ANALYTICAL ESTIMATION OF MAXIMUM BEAM-BEAM TUNE SHIFTS FOR ELECTRON-POSITRON AND HADRON CIRCULAR COLLIDERS*

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Abstract

In this paper we will make a brief review of the existing analytical formulae for the beam-beam tune shift limits for electron-positron and hadron circular colliders. The comparison of the estimated beam-beam tune shifts from these formulae with those obtained from existing machines has been made and the validity comparison among these formulae are given as well. Finally, the formulae from J. Gao have been applied in CEPC and SppC parameter optimizations.

INTRODUCTION

The luminosity of an electron-positron circular collider can be expressed as

$$L = \frac{I_{beam}\gamma\xi_y}{2er_e\beta_y^*}\left(1+\frac{\sigma_y^*}{\sigma_x^*}\right)F_h \tag{1}$$

where r_e is the electron radius (2.818×10^{-15} m), β_y^* is the beta function value at the interaction point, γ is the normalized beam energy, σ_x^* and σ_y^* are the bunch transverse dimensions at the interaction point, respectively, I_{beam} is the circulating current of one beam, F_h is Hourglass reduction factor, and ξ_y is defined as

$$\xi_y = \frac{N_e r_e \beta_y^*}{2\pi\gamma\sigma_y^*(\sigma_x^*+\sigma_y^*)} \tag{2}$$

is the vertical beam-beam tune shift, N_e is the particle population inside a bunch.

$$L = 2.17\times10^{34}(1+r)\xi_y \times \frac{E_0(GeV)N_b I_{bunch}(A)F_h}{\beta_y^*(cm)}[cm^{-2}s^{-1}] \tag{3}$$

where E_0 is the beam energy, $r=\sigma_y^*/\sigma_x^*$, N_b is the number of bunches inside a beam, I_{bunch} is the average current of a bunch, and $I_{beam}=N_b I_{bunch}$.

In fact, since ACO [1], it is found that for all circular colliders ξ_y is not a free parameter, and for a given collider, there is a maximum ξ_y, or $\xi_{y,max}$, which could not be surpassed no matter how to make working point optimization [2], and beyond $\xi_{y,max}$, the colliding bunch transverse dimensions blow-up and bunch lifetime drops drastically (exponentially in fact). These beam-beam interaction induced phenomena are called beam-beam effects. To understand the beam-beam effects is one of the key subjects for particle accelerator physicists. For a long time, in a collider design, $\xi_{y,max}$ is chosen as a constant value according to some experiences from previous machines independent of specific machine parameters, i.e., regardless whether $\xi_{y,max}$ is a function of the machine energy, damping time, number of interaction points and particle revolution period, etc. In fact, as we know from Ref. [3], for flat colliding electron-positron beams, $\xi_{y,max}$ can be expressed as (without top-up injection)

$$\xi_{y,max} = \frac{H_0}{2\pi}\sqrt{\frac{T_0}{\tau_y\gamma N_{IP}}} \tag{4}$$

where $H_0 = 2845$, τ_y is the transverse damping time, T_0 is the revolution period, and N_{IP} is the number of interaction points. Or, for isomagnetic case, one has

$$\xi_{y,max,iso} = H_0\gamma\sqrt{\frac{r_e}{6\pi R N_{IP}}} \tag{5}$$

where R is the local dipole bending radius.

Knowing the analytical expression of maximum beam-beam tune shift, $\xi_{y,max}$, one could has luminosity expressed as

$$L_{max}[cm^{-2}s^{-1}] = 2.17\times10^{34}(1+r)\xi_{y,max}\times \frac{E_0[\text{GeV}]N_b I_{bunch}[\text{A}]F_h}{\beta_y^*[\text{mm}]} \tag{6}$$

or

$$L_{max}[cm^{-2}s^{-1}] = \frac{0.158\times10^{34}(1+r)}{\beta_y^*[\text{mm}]}\times I_{beam}[\text{mA}]\sqrt{\frac{U_0[\text{GeV}]}{N_{Ib}}}F_h \tag{7}$$

where U_0 is the energy loss due to synchrotron radiation per turn, or

$$L_{max}[cm^{-2}s^{-1}] = \frac{0.158\times10^{34}(1+r)}{\beta_y^*[\text{mm}]}\times \sqrt{\frac{I_{beam}[\text{mA}]P_{sr}[\text{MV}]}{N_{Ib}}}F_h \tag{8}$$

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where P_{sr} is the synchrotron radiation power of one colliding beam.

If the collider has N_{IP} interaction points, and the total luminosity of the collider is denoted as L_{total}, it is clear that $L_{total} = N_{IP} L_{max} \propto \sqrt{N_{IP}}$.

OTHER TWO MAXIMUM BEAM-BEAM LIMIT SCALING LAWS IN ELECTRON POSITRON STORAGE RING COLLIDERS

Apart from the analytical expression given in eq. 4, which was obtained by improving the corresponding expression in ref. [4], another beam-beam tune limit scaling law was established according to LEP operation experiences at CERN [5]

$$\xi_y = I_{bunch}\sqrt{\frac{1}{A + (I_{bunch}B)^2}} \tag{9}$$

$$A = \left(\frac{2\pi e f_{rev}\gamma}{r_e}\right)^2 \frac{\beta_x^*}{\beta_y^*}\epsilon_x^0\epsilon_y^0 \tag{10}$$

$$B = \frac{1}{\xi_{y,max}} \tag{11}$$

where e is the electron charge, f_{rev} is the revolution frequency, ϵ_x^0 and ϵ_y^0 are zero current emittance, respectively, if B is not zero, means that there exists beam-beam tune limit, $\xi_{y,max}$. It is obvious to see that till now there is no information of how $\xi_{y,max}$ depends on the machine parameters, such as beam energy, revolution frequency, damping time, and number of interaction points. In ref. [5], two papers are cited to indicate that first [6]

$$\xi_{y,max} = f\left(\frac{T_0}{\tau_y N_{IP}}\right) \tag{12}$$

where f denotes a function, and the second [7]

$$\xi_{y,max} \propto \left(\frac{T_0}{\tau_y}\right)^{0.3} \tag{13}$$

Comparing eq. 4 with eq. 12, it is obvious that the γ dependance is missing in eq. 12. Comparing eq. 4 with eq. 13, it is obvious that the power dependance on damping decrement, T_0/τ_y, are different, the power dependance on damping decrement in eq. 4 is 0.5 instead of 0.3.

Talman's developed another formulation to estimate analytically the beam-beam tune shift limit [8], which will not be repeated here due to the complication of the formulation.

COMPARISONS OF THREE ANALYTICAL BEAM-BEAM TUNE LIMIT SCALING LAWS WITH EXPERIMENTS

After reviewing the three analytical formulations to estimate analytically the beam-beam tune limit in the previous two sections, it is high time now to make comparisons between analytical formulae and the experimental results from existing e^+e^- storage ring colliders. As shown in Tabs. 1-3.

From comparison results shown in Tab. 4 and Fig. 1, it is clear to see that Gao's analytical formula expressed in eq. 4 is quite close to the experimental results of machines working on both low and high energy domains, and it is used in CEPC parameter optimizations [10], as shown in Tab. 5.

According to eq. 4 above and eq. 48 in Ref. [9], for CEPC [11],one finds that $\xi_{x,max} = 0.1$ and $\xi_{y,max} = 0.073$. The $\xi_{y,max} = 0.082$ is chosen little overshot.

ANALYTICAL BEAM-BEAM TUNE SHIFT LIMIT IN HADRON CIRCULAR COLLIDERS

As for hadron circular colliders, one might want to use eq. 4 by substituting simply r_e in eq. 4 to the radius of the corresponding hadron, for example, proton's classical radius, r_p. To many peoples surprise, one can easily find out that it will give ridiculous results! Eq. 4, unfortunately, cannot be applied directly to the hadron particle cases. In the following part of this paper, as one step further, analytical formulae for the beam-beam tune shift limit for hadron circular colliders [12] will be introduced.

In fact, the physical reason for the difference between lepton and hadron circular colliders is very simple. In lepton circular colliders, due to strong strong synchrotron radiation effect, the two colliding buches could be regarded as two bunch of gases, and particles inside are in total mixing. As far as a hadron circular collider is concerned, usually, the stochastic motion will start for some particles only with strong nonlinear beam-beam forces, and the number of these particles moving in a stochastic way is smaller than the particle number in the whole bunch. The question now is to estimate how many particles located in the outer part of the bunch away from the bunche center are moving in a nonlinear beam-beam force driven stochastic motion for a given bunch current. Assuming a round colliding bunch of Gaussian transverse distribution,the number of these "heated" particles, $N_{p,heat}$, can be estimated by $N_{p,heat} = fN_{p,bunch}$, with $N_{p,bunch}$ being the particle number of the bunch. Obviously, for a lepton machine, $f = 1$.

According to ref. [12], one has the general analytical beam-beam tune shift, $\xi_{h,y,max}$ for a hadron circular collider, expressed as follows

$$\xi_{h,y,max} = \frac{H_0\gamma}{f(x_*)}\sqrt{\frac{r_p}{6\pi R N_{IP}}} \tag{14}$$

or

$$\xi_{h,y,max} = \frac{H_0}{2\pi f(x_*)}\sqrt{\frac{T_0}{\tau_y\gamma N_{IP}}} \tag{15}$$

where

$$f(x) = 1 - \frac{2}{\sqrt{2\pi}}\int_0^x \exp\left(-\frac{t^2}{2}\right)dt \tag{16}$$

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Table 1: The Comparison Result of Gao's Formula with Different Machine Experimental Values

Machine	E（GeV）	R(m)	B(T)	γ	N_{IP}	ξ_y (Gao)	ξ_y (experimental value)	ξ_y (parameter list)
DAFNE	0.51	1.42	1.2	998	1	0.0292	0.02	0.044
BEPC	1.89	9.23	0.903	3698	1	0.0423	0.04	0.035
BEPCII	1.89	9.31	0.677	3698	1	0.0422	0.04	0.0327
PEP-II(L)	3.12	13.87	0.75	6106	1	0.0570	0.06	0.0510/0.0727
PEP-II(H)	8.99	166.48	0.18	17593	1	0.0474	0.048	0.0703/0.0498
KEKB(L)	3.5	16.20	0.72	6849	1	0.0592	0.069	0.127/0.129
KEKB(H)	8.0	106.667	0.25	15656	1	0.0527	0.052	0.122/0.09
SuperKEKB(L)	4.0	70.18	0.19	7828	1	0.0325		0.0028/0.0881
SuperKEKB(H)	7.0	106.06	0.22	13699	1	0.0463		0.0012/0.0807
SuperB(L)	4.2	56	0.25	8219	1	0.0382		0.002/0.095
SuperB(H)	6.7	42.95	0.52	13111	1	0.0696		0.002/0.095
LEP-I	45.6	3096.175	0.0491	88062	4	0.0275	0.033	
LEP-II	104.5	3096.175	0.1112	191781	4	0.0639	0.079	0.025/0.065
LEP3	120	2620	0.153	234834	4	0.0798		0.09/0.08
CEPC	120	6094	0.066	234834	2	0.0739		0.104/0.074

Table 2: The Comparison Result of Assmann-Cornelis' Formula with Different Machine Experimental Values

Machine	E(GeV)	γ	C (km)	I_b (mA)	Bunch number	f_{rev}	β^*_x	β^*_y	ε^0_x (10^-9π rad-m)	ε^0_y (10^-9π rad-m)	ξ_y (calculated value)	ξ_y (experimental value)	ξ_y (parameter list)
DAFNE	0.510	998	0.098	1000	120	3.06*10^6	0.26	0.009	260	2.6	0.0549	0.02	0.044
BEPC	1.89	3698	0.2404	40	1	1.25*10^6	1.2	0.05	660	28	0.0364	0.04	0.035
BEPCII	1.89	3698	0.2375	725	88	1.26*10^6	1.0	0.015	144	2.2	0.0341	0.04	0.0327
PEP-II(L)	3.12	6106	2.2	3026	1732	136364	0.5	0.012	24	1.8	0.1386	0.06	0.0510/0.0727
PEP-II(H)	8.99	17593	2.2	1960	1732	136364	0.5	0.012	48	1.8	0.02204	0.048	0.0703/0.0498
KEKB(L)	3.5	6849	3.016	1637	1585	99469	1.2	0.0059	18	0.56	0.09385	0.069	0.127/0.129
KEKB(H)	8.0	15656	3.016	1188	1585	99469	1.2	0.0059	24	0.61	0.02472	0.052	0.122/0.09
SuperKEKB(L)	4.0	7828	3.016	3600	2500	99469	0.032	0.00027	3.2	0.0086	2.8704?		0.0028/0.0881
SuperKEKB(H)	7.0	13699	3.016	2600	2500	99469	0.025	0.0003	4.6	0.013	0.9584		0.0012/0.0807
SuperB（L）	4.2	8219	1.258	2400	978	238474	0.026	0.00025	2.0	0.005	3.4413?		0.002/0.095
SuperB（H）	6.7	13111	1.258	1900	978	238474	0.032	0.00021	2.5	0.006	1.1520		0.002/0.095
LEP-I	45.6	88062	26.66	1.28	4	11253	2.0	0.05	55.6	0.25	0.0383	0.033	
LEP-II	104.5	204501	26.66	4	4	11253	1.5	0.05	48	0.25	0.0642	0.079	0.025/0.065
LEP3	120	234834	26.66	7.2	4	11253	0.2	0.001	25	0.10	0.0854		0.09/0.08
CEPC	120	234834	53.6	16.6	50	5597	0.8	0.0012	6.79	0.0204	0.07368		0.104/0.074

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Table 3: The Comparison Result of Talman's Formula with Different Machine Experimental Values [8]

TABLE III. Parameters of some circular, flat beam, e^+e^- colliding rings, and the saturation tune shift values predicted by Eq. (41). For points not excluded by one of these factors (see table footnotes) the mean and standard deviations of theory/experiment (the last column) are 1.26 ± 0.45.

Ring	IPs	Q_x/IP	Q_y/IP	Q_s/IP	σ_s	β_y^*	$10^4\delta_y$	$\xi_{\rm th}$	$\Delta Q_{y,\rm exp}$	$\xi_{\rm th}/\Delta Q_{y,\rm exp}$
VEPP4	1	8.55	9.57	0.024	0.06	0.12	1.68	0.028	0.046	0.61
PEP-1IP	1	21.296	18.205	0.024	0.021	0.05	6.86	0.076	0.049	1.55
PEP-2IP	2	5.303	9.1065	0.0175	0.020	0.14	4.08	0.050	0.054	0.93
CESR-4.7	2	4.697	4.682	0.049	0.020	0.03	0.38	0.037	0.018	2.06
CESR-5.0	2	4.697	4.682	0.049	0.021	0.03	0.46	0.034	0.022	1.55
CESR-5.3	2	4.697	4.682	0.049	0.023	0.03	0.55	0.029	0.025	1.16
CESR-5.5	2	4.697	4.682	0.049	0.024	0.03	0.61	0.027	0.027	1.00
CESR-2000	1	10.52	9.57	0.055	0.019	0.02	1.113	0.028	0.043	0.65
KEK-1IP	1	10.13	10.27	0.037	0.014	0.03	2.84	0.046	0.047	0.98
KEK-2IP	2	4.565	4.60	0.021	0.015	0.03	1.42	0.048	0.027	1.78
LEP-46	4[c]	22.58	19.04	0.016	0.0076	0.05	0.958	0.128	0.034	
LEP-65	4[c]	22.57	19.04	0.019	0.009	0.05	2.7	0.086		
LEP-98	4[c]	24.59	24.05	0.029	0.0110	0.05	8.6	0.12[b]	0.052	
PEP-LER	1	38.65	36.58	0.027	0.0123	0.0125	1.17	0.044	0.044	1.00
PEP-HER	1	24.57	23.64	0.045	0.0115	0.0125	1.98	0.056	0.026[a]	
KEK-LER	1	45.518	44.096	0.021	0.0057	0.007	2.34	0.042	0.032	1.31
KEK-HER	1	44.525	42.135	0.019	0.055	0.007	2.18	0.060	0.018[a]	
BEPC	1	5.80	6.70	0.020	0.05	0.05	0.16	0.068	0.039	1.74

[a]Ion effect blowup of low energy beam may prevent beam-beam saturation.
[b]Theory value is erratic
[c]Unequally spaced IPs.

Table 4: The Comparison Result of The Errors of Three Different Formulae with Different Machine Experimental Values

Machine	ξ_y Gao's Theory	ξ_y Assmann's theory	ξ_y Talman's theory	ξ_y (experimental value)	ξ_y (parameter list)
DAFNE	0.0292	0.0549		0.02	0.044
BEPC	0.0423	0.0364	0.068	0.04	0.035
BEPCII	0.0422	0.0341		0.04	0.0327
PEP-II(L)	0.0570	0.1386	0.044	0.06	0.0510/0.0727
PEP-II(H)	0.0474	0.02204	0.056	0.048	0.0703/0.0498
KEKB(L)	0.0592	0.09385	0.042	0.069	0.127/0.129
KEKB(H)	0.0527	0.02472	0.060	0.052	0.122/0.09
SuperKEKB(L)	0.0325	2.8704?			0.0028/0.0881
SuperKEKB(H)	0.0463	0.9584			0.0012/0.0807
SuperB（L）	0.0382	3.4413?			0.002/0.095
SuperB（H）	0.0696	1.1520			0.002/0.095
LEP-I	0.0275	0.0383	0.128	0.033	
LEP-II	0.0639	0.0642	0.12	0.079	0.025/0.065
LEP3	0.0798	0.0854			0.09/0.08
CEPC	0.0739	0.07368			0.104/0.074

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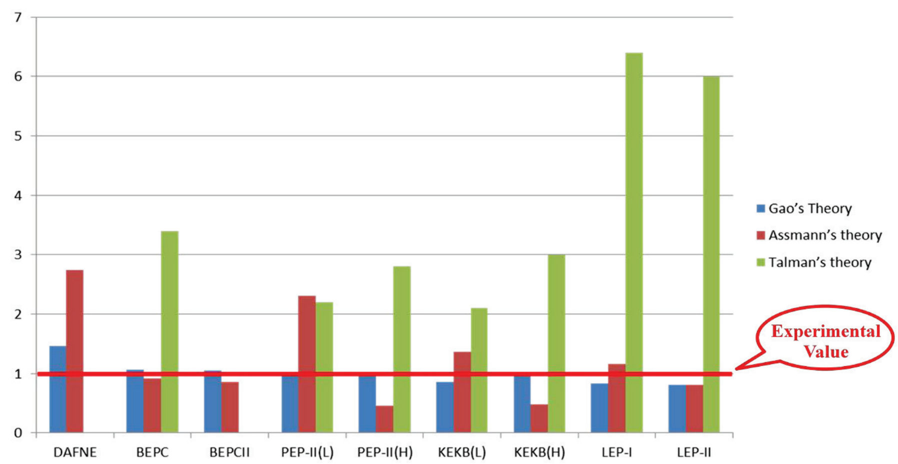

Figure 1: The errors between the analytical estimations and the experimental results vs different machines working on different energys.

Table 5: The CEPC Parameter List

Parameter	Unit	Value	Parameter	Unit	Value
Beam energy [E]	GeV	120	Circumference [C]	m	54420
Number of IP[N_{IP}]		2	SR loss/turn [U_0]	GeV	3.11
Bunch number/beam[n_B]		50	Bunch population [Ne]		3.71E+11
SR power/beam [P]	MW	51.7	Beam current [I]	mA	16.6
Bending radius [r]	m	6094	momentum compaction factor [a_p]		3.39E-05
Revolution period [T_0]	s	1.82E-04	Revolution frequency [f_0]	Hz	5508.87
emittance (x/y)	nm	6.12/0.018	b_{IP}(x/y)	mm	800/1.2
Transverse size (x/y)	mm	69.97/0.15	$x_{x,y}$/IP		0.116/0.082
Beam length SR [$s_{s.SR}$]	mm	2.17	Beam length total [$s_{s.tot}$]	mm	2.53
Lifetime due to Beamstrahlung	min	80	lifetime due to radiative Bhabha scattering [t_L]	min	52
RF voltage [V_{rf}]	GV	6.87	RF frequency [f_{rf}]	MHz	650
Harmonic number [h]		117900	Synchrotron oscillation tune [n_s]		0.18
Energy acceptance RF [h]	%	5.98	Damping partition number [Je]		2
Energy spread SR [$s_{d.SR}$]	%	0.13	Energy spread BS [$s_{d.BS}$]	%	0.08
Energy spread total [$s_{d.tot}$]	%	0.16	n_g		0.23
Transverse damping time [n_x]	turns	78	Longitudinal damping time [n_e]	turns	39
Hourglass factor	Fh	0.692	Luminosity /IP[L]	$cm^{-2}s^{-1}$	2.01E+34

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Table 6: Comparison of Gao's Formulae to Some Hadron Circular Colliders

Machine	E(TeV)	R(m)	γ	N_{IP}	f(x)	ξ_y(Gao)	ξ_y(exp. value)	ξ_y(para.list)
Tevatron	0.98	682	1048	2	0.0141217	0.00149268		0.012
HERA(p)	0.92	588	984	2	0.0140073	0.00147705		0.0009
LHC	7	2801	7458	2	0.0252262	0.00320658	0.0034	0.005
SSC	22	9824	23400	2	0.0313979	0.00431618		0.0021
HL-LHC	7	2801	7458	2	0.0252262	0.00320658		0.0075
HE-LHC	16.5	2750	17581	2	0.0363833	0.00528936		0.005
FCC-hh	50	10416	53277	2	0.0437486	0.0068494		0.005
SppC	37.4	6236	39872	2	0.0431489	0.0067169		0.006

Table 7: SppC Parameter (1)

Parameter	Value	Unit
Main parameters		
Circumference	56	km
Beam energy	37.4	TeV
Lorentz gamma	39891	
Dipole field	20	T
Dipole curvature radius	6236	m
Arc filling factor	0.79	
Total dipole magnet length	39184	m
Arc length	49600	m
Total straight section length	6400	m
Energy gain factor in collider rings	17.8	
Injection energy	2.1	TeV
Number of IPs	2	
Revolution frequency	5.36	kHz
Physics performance and beam parameters		
Peak luminosity per IP	1.3E+35	$cm^{-2}s^{-1}$
Beta function at collision	0.75	m
Circulating beam current	1.0	A

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Table 8: SppC Parameter (2)

Max beam-beam tune shift per IP	**0.006**	
Bunch separation	25	ns
Number of bunches	5973	
Bunch population	2.0E+11	
Accumulated particles per beam	1.2E+15	
Normalized rms transverse emittance	4.1	mm
Beam life time due to burn-off	9.3	hour
Total / inelastic cross section	140	mbarn
Reduction factor in luminosity	0.96	
Full crossing angle	71	mrad
rms bunch length	75.5	mm
rms IP spot size	9.0	mm
Beta at the 1st parasitic encounter	19.5	m
rms spot size at the 1st parasitic encounter	46.1	mm
Stored energy per beam	6.3	GJ
SR power per beam	2.1	MW
SR heat load at arc dipoles	63.9	W/m
Energy loss per turn	2.45	MeV

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$$x^2 = \frac{4f(x)}{\pi \xi_{y,max} N_{IP}} \quad (17)$$

and x_* in eq. 15 could be solved by the following equation

$$x_*^2 = \frac{4f(x_*)^2}{H_0 \pi \gamma} \sqrt{\frac{6\pi R}{r_p N_{IP}}} \quad (18)$$

Before making any application of eqs. 15 and 18 to estimate a hadron circular collider beam-beam tune limit, we made a comparison with the existing machines and some machines under design as shown in Tab. 6 [13]. It is obvious that the analytical estimation gives good prediction.

By using SppC's parameter [11], shown in Tabs. 7 and 8, one gets from eqs. 15 and 18 that $\xi_{SppC,y,max} = 0.06$.

CONLCUSION

In this paper, we reviewed three analytical formulae of the beam-beam tune limit estimation for e^+e^- circular colliders, and one for hadron colliders. It is shown that Gao's formulae, both for lepton and hadron circular colliders, provide reasonable estimations, and they are used to make CEPC and SppC parameter optimizations.

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