LATTICE OPTIMIZATION FOR TOP-OFF INJECTION

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Abstract

This paper discusses Higgs factory injection. Full energy, top-off injection is assumed. Vertical injection seems preferable to horizontal and kicker-free, bunch-by-bunch injection concurrent with physics running may be feasible. Achieving high efficiency injection justifies optimizing injector and/or collider lattices for maximum injection efficiency. Stronger focusing in the injector and weaker focusing in the collider improves the injection efficiency. Scaling formulas (for the dependence on ring radius R) show injection efficiency increasing with increasing ring circumference. Scaling up from LEP, more nearly optimal parameters for both injector and collider are obtained. Maximum luminosity favors adjusting the collider cell length L_c for maximum luminosity and choosing a shorter injector cell length, $L_i < L_c$, for maximizing injection efficiency.

INJECTION STRATEGY: STRONG FOCUSING INJECTOR, WEAK FOCUSING COLLIDER

Introduction

I take it as given that full energy top-off injection will be required for the FCC electron-positron Higgs factory. Without reviewing the advantages of top-off injection, one has to be aware of one disadvantage. The cost in energy of losing a full energy particle due to injection inefficiency is the same as the cost of losing a circulating particle to Bhabha scattering or to beamstrahlung or to any other loss mechanism. Injection efficiency of 50% is equivalent to doubling the irreducible circulating beam loss rate. To make this degradation unimportant one should therefore try to achieve 90% injection efficiency.

Achieving high efficiency injection is therefore sufficiently important to justify optimizing one or both of injector and collider lattices to improve injection. The aspect of this optimization to be emphasized here is shrinking the injector beam emittances and expanding the collider beam acceptances by using stronger focusing in the injector than in the collider. What are the dynamic aperture implications? They will be shown to be progressively more favorable as the ring radius R is increased relative to the LEP value. The dynamic-aperture/beam-width ratio increases as $R^{1/2}$ and is the same for injector and collider. Before addressing this optimization other injection issues will be surveyed.

Handling the synchrotron radiation at a Higgs Factory is difficult and replenishing the power loss is expensive. Otherwise the RF power loss is purely beneficial, especially for injection. Betatron damping decrements δ (fractional amplitude loss per turn) are approximately half the energy loss per turn divided by the beam energy, (e.g. $\delta \approx 0.5\times 2.96/120 = 1.25\,\%$.) Also the energy dependence is large enough for injection efficiency to improve significantly with increasing energy.

According to Liouville's theorem, increasing the beam particle density by injection is impossible for a Hamiltonian system. The damping decrement δ measures the degree to which the system is *not* Hamiltonian. Usually bumpers and kickers are needed to keep the already stored beam captured while the injected beam has time to damp. If δ is large enough one can, at least in principle, inject with no bumpers or kickers.

Advantages of Vertical Injection and Bumper-Free, Kicker-Free, Top-Off Injection

The most fundamental parameter limiting injection efficiency is the emittance of the injected beam. The vertical emittance in the booster accelerator can be very small, perhaps $\epsilon_y < 10^{-10}$ m. This may require a brief flat top at full energy in the booster. For injection purposes the beam height can then be taken to be effectively zero. The next most important injector parameter is the septum thickness. For horizontal injection this septum also has to carry the current to produce a horizontal deflection. Typically this requires the septum thickness to be at least 1 mm. For vertical injection, with angular deflection not necessarily required, the septum can be very thin, even zero. The remaining (and most important) injection uncertainty is whether the ring dynamic aperture extends out to the septum. If not, it may be possible to improve the situation by moving the closed orbit closer to the wall using DC bumpers. (However this may be disadvantageous for vertical injection as vertical bends contribute unwanted vertical emittance to the stored beams.)

A virtue of top-off injection is that, with beam currents always essentially constant, the linear part of the beam-beam tune shift can be designed into the linear lattice optics. One beam "looks", to a particle in the other beam, like a lens (focusing in both planes). Large octupole moments makes this lens far from ideal. Nevertheless, if the beam currents are constant the pure linear part can be subsumed into the linear lattice model. And the octupole component, though nonlinear, does not necessarily limit the achievable luminosity very severely.

With injection continuing during data collection there would be no need for cycling between injection mode and data collection mode. This could avoid the need for the always difficult "beta squeeze" in transitioning from injection mode to collision mode. Runs could then last for days, always at maximum luminosity. This would improve both average luminosity and data quality.

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Kicker-free vertical injection is indicated schematically in Figure 1. Let $n_{\rm inj.}$ be a small integer indicating the number of turns following injection before the injected beam threatens to wipe out on the injection septum. The fractional shrinkage of the Courant-Snyder invariant after $n_{\rm inj.}$ turns is $n_{\rm inj.}\delta$. By judicious choice of vertical, horizontal, and synchrotron tunes this shrinkage may be great enough that less than, say, 10 % of the beam is lost on the septum.

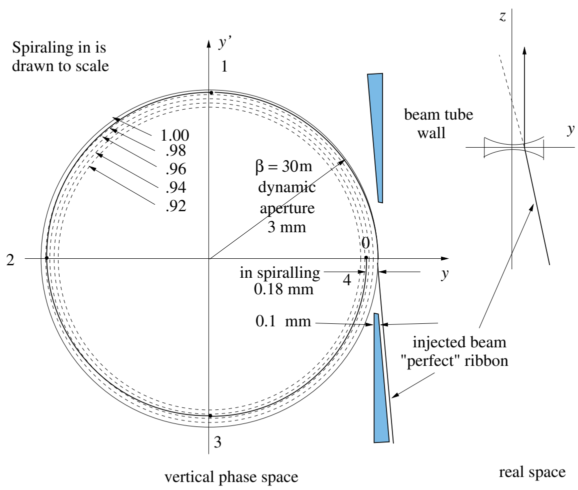

Figure 1: A cartoon of kicker-free, vertical injection. The dashed line shows the Courant-Snyder amplitude of the injected particle with the fractional shrinking per turn drawn more or less to scale.

CONSTANT DISPERSION SCALING WITH BEND RADIUS *R*

Linear Lattice Optics

Most of the following scaling formulas come from Jowett [1] or Keil [2] or from reference [3]. For simplicity, even if it is not necessarily optimal, assume the Higgs factory arc optics can be scaled directly from LEP values, which are: phase advance per cell $\mu_x = \pi/2$, full cell length $L_c = 79$ m. (The subscript "c" distinguishes the collider lattice cell length from the injector lattice cell length L_i.) At constant phase advance, the beta function β_x scales as L_c and dispersion D scales as bend angle per cell $\phi = L_c/R$ multiplied by cell length L_c;

$$D \propto \frac{L_c^2}{R}. \tag{1}$$

Holding L_c constant as R is increased would decrease the dispersion, impairing our ability to control chromaticity. Let us therefore *tentatively adopt the scaling*

$$L_c \propto R^{1/2}, \quad \text{corresponding to} \quad \phi \propto R^{-1/2}. \tag{2}$$

This can be seen to be tantamount to holding dispersion D constant, and is consistent with electron storage ring design trends over the decades.

These quantities and "Sands curly H" $\mathcal{H}$ then scale as

$$\beta_x \propto R^{1/2}, \quad D \propto 1, \quad \mathcal{H} \propto \frac{D^2}{\beta_x} \propto \frac{1}{R^{1/2}}. \tag{3}$$

Copied from Jowett [1], the fractional energy spread is given by

$$\sigma_\epsilon^2 = \frac{55}{32\sqrt{3}}\frac{\hbar}{m_e c}\gamma_e, \quad \text{where}$$
$$F_\epsilon = \frac{<1/R^3>}{J_x<1/R^2>} \propto \frac{1}{R}, \tag{4}$$

and the horizontal emittance is given by

$$\epsilon_x = \frac{55}{32\sqrt{3}}\frac{\hbar}{m_e c}\gamma_e F_x, \quad \text{where}$$
$$F_x = \frac{<\mathcal{H}/R^3>}{J_x<1/R^2>} \propto \frac{1}{R^{3/2}}. \tag{5}$$

The betatron contribution to beam width scales as

$$\sigma_{x,\rm betatron} \propto \sqrt{\beta_x\epsilon_x} \propto 1/R^{1/2}. \tag{6}$$

Similarly, at fixed beam energy, the fractional beam energy (or momentum) spread σ_δ scales as

$$\sigma_\delta \propto \sqrt{B} \propto 1/R^{1/2}. \tag{7}$$

Scaling with R of Arc Sextupole Strengths and Dynamic Aperture

At this stage in the Higgs Factory design, it remains uncertain whether the IP-induced chromaticity can be cancelled locally, which would promise well over than a factor of two increase in luminosity, but would require strong bends close to the IP. For the time being I assume the IP chromaticity is cancelled in the arcs. Individual sextupole strengths can be apportioned as

$$S = S^{\rm arc\ chr.} + S^{\rm IP\ chr.} \tag{8}$$

The IP-compensating sextupole portion $S^{\rm IP\ chr.}$ depends on the IP-induced chromaticity. A convenient rule of thumb has the IP chromaticity equal to the arc chromaticity. By this rule doubling the arc-compensating sextupole strengths cancels both the arc and the IP chromaticity.

With dispersion $D \propto 1$, quad strength $q \propto 1/R^{1/2}$, and $S^{\rm arc\ chr.} \propto q/D$, one obtains the scaling of sextupole strengths and dynamic aperture scaling;

$$S \propto \frac{1}{R^{1/2}}, \quad \text{and} \quad x^{\rm dyn.\ ap.} \propto \frac{q}{S^{\rm arc\ chr.}} \propto 1. \tag{9}$$

The most appropriate measure of dynamic aperture is

$$\frac{x^{\rm dyn.\ ap.}}{\sigma_x} \propto \frac{1}{1/R^{1/2}} \propto R^{1/2}. \tag{10}$$

The increase of dynamic aperture divided by beam size with increasing R would allow the IP optics to be more aggressive for the Higgs factory than for LEP. Unfortunately it is

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the chromatic mismatch between IP and arc that is thought to be more important in limiting the dynamic aperture than is the simple compensation of total chromaticity. The constant dispersion scaling formulas derived so far are given in Table 1.

INJECTION-OPTIMIZED SCALING FOR INJECTOR AND COLLIDER

Revised Injector and Collider Parameters

What has been discussed so far has been "constant dispersion scalling". But, as already stated, we wish to differentiate the injector and collider optics such that the injector emittances are smaller and the collider acceptances are larger. This can be accomplished by shortening injector length L_i and lengthening collider cell length L_c. The resulting R-dependencies and scaling formulas are shown in Table 2. They yield the lattice parameters in Table 3 for both the 50 km and 100 km circumference options.

Implications of Changing Lattices for Improved Injection

According to these calculations there is substantial advantage and no disadvantage to strengthening the injector focusing and weakening the collider focusing. This is achieved by shortening the injector cell length L_i and increasing the collider cell length L_c. Weakening the collider focusing has the effect of increasing the transverse beam sizes

As indicated in the caption to Table 3, the improvement in the injector emittance/collider acceptance ratio is probably unnecessaily large, at least for a 100 km ring, where the improvement is seven-fold.

Furthermore there is at least one more constraint that needs to be met. Maximum luminosity results only when the beam aspect ratio at the crossing point is optimal. Among other things this imposes a condition of the horizontal emittance ϵ_x. At the moment the preferred method for controlling ϵ_x is by the appropriate choice of cell length L_c. With lattice manipulations other than changing the cell length it may be possible to increase, but probably not decrease ϵ_x.

According to Table 3 of my WG 2 paper, "Ring Circumference and Two Rings vs One Ring", with $\beta_y^* = 5$ mm the optimal choice of ϵ_x is 3.98 nm. According to Table 3 of the present report the actual value will be $\epsilon_x = 7.82$ nm. These values can be considered "close enough for now", or they can be considered different enough to argue that further design refinement is required (which is obvious in any case). But the suggestion is that the $L_c = 213$ m collider cell length choice in this paper may be somewhat too long.

Unfortunately the optimal value of ϵ_x depends strongly on the optimal value of β_y^*, which is presently unkown. These considerations show that the arc and intersection region designs cannot be separately optimized. Rather a full ring optimization is required.

REFERENCES

[1] J. Jowett, *Beam Dynamics at LEP,* CERN SL/98-029 (AP), 1998

[2] E. Keil, *Lattices for Collider Storage Rings,* Section in Accelerator Handbook, edited by A. Chao and M. Tigner.

[3] R. Talman, *Accelerator X-Ray Sources,* Wiley-VCH Verlag, 2006

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Table 1: *Constant dispersion* scaling is the result of choosing cell length $L \propto R^{1/2}$. This is empasized by the shaded row, where the 1 in the final column indicates constancy as the ring radius is changed. The table gives the scaling with R of other lattice and beam parameters.

Parameter	Symbol	Proportionality	Scaling
phase advance per cell	μ		1
cell length	L		$R^{1/2}$
bend angle per cell	ϕ	$= L/R$	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L$	$R^{-1/2}$
dispersion	D	ϕL	1
beta	β	L	$R^{1/2}$
tunes	Q_x, Q_y	R/β	$R^{1/2}$
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-1/2}$
partition numbers	$J_x/J_y/J_\epsilon$	= 1/1/2	1
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-3/2}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$\sqrt{\beta\epsilon_x}$	$R^{-1/2}$
-synchrotron	$\sigma_{x,synch.}$	$D\sigma_\delta$	$R^{-1/2}$
sextupole strength	S	q/D	$R^{-1/2}$
dynamic aperture	$x^{\max}$	q/S	1
relative dyn. aperture	$x^{\max}/\sigma_x$		$R^{1/2}$
pretzel amplitude	x_p	σ_x	$R^{-1/2}$

Table 2: To improve injection efficiency (compared to constant dispersion scaling) the injector cell length can increase more weakly, for example $L_i \propto R^{1/4}$, and the collider cell length can increase more strongly, for example $L_i \propto R^{3/4}$. The shaded entries assume circumference C=100 km, R/R_{LEP}=3.75.

Parameter	Symbol	Proportionality	$L \propto R^{1/4}$ injector	$L \propto R^{1/2}$	$L \propto R^{3/4}$ collider
phase advance per cell	μ_x		90°	90°	90°
cell length	L		$R^{1/4}$	$R^{1/2}$	$R^{3/4}$
			110 m	153 m	213 m
bend angle per cell	ϕ	$= L/R$	$R^{-3/4}$	$R^{-1/2}$	$R^{-1/4}$
momentum compaction		ϕ^2	$R^{-3/2}$	R^{-1}	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L$	$R^{-1/4}$	$R^{-1/2}$	$R^{-3/4}$
dispersion	D	ϕL	$R^{-1/2}$	1	$R^{1/2}$
beta	β	L	$R^{1/4}$	$R^{1/2}$	$R^{3/4}$
tune	Q_x	R/β	$R^{3/4}$	$R^{1/2}$	$R^{1/4}$
			243.26	174.26	125.26
tune	Q_y	R/β	$R^{3/4}$	$R^{1/2}$	$R^{1/4}$
			205.19	147.19	106.19
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-5/4}$	$R^{-1/2}$	$R^{1/4}$
partition numbers	$J_x/J_y/J_\epsilon$	1/1/2	1/1/2	1/1/2	1/1/2
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-9/4}$	$R^{-3/2}$	$R^{-3/4}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$	$R^{-1/2}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$= \sqrt{\beta\epsilon_x}$	R^{-1}	$R^{-1/2}$	1
-synchrotron	$\sigma_{x,synch.}$	$= D\sigma_\delta$	R^{-1}	$R^{-1/2}$	1
sextupole strength	S	q/D	$R^{1/4}$	$R^{-1/2}$	$R^{-5/4}$
dynamic aperture	x^{da}	q/S	$R^{-1/2}$	1	$R^{1/2}$
relative dyn. aperture	x^{da}/σ_x		$R^{1/2}$	$R^{1/2}$	$R^{1/2}$
separation amplitude	x_p	σ_x	N/A	$R^{-1/2}$	1

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Table 3: Lattice parameters for improved injection efficiency. The shaded row indicates how successfully the injector emittance has been reduced relative to the collider emittance. The factor of seven improvement, 7.82/1.08, in this ratio for a 100 km ring, seems unnecessarily large, indicating that less radical scaling should be satisfactory.

Parameter	Symbol	LEP(sc)	Unit	Injector		Collider	
mean bend radius	R	**3026**	m	**5675**	**11350**	**5675**	**11350**
beam Energy		120	GeV	120	120	120	120
circumference	C	26.7	km	50	100	50	100
cell length	L	79	m	92.4	110	127	213
momentum compaction	α_c	1.85e-4	m	0.72e-4	0.25e-4	1.35e-4	0.96e-4
tunes	Q_x	90.26		144.26	243.26	105.26	125.26
	Q_y	76.19		122.19	205.19	89.19	106.19
partition numbers	$J_x/J_y/J_\epsilon$	1/1.6/1.4		1/1/2	1/1/2	1/1/2	1/1/2
main bend field	B_0	0.1316	T	0.0702	0.0351	0.0702	0.0351
energy loss per turn	U_0	6.49	GeV	3.46	1.73	3.46	1.73
radial damping time	τ_x	0.0033	s	0.0061	0.0124	0.0061	0.0124
	τ_x/T_0	37	turns	69	139	69	139
fractional energy spread	σ_δ	0.0025		0.0018	0.0013	0.0018	0.0013
emittances (no BB), x	ϵ_x	**21.1**	nm	**5.13**	**1.08**	**13.2**	**7.82**
y	ϵ_y	1.0	nm	0.25	0.05	0.66	0.39
max. arc beta functs	$\beta_x^{\max}$	125	m	146	174	200	337
max. arc dispersion	$D^{\max}$	0.5	m	0.37	0.26	0.68	0.97
quadrupole strength	$q \approx \pm 2.5/L_p$	0.0316	1/m	0.027	0.0227	0.0197	0.0117
max. beam width (arc)	$\sigma_x = \sqrt{2\beta_x^{\max}\epsilon_x}$	$1.6\sqrt{2}$	mm	$0.865\sqrt{2}$	$0.433\sqrt{2}$	$1.62\sqrt{2}$	$1.62\sqrt{2}$
(ref) sextupole strength	$S = q/D$	0.0632	1/m^2	0.0732	0.0873	0.0290	0.0121
(ref) dynamic aperture	$x^{\rm da} \sim q/S$	~0.5	m	~0.370	~0.260	~0.679	~0.967
(rel-ref) dyn.ap.	$x^{\rm da}/\sigma_x$	~0.313		~0.428	~0.600	~0.417	~0.621
separation amplitude	$\pm 5\sigma_x$	$\pm 8.0\sqrt{2}$	mm			$\pm 8.1\sqrt{2}$	$\pm 7.8\sqrt{2}$

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