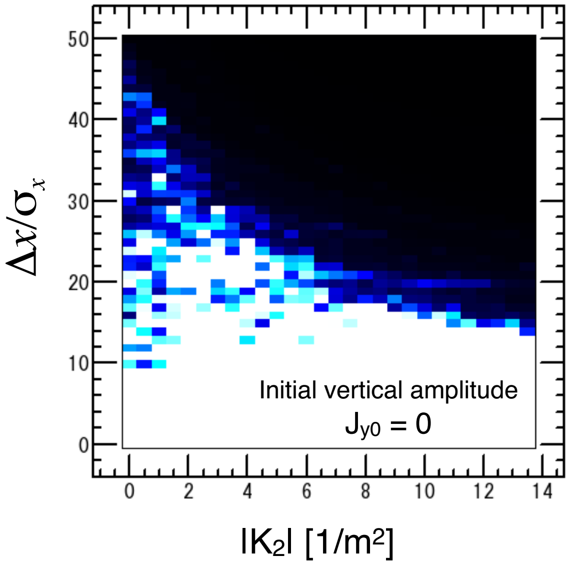

Figure 11: Dynamic aperture in the transverse plan for the crab-waist lattice with beam-beam effects.

SUMMARY

We present the issues of the dynamic aperture in SuperKEKB. The dynamic aperture is optimized for the both of LER and HER. The Touschek lifetime of 600 sec has been accomplished without machine error and beam-beam interactions. The variables to optimize the dynamic aperture are sextupoles, skew sextupoles, and octupoles which are more than 50 variables. The optimization is based on the chromaticity corrections for the off-momentum optics and the down-hill simplex method by using tracking simulations.

The dynamic aperture under the influence of the beam-beam interactions will be reduced in the nano-beam scheme. Especially, a particle with a large horizontal amplitude will receive a large beam-beam kick in the vertical plane. The Touschek lifetime in the LER is ∼200 sec so far with the beam-beam effects after choosing better working point and re-optimization. The crab-waist scheme is one of solutions to cure the beam-beam effect. However, the crab-waist has a big issue which comes from nonlinear terms between two crab-waist sextupoles without the beam-beam effect. No solution is found to alleviate the nonlinear kick due to the sextupoles in the crab-waist scheme since the transfer map between sextupoles includes strong nonlinear components such as the final focus system.

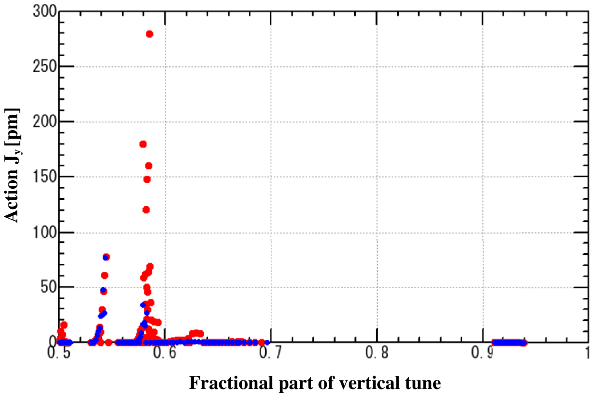

Figure 12: Action as a function of fractional part of tune in the vertical direction in LER. Blue plots show crab-waist turned off and red plots show crab-waist turned on. The field strength of the sextupole, K_2, is 11 m^{-2} for the crab-waist on.

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