

# IMPEDANCE AND COLLECTIVE EFFECTS STUDIES IN CEPC\*

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## Abstract

Circular electron-positron collider (CEPC) is a 120GeV storage ring-based collider. Due to the small beam size and high single bunch population, the collective effects may bring new challenges to the physical design of the machine. A thorough evaluation of the coupling impedance is necessary in controlling the total impedance of the ring, which can accordingly prevent the occurrence of the beam instability. The primary studies on the impedance and collective effects in CEPC are presented.

## INTRODUCTION

Interaction of an intense charged particle beam with the vacuum chamber surroundings may lead to collective instabilities. These instabilities can induce beam quality degradation or beam loss, and finally restrict the luminosity of the machine. Therefore, beam instability study is essential for designing a new machine. In this paper, the primary calculations of the impedances are first given. Based on the impedance studies, beam instabilities due to single bunch and multi bunch effects are estimated. Instabilities due to interaction of electron beam with the residual ions and positron beam with the electron cloud are also investigated. The main parameters used in the calculation are listed in Table 1.

Table 1: Main Parameters of CEPC

Parameter	Symbol, unit	Value
Beam energy	$E$ , GeV	120
Circumference	$C$ , m	54752
Beam current	$I_0$ , mA	16.6
Bunch number	$n_b$	50
Bunch length	$\sigma_z$ , mm	2.65
RF frequency	$f_{rf}$ , GHz	0.65
Energy spread	$\sigma_e$	$1.63 \times 10^{-3}$
Slipping factor	$\alpha_p$	$3.36 \times 10^{-5}$
Betatron tune	$\nu_x/\nu_y$	179.08/179.22
Synchrotron tune	$\nu_s$	0.18
Damping time	$\tau_x/\tau_y/\tau_z$ , ms	14/14/7

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## IMPEDANCE

Since most of the engineering designs of the vacuum objects are not done yet, only the RF cavities and the resistive wall impedance are considered here. A more complete impedance budget will be obtained as more vacuum components are designed.

### RF Cavities

A five cell superconducting RF cavity structure with RF frequency of 650 MHz will be used in CEPC. Given an accelerating gradient of 15.5 MV/m, 384 cavities will be needed. Since the RF cavities are axisymmetric, the impedance and wake are calculated with the code ABCI [1]. The short range wake at nominal bunch length is shown in Fig. 1. We fit the bunch wake with the analytical model [2]

$$W(s) = -Rc\lambda(s) - Lc^2\lambda'(s), \quad (1)$$

where  $L$  and  $R$  are effective inductance and resistance, respectively. The calculated loss factor for one RF cavity is  $k_f=2.332$  V/pC.

### Resistive Wall

The resistive wall wake for a Gaussian bunch in a cylindrical beam pipe is calculated analytically [3]

$$W(s) = \frac{cl}{8\sqrt{2}\pi i} \frac{1}{\sigma_z^{3/2}} \sqrt{\frac{Z_0}{\sigma_c}} f(s/c), \quad (2)$$

where

$$f(x) = \sqrt{|x|^3} e^{-x^2/4} (I_{1/4} - I_{-3/4} \pm I_{-1/4} \mp I_{3/4})_{|x^2/4}, \quad (3)$$

and  $I_n(x)$  is the modified Bessel function of the first kind.

Aluminium beam pipes will be used in CEPC. The beam pipe has an elliptical cross section with half height of dimension of  $a_x=52$  mm and  $a_y=28$  mm. We use the vertical aperture in the calculation and obtain the longitudinal wake as shown in Fig. 1.

Impedance budget of the objects considered is given in Table 2.

Table 2: Summary of the Impedance Budget

Objects	$R$ , k $\Omega$	$L$ , nH	$k_{\text{loss}}$ , V/pC
RF cavities	28.1	--	895.5
Resistive wall	9.7	126.8	309.6
Total	37.8	126.8	1205.1

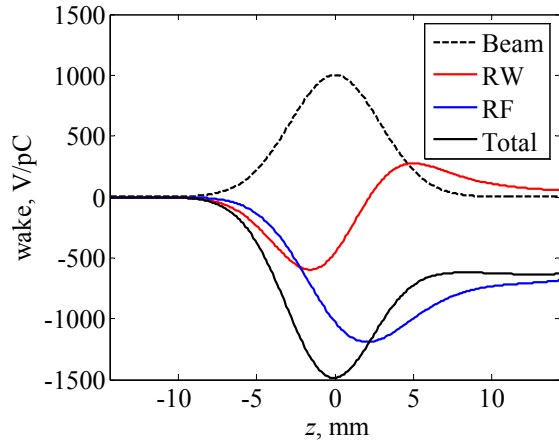


Figure 1: Longitudinal short range wake of different vacuum components at nominal bunch length.

### SINGLE BUNCH EFFECTS

#### Bunch Lengthening

Interaction of the beam with broadband impedance can change the bunch length and longitudinal distribution due to potential well distortion. The longitudinal bunch density distribution is obtained by numerically solving the Haissinski equation [4, 5]

$$\rho(z) = \rho(0) \exp \left[ -\frac{1}{2} \left( \frac{\omega_s z}{\eta c \sigma_s} \right)^2 + \frac{r_0}{\eta \sigma_s^2 \gamma C} \int_0^z dz'' \int_{z''}^{\infty} dz' \rho(z') W_0'(z'' - z') \right]. \quad (4)$$

The Pseudo-Green function wake with bunch length of 0.5mm as shown in Fig. 2 is used in the instability calculation. The longitudinal bunch density with the influence of the wake is shown in Fig. 3. We can see that the bunch is shortened due to the capacitive property of the RF cavity.

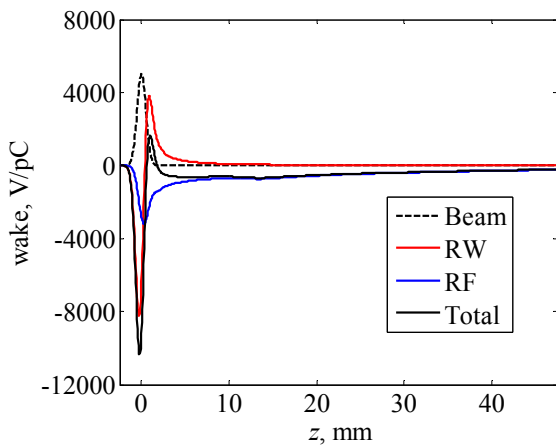


Figure 2: Pseudo-Green function wake with  $\sigma_z=0.5$ mm.

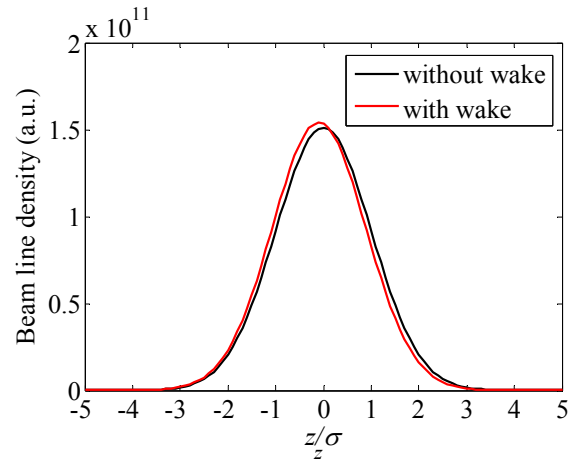


Figure 3: Steady-state longitudinal bunch distribution.

Here, only the impedances of resistive wall and RF cavity are considered in the calculation. Since it is impossible to get a complete impedance model at the present stage, we used SuperKEKB's wake model to do the estimation. The total wake is the SuperKEKB HER or LER's wake scaled by the ratio of the circumference of CEPC to SuperKEKB. The bunch lengthening with different bunch population is simulated. The result is shown in Fig. 4. The red and green curves are the cases with simple scaling by the ratio of the circumference, while the blue and pink ones with a more careful scaling, in which the total wake includes the wake of

- CEPC RF+RW
- bellows, flanges, pumping ports, SR masks, BPMs scaled by the ratio of the circumferences
- feedback kicker and longitudinal kicker without scaling
- collimators, IR duct scaled by the number of IP

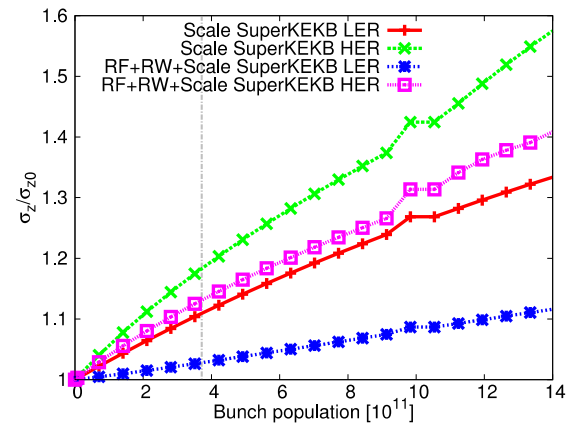


Figure 4: Bunch lengthening vs. bunch population. The grey dashed line indicates the design bunch population.

#### Microwave Instability

The average threshold current for the longitudinal microwave instability is estimated according to the Bousard or Keil-Schnell criterion [6, 7]

$$I_{th} = \frac{\sqrt{2\pi}\alpha_p \frac{E}{e} \sigma_e^2 \sigma_z}{R \left| \frac{Z}{n} \right|_{eff}}. \quad (5)$$

For nominal design current, the threshold impedance is 0.025Ω. The microwave instability is also simulated with the scaled SuperKEKB's wake. The result is shown in Fig. 5. The red and green curves correspond to rough scaling, while the blue and pink ones correspond to careful scaling. We can see that, with LER wake, the threshold bunch population is higher than 10<sup>12</sup>, while with the HER wake, the threshold bunch population is about 9×10<sup>11</sup>.

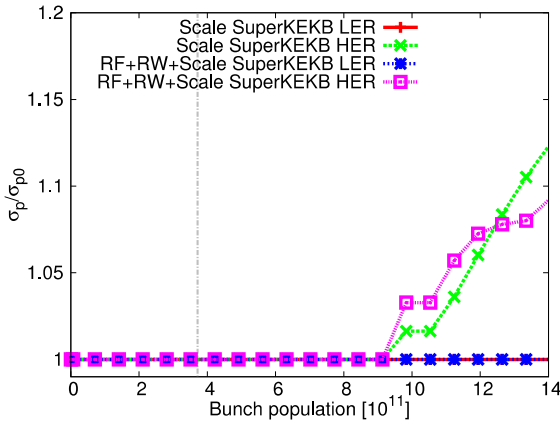


Figure 5: Energy spread vs. bunch population. The grey dashed line indicates the design bunch population.

### Transverse Mode Coupling Instability

The threshold bunch current for the transverse mode coupling instability is estimated using eigen mode analysis. Figure 6 shows the dependence of the frequency shift of the head-tail modes with the bunch current. As a primary study, only the resistive wall impedance is considered. The threshold bunch current is 3.4 mA.

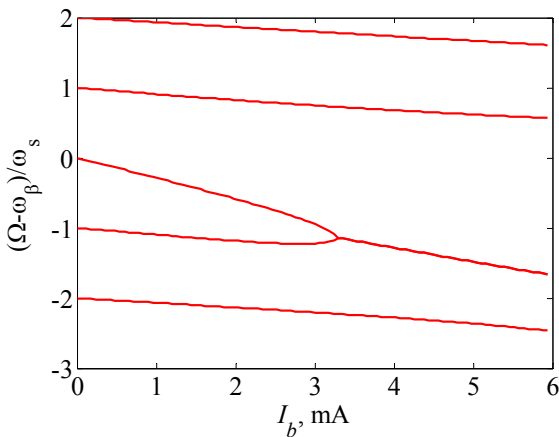


Figure 6: Dependences of the head-tail mode frequencies on the bunch current.

### Beam Tilt Due to Transverse Wake Fields

When a beam passes through an impedance with a transverse offset, the tail particles will receive transverse kicks and induce bunch shape distortion. The transverse kick experienced by a particle located at longitudinal position  $z$  is given by [4]

$$\Delta y'(z) = \frac{Ne^2}{E} \int_0^\infty dz' \rho(z'+z) W_\perp(y_b, z'). \quad (6)$$

This will lead to a transverse displacement of the bunch tail at IP [8, 9]

$$\Delta y = \sqrt{0.5\beta_y^* \beta_y} \Delta y', \quad (7)$$

where  $\beta_y^*$  and  $\beta_y$  are the vertical beta function at the IP and at the location of the impedance, respectively.

Considering a pretzel orbit of 5 mm in the horizontal plane and closed orbit of 1mm in the vertical plane, the transverse kicks along the bunch excited by the impedance of one RF cavity in both planes are shown in Fig. 7 and Fig. 8.

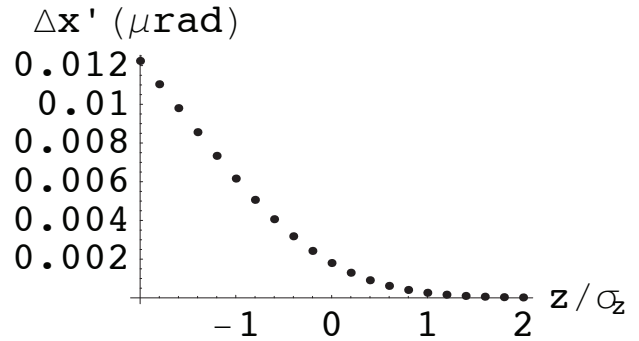


Figure 7: Transverse kick along the bunch in horizontal.

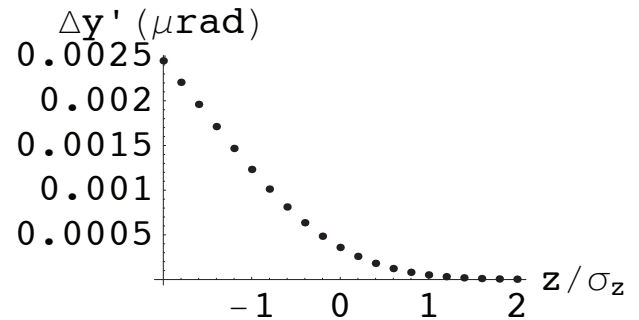


Figure 8: Transverse kick along the bunch in vertical.

The maximum kick angle excited by the transverse impedance of one RF cavity is 12nrad in horizontal and 2nrad in vertical. The corresponding displacements at IP are 54.0nm (horizontal) and 0.42nm (vertical). Since there are 384 cavities located in 8 positions in the ring, the

displacement at IP is  $48\sqrt{8} \times 54.0\text{nm} = 7.3\mu\text{m}$  in horizontal and  $48\sqrt{8} \times 0.42\text{nm} = 57\text{nm}$  in vertical.

### Coherent Synchrotron Radiation

In evaluating the coherent synchrotron radiation (CSR) effect, the beam is assumed to be moving in a circle of radius  $\rho$  between two parallel plates at locations  $y = \pm h$ . From the linear theory, the condition for the onset of coherent synchrotron radiation is given by the threshold current  $S_{th}$ , which is given as a function of shielding parameter  $\Pi$  [10]

$$S_{th} = 0.50 + 0.12\Pi, \quad (8)$$

where

$$S = \frac{r_e N_b \rho^{1/3}}{2\pi v_s \gamma \sigma_\delta \sigma_z^{4/3}}, \quad \Pi = \frac{\sigma_z \rho^{1/2}}{h^{3/2}}. \quad (9)$$

The CEPC design parameters give  $\Pi=16$ , which means CSR is well shielded. The threshold bunch population is about  $7.3 \times 10^{12}$ , which is much higher than the designed value of  $3.79 \times 10^{11}$ .

## MULTI-BUNCH EFFECTS

### Transverse Resistive Wall Instability

One of the main origins for exciting the transverse multi-bunch instability is due to the interaction of the beam with the resistive wall impedance. Considering  $n_b$  uniformly distributed bunches, the rise time of the transverse multi-bunch instability can be estimated by [4]

$$\frac{1}{\tau_\perp} = \frac{n_b I_b c}{4\pi(E/e)v_{x,y}} \sum_{p=-\infty}^{\infty} e^{-(\omega_p - \xi\omega_0/\alpha_p)^2 \sigma_\tau^2} \text{Re} Z_\perp(\omega_p). \quad (10)$$

where  $\omega_p = (pn_b + \mu + \nu\beta)\omega_0$ .

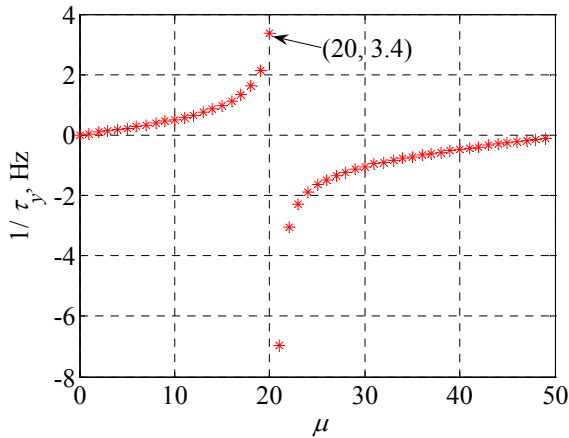


Figure 9: Resistive wall instability growth rate versus oscillation mode number.

Figure 9 shows the growth rate of the transverse resistive wall instability with different mode numbers. We can

see that the rise time for the most dangerous mode is about 0.3s. The rise time is much higher than the transverse radiation damping rate.

### Coupled Bunch Instability Induced by the RF HOM's

Another dominant contribution to the coupled bunch instability is the higher order modes (HOM) of the accelerating cavities. In the resonant condition, i.e. when the resonant frequency is coincident with the beam spectrum, the growth time can be given by [4]

$$\frac{1}{\tau_\parallel} = \frac{\alpha_p I_0 \omega_R R_\parallel}{4\pi v_s (E/e)}, \quad (11)$$

and

$$\frac{1}{\tau_\perp} = \frac{\beta_{x,y} I_0 \omega_0 R_\perp}{4\pi(E/e)}, \quad (12)$$

To keep the beam stable, the radiation damping time should be less than the rise time of any of the oscillation modes. Then we obtain the threshold for the longitudinal impedance is shown in Fig. 10. The threshold for the transverse impedance is  $3.9\text{M}\Omega/\text{m}$ .

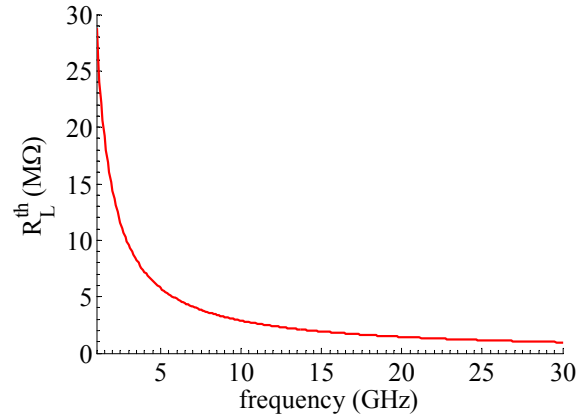


Figure 10: Longitudinal impedance threshold of the RF HOM's.

## ELECTRON CLOUD INSTABILITY

The threshold volume density of the electron cloud for the head-tail instability is given by [11, 12]

$$\rho_{e,th} = \frac{2\gamma v_s \omega_e \sigma_z / c}{\sqrt{3KQr_0\beta L}}, \quad (13)$$

where  $K = \omega_e \sigma_z / c$ ,  $Q = \min(Q_{nl}, \omega_e \sigma_z / c)$ ,  $Q_{nl}$  depends on the nonlinear interaction, and  $\omega_e$  the electron oscillation frequency. Here, we take  $Q_{nl} = 7$  for analytical estimation, and get the threshold density for the single bunch instability is  $9.3 \times 10^{11} \text{m}^{-3}$ .

For the multi-bunch instability, the electron cloud is considered as a rigid Gaussian beam with the chamber size. The characteristic frequency is

$$\omega_{G,y}^2 = \frac{2\lambda_b r_e c^2}{(\Sigma_x + \Sigma_y) \Sigma_y}, \quad (14)$$

where  $\Sigma_x$  and  $\Sigma_y$  are horizontal and vertical electron cloud sizes, and  $\Sigma_{x(y)} \gg \sigma_{x(y)}$ . The phase angle between adjacent bunches is  $\omega_G L_{sep}/c = 32.3$ . So the electrons are not supposed to accumulate and the multipacting effect is low.

## BEAM ION INSTABILITY

In an electron ring, instabilities can be excited by the ions of the residual gas accumulated in the potential well of the electron beam. With uniform filling, the ions with relative molecular mass greater than  $A_{x,y}$  will be trapped

$$A_{x,y} = \frac{N_b r_p S_b}{2(\sigma_x + \sigma_y) \sigma_{x,y}}. \quad (15)$$

Figure 11 shows the critical mass number  $A_{x,y}$  along the ring. As the threshold is quite high, the ions will not be trapped by the beam.

Fast beam ion instability is a transient beam instability excited by the beam generated ions accumulated in a single passage of the bunch train. The phase angle between adjacent bunches is  $\omega_i L_{sep}/c = 42$ . So the ions will not accumulate due to the over-focus inside the bunch train.

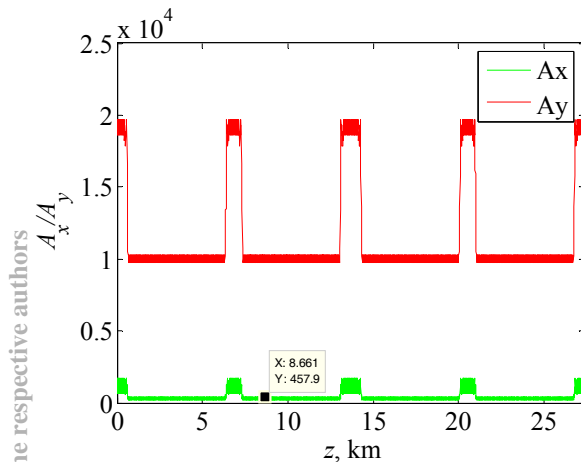


Figure 11: Critical mass number along half of the ring.

## SUMMARY

According to the primary analysis, CEPC should be safe from the microwave instability and transverse mode coupling instability. With only the impedance of resistive wall and RF cavities, the bunch length is reduced due to the capacitive property of the RF cavity. Analysis based on SuperKEKB's geometry impedance shows bunch lengthening of about 10%.

Bunch shape distortion due to the transverse wake is another potential restriction to the high luminosity. More detail analysis take into account impedance localization are needed.

Coupled bunch instabilities are less serious compare to the single bunch effects since the bunch spacing is large. Electron cloud and ion instability should not be a problem due to the overfocus inside the bunch train.

The above analyses are based on a very rough impedance model. A complete impedance model is needed to get more accurate instability estimations. On the other hand, the impedance should be carefully studied and well controlled to suppress the single bunch effects.

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