RING CIRCUMFERENCE AND TWO RINGS VS ONE RING

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Abstract

The natural next future circular collider is a circular e+e- Higgs Factory and, after that, a post-LHC p,p collider in the same tunnel. The main Higgs factory cost-driving parameter choices include: tunnel circumference C, whether there is to be one ring or two, what is the installed power, and what is the "Physics" for which the luminosity deserves to be maximized. This paper discusses some of the trade-offs among these choices, and attempts to show that the optimization goals for the Higgs factory and the later p,p collider are consistent.

GENERAL COMMENTS

The quite low Higgs mass (125 GeV) makes a circular e+e- collider (FCC-ep) ideal for producing background-free Higgs particles. There is also ample physics motivation for planning for a next-generation proton-proton collider with center of mass energy approaching 100 TeV. This suggests a two-step plan: first build a circular e+e- Higgs factory; later replace it with a 100 TeV pp collider (or, at least, center of mass energy much greater than LHC). This paper is devoted almost entirely to the circular Higgs factory step, but keeping in mind the importance of preserving the p,p collider potential.

The main Higgs factory cost-driving parameter choices include: tunnel circumference C, whether there is to be one ring or two, what is the installed power, and what are the physics priorities. From the outset I confess my prejudice towards a single LEP-like ring, optimized for Higgs production at $E = 120$ Gev, with minimum *initial* cost, and highest possible eventual p,p energy. This paper discusses some of the trade-offs among these choices, and attempts to show that electron/positron and proton/proton optimization goals are consistent.

Both Higgs factory power considerations and eventual p,p collider favor a tunnel of the largest possible radius R. Obviously one ring is cheaper than two rings. For 120 GeV Higgs factory operation (and higher energies) it will be shown that one ring is both satisfactory and cheaper than two. But higher luminosity (by a factor of five or so) at the (45.6 GeV) Z_0 energy, requires two rings.

Unlike the Z_0, there is no unique "Higgs Factory energy". Rather there is the threshold turn-on of the cross section shown, for example, in Figure 1 of my WG 2 paper "Single Ring Multibunch Operation and Beam Separation".

We arbitrarily choose 120 GeV per beam as the Higgs particle operating point and identify the single beam energy this way in subsequent tables. Similarly identified are the Z_0 energy (45.6 GeV), the W-pair energy of 80 GeV, the LEP energy (arbitrarily taken to be 100 GeV) and the $t\bar{t}$ energy of 175 GeV to represent high energy performance.

SCALING UP FROM LEP TO HIGGS FACTORY

Scaling Radius and Power Inversely Conserves Luminosity

Most of the conclusions in this paper are based on scaling laws, either with respect to bending radius R or with respect to beam energy E. Scaling with bend radius R is equivalent to scaling with circumference C. (Because of limited "fill factor", RF, straight sections, etc., $R \approx C/10$.)

Higgs production was just barely beyond the reach of LEP's top energy, by the ratio 125 GeV/105 GeV = 1.19. This should make the extrapolation from LEP to Higgs factory quite reliable. In such an extrapolation it is increased radius more than increased beam energy that is mainly required.

One can note that, for a ring three times the size of LEP, the ratio of E^4/R (synchrotron energy loss per turn) is $1.19^4/3 = 0.67$—i.e. *less than final LEP operation*. Also, for a given RF power $P_{\rm rf}$, the total number of stored particles is proportional to R^2—doubling the ring radius cuts in half the energy loss per turn and doubles the time interval over which the loss occurs. These comments deflate a long-held perception that LEP had the highest practical energy for an electron storage ring.

There are three distinct upper limit constraints on the luminosity. Maximum luminosity results when the parameters have been optimized so the three constraints yield the same upper limit for the luminosity. For now we concentrate on just the simplest luminosity constraint $\mathcal{L}^{\rm RF}_{\rm pow}$, the maximum luminosity for given RF power $P_{\rm rf}$. With n_1 being number of stored particles per MW; f the revolution frequency; N_b the number of bunches, which is proportional to R; σ^*_y the beam height at the collision point; and aspect ratio σ^*_x/σ^*_y fixed (at a large value such as 15);

$$\mathcal{L}^{\rm RF}_{\rm pow} \propto \frac{f}{N_b}\Big(\frac{n_1 P_{\rm rf}[\rm MW]}{\sigma^*_y}\Big)^2. \tag{1}$$

Consider variations for which

$$P_{\rm rf} \propto \frac{1}{R}. \tag{2}$$

Dropping "constant" factors, the dependencies on R are, $N_b \propto R$, $f \propto 1/R$, and $n_1 \propto R^2$. With the $P_{\rm rf} \propto 1/R$ scaling of Eq. (2), $\mathcal{L}$ is independent of R. In other words, the luminosity depends on R and $P_{\rm rf}$ only through their product

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$RP_{\rm rf}$. Note though, that this scaling relation *does not* imply that $\mathcal{L} \propto P_{\rm rf}^2$ at fixed R.

This radius/power scaling formula can be checked numerically by comparing Tables 5 and 6 in the present paper, which assume 100 km circumference, 25 MW/beam, with the corresponding tables in my "Single Ring Multi-bunch Operation and Beam Separation" paper which assume 50 km circumference, 50 MW/beam. The comparison is only approximate since other parameters and the scalings from LEP are not exactly the same in the two cases.

Parameter Scaling with Radius

For simplicity, even if it is not necessarily optimal, let us assume the Higgs factory arc optics can be scaled directly from LEP values, which are: phase advance per cell $\mu_x = \pi/2$, full cell length $L_c = 79$ m. (The subscript "c" distinguishes the Higgs factory collider lattice cell length from injector lattice cell length L_i.)

Constant dispersion scaling formulas are given in Table 1. These formulas are derived in my WG 6 paper "Lattice Optimization for Top-Off Injection", at this meeting. These formulas are then applied to extrapolate from LEP to find the lattice parameters for Higgs factories of circumference 50 km and 100 km, shown in Table 4.

Table 1: Constant ispersion (see shaded row) caling with R of arious attice and eam arameters

Parameter	Symbol	Proportionality	Scaling
phase advance per cell	μ		1
collider cell length	L_c		$R^{1/2}$
bend angle per cell	ϕ	$= L_c/R$	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L_c$	$R^{-1/2}$
dispersion	D	ϕL_c	1
beta	β	L_c	$R^{1/2}$
tunes	Q_x, Q_y	R/β	$R^{1/2}$
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-1/2}$
partition numbers	$J_x/J_y/J_\epsilon$	= 1/1/2	1
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-3/2}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$\sqrt{\beta\epsilon_x}$	$R^{-1/2}$
-synchrotron	$\sigma_{x,synch.}$	$D\sigma_\delta$	$R^{-1/2}$
sextupole strength	S	q/D	$R^{-1/2}$
dynamic aperture	$x^{\rm max}$	q/S	1
relative dyn. aperture	$x^{\rm max}/\sigma_x$		$R^{1/2}$
pretzel amplitude	x_p	σ_x	$R^{-1/2}$

STAGED OPTIMIZATION

A Cost Model

To maximize both the likelihood of initial approval and the eventual p,p performance, the cost of the first step has to be minimized and the tunnel circumference maximized. Surprisingly, these requirements are quite consistent. Consider optimization principles for three FCC stages:

- **Stage I, e+e-:** Starting configuration. Minimize cost at "respectable" luminosity, e.g. 10^{34}. Constrain the number of rings to 1, and the number of IP's to $N^* = 2$.
- **Stage II, e+e-:** Maximize luminosity/cost for production Higgs (etc.) running. Upgrade the luminosity by some combination of: $P_{\rm rf} \rightarrow 2P_{\rm rf}$ or $4P_{\rm rf}$, one ring $\rightarrow$ two rings, increasing N^* from 2 to 4, or decreasing β_y^*.
- **Stage III, pp:** Maximize the ultimate physics reach, i.e. center of mass energy, i.e. maximize tunnel circumference.

Cost Optimization

Treating the cost of the 2 detectors as fixed, and letting C be the cost exclusive of detectors, the cost can be expressed as a sum of a term proportional to size and a term proportional to power;

$$C = C_R + C_P \equiv c_R R + c_P P_{\rm rf} \tag{3}$$

where c_R and c_P are unit cost coefficients. As given by Eq. (2), for constant luminosity, the RF power, luminosity, and ring radius, for small variations, are related by

$$P_{\rm rf} = \frac{\mathcal{L}}{k_1 R}. \tag{4}$$

Minimizing C at fixed $\mathcal{L}$ leads to

$$R_{\rm opt} = \sqrt{\frac{1}{k_1}\frac{c_P}{c_R}\mathcal{L}}. \tag{5}$$

Conventional thinking has it that c_P is universal world wide but, at the moment, c_R is thought to be somewhat cheaper in China than elsewhere. If so, the optimal radius should be somewhat greater in China than elsewhere. Exploiting $P_{\rm rf} \propto \mathcal{L}/R$, some estimated costs (in arbitrary cost units) and luminosities for Stage I and (Higgs Factory)Stage-II are given in Table 2. The luminosity estimates are from Table 6 and are explained in later sections and in my WG 6 paper, "Lattice Optimization for Top-Off Injection".

Table 2: Estimated costs, one ring in the upper table, two in the lower. *A crude LEP spread sheet shows that doubling the radius and halving the power leaves the accelerator cost not very much changed. Also bending magnet costs are assumed to be proportional to stored magnetic energy.

	R km	$P_{\rm rf}$ MW	$C_{\rm tun}$ arb.	$C_{\rm acc}$ arb.	Phase-I cost arb.	$\mathcal{L}^I$ (Higgs) 10^{34}	$\mathcal{L}^I$ (Z_0) 10^{34}	$\mathcal{L}^{II}$ (Higgs) 10^{34}
1	5	50	0.5	2.5	3.0	1.2	2.6	2
	10	25	1.0	2.5*	**3.5**	1.2	5.2	**5**
	10	50	1.0	4.0	5.0	2.3	10.4	5
2	5	50	0.5	4.5	5.0	1.2	21	2
	10	25	1.0	4.5*	5.5	1.2	21	5
	10	50	1.0	7.0	8.0	2.3	42	5

Note that doubling the radius, while cutting the power in half, increases the cost only modestly, while leaving generous options for upgrading to maximize Higgs luminosity, as well as maximizing the potential p,p physics reach. The shaded row in Table 2 seems like the best deal. Both Higgs factory and, later, p,p luminosities are maximized, and the initial cost is (almost) minimized. Of course this optimization has been restricted to a simple choice between 50 km and 100 km circumference.

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LUMINOSITY LIMITING PHENOMENA

Saturated Tune Shift

My electron/positron beam-beam simulation [2] dead reckons the saturation tune shift $\xi_{\max}$ which is closely connected to the maximum luminosity. For an assumed $R \propto E^{5/4}$ scaling, $\xi_{\max}$ is plotted as a function of machine energy E in Figure 1. This plot assumes that the r.m.s. bunch-length σ_z is equal to β_y^*, the vertical beta function at the intersection point (IP).

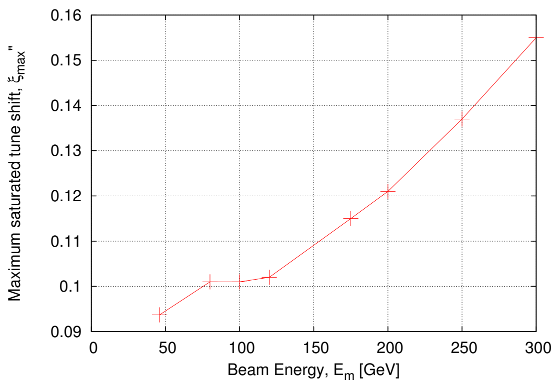

Figure 1: Plot of maximum tune shift $\xi_{\max}$ as a function of maximum beam energy for rings such that $E \propto R^{5/4}$. The non-smoothness has to be blamed on statistical fluctuations in the Monte Carlo program calculation. The maximum achieved tune shift parameter 0.09 at 100 GeV at LEP was less than shown, but their torturous injection and energy ramping seriously constrained their operations.

The physics of the simulation assumes there is an equilibrium established between beam-beam heating versus radiation cooling of vertical betatron oscillations. Under ideal single beam conditions the beam height would be $\sigma_y \approx 0$. This would give infinite luminosity in colliding beam operation —*this is unphysical*. In fact beam-beam forces cause the beam height to grow into a new equilibrium with normal radiation damping. It is parametric modulation of the vertical beam-beam force by horizontal betatron and longitudinal synchrotron oscillation that modulates the vertical force and increases the beam height. The resonance driving strength for this class of resonance is proportional to $1/\sigma_y$ and would be infinite if σ_y=0—*which is also unphysical*. Nature, "abhoring" both zero and infinity, plays off beam-beam emittance growth against radiation dampling. However amplitude-dependent detuning limits the growth, so there is only vertical beam growth but no particle loss (at least from this mechanism). In equilibrium the beam height is proportional to the bunch charge. The simulation automatically accounts for whatever resonances are nearby.

To estimate Higgs factory luminosity the tune plane is scanned for various vertical beta function values and bunch lengths, as well as other, less influential, parameters. The resulting ratio ($\xi^{\rm sat}/\beta_y^*$) is plotted in Figure 2. The ratio $\xi^{\rm sat.}/\beta_y^*$ determines the beam area A_{β_y} just sufficient for

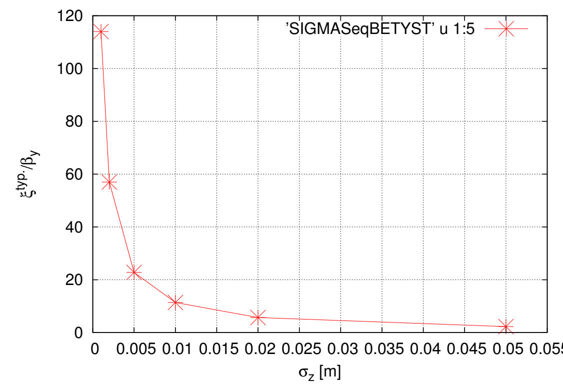

Figure 2: Plot of $\xi^{\rm sat.}/\beta_y$ as a function of σ_z, with $\beta_y^* = \sigma_z$, $\delta = 0.00764$, and synchrotron tune advance between collisions $Q_s = 0.0075$.

vertical saturation according to the formula,

$$A_{\beta_y} = \pi\sigma_x\sigma_y = \frac{N_p r_e}{2\gamma}\frac{1}{(\xi^{\rm sat.}/\beta_y)}. \tag{6}$$

This fixes the tune-shift-saturated charge density (per unit transverse area). It is only the product $\sigma_x\sigma_y$ that is fixed but there is a broad optimum in luminosity for aspect ratio $a_{xy} = \sigma_x/\sigma_y \approx 15$. To within this ambiguity all transverse betatron parameters are then fixed. β_x^* is adjusted to make horizontal and vertical beam-beam tune shifts approximately equal. The lattice optics is adjusted so that the (arc-dominated) emittance ϵ_x gives the intended aspect ratio a_{xy}; $\epsilon_x = \sigma_x^2/\beta_x^*$.

(Incidentally, it will not necessarily be easy to optimize ϵ_x for each beam energy. My W6 paper "Lattice Optimization for Top-Off Injection" discusses tailoring cell length L_c to adjust ϵ_x. Unfortunately other considerations influence the choice of L_c and, in any case, once optimized for one energy, L_c remains fixed at all energies.)

Beamstrahlung

"Beamstrahlung" is the same as synchrotron radiation, except that it occurs when a particle in one beam is deflected by the electric and magnetic fields of the other beam. Emission of the occasional single *hard* x-ray is inevitable and the lost energy has to be paid for. Much worse is the possibility that the reduction in momentum causes the particle itself to be lost, greatly magnifying the energy loss. It is this process that makes beamstrahlung so damaging. The damage is quantified by the beamstrahlung-dominated beam lifetime $\tau_{\rm bs}$. The important parameter governing beamstrahlung is the "critical energy" u_c^* which is proportional to 1/bunch-length σ_z; beamstrahlung particle loss increases exponentially with u_c^*. To decrease beamstrahlung by increasing σ_z also entails increasing β_y^* which reduces luminosity. A favorable compromise can be to increase charge per bunch along with β_y^*.

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Reconciling the Luminosity Limits

The number of electrons per bunch N_p itself is fixed by the available RF power and the number of bunches N_b. For increasing the luminosity N_b wants to be *reduced*. To keep beamstrahlung acceptably small N_b wants to be *increased*. The maximum achievable luminosity is determined by this compromise between beamstrahlung and available power.

Three luminosities can be defined: $\mathcal{L}^{\rm RF}_{\rm pow}$ is the RF power limited luminosity (introduced earlier to analyse constant luminosity scaling); $\mathcal{L}^{\rm bb}_{\rm sat}$ is the beam-beam saturated luminosity; $\mathcal{L}^{\rm bs}_{\rm trans}$ is the beamstrahlung-limited luminosity. Single beam dynamics gives $\sigma_y = 0$ which implies $\mathcal{L}^{\rm RF}_{\rm pow} = \infty$? Nonsense. Recalling the earlier discussion, the resonance driving force, being proportional to $1/\sigma_y$ would also be infinite. As a result the beam-beam force expands $\sigma_y = 0$ as necessary. *Saturation is automatic* (unless the single beam emittance is already too great for the beam-beam force to take control—it seems this condition was just barely satisfied in highest energy LEP operation). Formulas for the luminosity limits are:

$$\mathcal{L}^{\rm RF}_{\rm pow} = \frac{1}{N_b}\, H(r_{yz})\, \frac{1}{a_{xy}}\, \frac{f}{4\pi}\, \Big(\frac{n_1 P_{\rm rf}[\rm MW]}{\sigma_y}\Big)^2, \tag{7}$$

$$\mathcal{L}^{\rm bb}_{\rm sat} = N_{\rm tot.} H(r_{yz})\, f\, \frac{\gamma}{2r_e}\, (\xi^{\rm sat.}/\beta_y), \tag{8}$$

$$\mathcal{L}^{\rm bs}_{\rm trans} = N_b H(r_{yz})\, a_{xy} \sigma_z^2\, f\, \Big(\frac{\sqrt{\pi}\; 1.96\times 10^5}{28.0\,{\rm m}\;\sqrt{2/\pi}}\Big)^2 \times$$
$$\times \frac{1}{r_e^2 \widetilde{E}^2}\, \Bigg(\frac{91\eta}{\ln\big(\frac{1/\tau_{\rm bs}}{f\, n^*_{\gamma,1}\, \mathcal{R}^{\rm Gauss}_{\rm unif.}}\big)}\Bigg)^2. \tag{9}$$

Here $H(r_{yz})$ is the hourglass reduction factor. If $\mathcal{L}^{\rm bs}_{\rm trans} < \mathcal{L}^{\rm bb}_{\rm sat}$ we must increase N_b. But $\mathcal{L}^{\rm bs}_{\rm trans} \propto N_b$, and $\mathcal{L}^{\rm RF}_{\rm pow} \propto 1/N_b$. We accept the compromise $N_{\rm b,new}/N_{\rm b,old} = \mathcal{L}^{\rm bb}_{\rm sat}/\mathcal{L}^{\rm bs}_{\rm trans}$ as good enough.

Parameter tables, scaled up from LEP, are given for 100 km circumference Higgs factories in Tables 5 and 6. The former of these tables assume the number of bunches N_b is unlimited. The latter table derates the luminosity under the assumtion that N_b cannot exceed 200. Discussion of the one ring vs two rings issue can therefore be based on Table 6.

Some parameters not given in tables are: Optimistic=1.5 (a shameless excuse for actual optimatization), $\eta_{\rm Telnov}$=0.01 (lattice fractional energy acceptance), $\tau_{\rm bs}$=600 s, $R_{\rm GauUnif}$= 0.300, P_{rf} = 25 MW, Over Voltage=20 GeV, aspect ratio a_{xy}=15, $r_{yz} = \beta^*_y/\sigma_z$=1, and $\beta_{\rm arc\ max}$=198.2 m.

With the exception of the final table, which is specific to the single ring option, the following tables apply equally to single ring or dual ring Higgs factories. The exception relates to N_b, the number of bunches in each beam. With N_b unlimited (as would be the case with two rings) all parameters are the same for one or two rings (at least according to the formulas in this paper).

ONE RING OR TWO RINGS?

With one ring, the maximum number of bunches is limited to approximately ≤ 200. (I have not studied crossing angle schemes which may permit this number to be increased.) For $N_b > 200$ the luminosity $\mathcal{L}$ has to be de-rated accordingly; $\mathcal{L} \rightarrow \mathcal{L}_{\rm actual} = \mathcal{L} \times 200/N_b$. This correction is applied in Table 6. This table, whose entries are simply drawn from Table 5, makes it easy to choose between one and two rings. Entries in this table have been copied into the earlier Table 2. When the optimal number of bunches is less than (roughly) 200, single ring operation is satisfactory, and hence favored. When the optimal number of bunches is much greater than 200, for example at the Z_0 energy, two rings are better.

Note though, that the Z_0 single ring luminosities are still very healthy. In fact, with β^*_y=10 mm, which is a more conservative estimate than most others in this paper and in other FCC reports, the Z_0 single ring penalty is substantially less.

Luminosities and optimal numbers of bunches in Phase II Higgs factory running are shown in Figure 3.

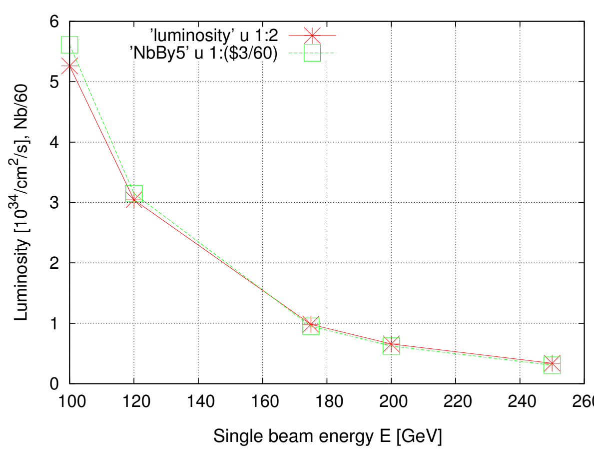

Figure 3: Dependence of luminosity on single beam energy (after upgrade to Stage II luminosity). The number of bunches (axis label to be read as $N_b/60$) is also shown, confirming that (as long as the optimal value of N_b is 1 or greater) the luminosity is proportional to the number of bunches. There is useful luminosity up to E = 500 GeV CM energy.

REFERENCES

[1] J. Jowett, *Beam Dynamics at LEP,* CERN SL/98-029 (AP), 1998

[2] R. Talman, *Specific Luminosity Limit of e+/e- Colliding Rings,* Phys. Rev. ST-AB, 2002

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Table 3: Single beam parameters, assuming 100 km circumference. The second last column (†) lists the value of ϵ_x appropriate only for $\beta_y^* = 5$ mm. Though determined by arc optics, ϵ_x has to be adjusted, according to the value of β_y^*, to optimize the beam shape at the IP. Other cases can be calculated from entries in other tables. U_1 is the energy loss per turn per particle. u_c is the critical energy for bending element synchrotron radiation. δ is the synchrotron radiation damping decrement.

name	E GeV	C km	R km	f KHz	U_1 GeV	eV_{excess} GeV	n_1 elec./MW	$\delta = \alpha_2$	u_c GeV	ϵ_x † nm	σ_x^{arc} mm
Z	46	100	10.6	3.00	0.04	20	5.81e+13	0.00020	0.00002	0.573	2
W	80	100	10.6	3.00	0.34	20	6.08e+12	0.00107	0.00011	1.771	1.19
LEP	100	100	10.6	3.00	0.83	19	2.49e+12	0.00209	0.00021	2.767	0.972
H	120	100	10.6	3.00	1.73	18	1.20e+12	0.00361	0.00036	3.984	0.824
tt	175	100	10.6	3.00	7.83	12	2.66e+11	0.01119	0.00112	8.473	0.585

Table 4: Higgs factory parameter values for 50 km and 100 km options. The entries are mainly extrapolated from Jowett's, 45.6 Gev report [1], and educated guesses. "N/Sc." indicates (important) parameters too complicated to be estimated by scaling. Duplicate entries in the third column, such as 45.6/91.5 are from Jowett [1]; subsequent scalings are based on the 45.6 Gev values.

Parameter	Symbol	Value	Unit	Energy-scaled	Radius-	scaled
Mean bend radius	R	**3026**	m	**3026**	**5675**	**11350**
	$R/3026$			1	1.875	3.751
Beam Energy	E	**45.6**/91.5	GeV	**120**	**120**	**120**
Circumference	C	26.66	km	26.66	50	100
Cell length	L_c		m	79	108	153
Momentum compaction	α_c	1.85e-4		1.85e-4	0.99e-4	0.49e-4
Tunes	Q_x	90.26		90.26	123.26	174.26
	Q_y	76.19		76.19	104.19	147.19
Partition numbers	$J_x/J_y/J_\epsilon$	1/1/2		1/1.6/1.4 !	1/1/2	1/1/2
Main bend field	B_0	0.05/0.101	T	0.1316	0.0702	0.0351
Energy loss per turn	U_0	0.134/2.05	GeV	6.49	3.46	1.73
Radial damping time	τ_x	0.06/0.005	s	0.0033	0.0061	0.0124
	τ_x/T_0	679/56	turns	37	69	139
Fractional energy spread	σ_δ	0.946e-3/1.72e-3		0.0025	0.0018	0.0013
Emittances (no BB), x	ϵ_x	22.5/30	nm	21.1	8.2	2.9
y	ϵ_y	0.29/0.26	nm	1.0	0.4	0.14
Max. arc beta functs	$\beta_x^{\max}$	125	m	125	171	242
Max. arc dispersion	$D^{\max}$	0.5	m	0.5	0.5	0.5
Beta functions at IP	β_x^*, β_y^*	2.0,0.05	m	1.25/0.04	N/Sc.	N/Sc.
Beam sizes at IP	σ_x^*, σ_y^*	211, 3.8	μm	178/11	N/Sc.	N/Sc.
Beam-beam parameters	ξ_x, ξ_y	0.037,0.042		0.06/0.083	N/Sc.	N/Sc.
Number of bunches	N_b	8		4	N/Sc.	N/Sc.
Luminosity	$\mathcal{L}$	2e31	cm^{-2}s^{-1}	1.0e32	N/Sc.	N/Sc.
Peak RF voltage	V_{RF}	380	MV	3500	N/Sc.	N/Sc.
Synchrotron tune	Q_s	0.085/0.107		0.15	N/Sc.	N/Sc.
Low curr. bunch length	σ_z	0.88	cm	$\frac{\alpha_c R \sigma_e}{Q_s E}$	N/Sc.	N/Sc.

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Table 5: The major factors influencing luminosity, assuming 100 km circumference and 25 MW/beam RF power. The predicted luminosity is the smallest of the three luminosities, $\mathcal{L}^{RF}$, $\mathcal{L}^{bs}_{trans}$, and $\mathcal{L}^{bb}$. All entries in this table apply to either one ring or two rings, except where the number of bunches N_b is too great for a single ring.

name	E GeV	ϵ_x nm	β^*_y mm	ϵ_y pm	ξ_{sat}	N_{tot} 10^{12}	σ_y μm	σ_x μm	u^*_c GeV	$n^*_{\gamma,1}$	$\mathcal{L}^{RF}$ 10^{34}	$\mathcal{L}^{bs}_{trans}$ 10^{34}	$\mathcal{L}^{bb}$ 10^{34}	N_b	β^*_x m	P_{rf} MW
Z	46	0.949	2	63.3	0.094	1500	0.356	5.34	0.000	2.01	52.5	103	52.5	65243	0.03	25
W	80	0.336	2	22.4	0.101	150	0.212	3.17	0.001	2.10	9.66	17.2	9.6	10980	0.03	25
LEP	100	0.223	2	14.9	0.101	62	0.172	2.59	0.002	2.13	4.95	8.46	4.94	5421	0.03	25
H	120	0.159	2	10.6	0.102	30	0.146	2.19	0.003	2.17	2.86	4.74	2.86	3044	0.03	25
tt	175	0.078	2	5.33	0.118	6.6	0.103	1.55	0.006	2.24	0.923	1.43	0.92	920	0.03	25
Z	46	17.2	5	1140	0.094	1500	2.39	35.89	0.001	2.16	21	35.1	21.	3605	0.075	25
W	80	6.11	5	408	0.101	150	1.43	21.42	0.003	2.26	3.86	5.83	3.86	602	0.075	25
LEP	100	4.07	5	271	0.101	62	1.16	17.47	0.005	2.31	1.98	2.86	1.97	296	0.075	25
H	120	2.92	5	195	0.102	30	0.987	14.80	0.008	2.35	1.15	1.6	1.14	166	0.075	25
tt	175	1.47	5	98.1	0.118	6.6	0.7	10.51	0.017	2.43	0.369	0.479	0.37	49	0.075	25
Z	46	155	10	10300	0.094	1500	10.2	152.3	0.002	2.29	10.5	15.5	10.5	400	0.15	25
W	80	55.4	10	3690	0.101	150	6.08	91.17	0.007	2.41	1.93	2.55	1.93	66	0.15	25
LEP	100	37.0	10	2470	0.101	62	4.97	74.48	0.011	2.46	0.989	1.25	0.99	32	0.15	25
H	120	26.6	10	1770	0.102	30	4.21	63.15	0.016	2.50	0.573	0.696	0.57	18.3	0.15	25
tt	175	13.5	10	898	0.118	6.6	3.0	44.94	0.036	2.60	0.185	0.207	0.19	5.5	0.15	25

Table 6: Luminosites achievable with a single ring for which the number of bunches N_b is limited to 200, assuming 100 km circumference and 25 MW/beam RF power. Entries in this table have been distilled down to include only the most important entries in Table 5, as corrected for the restricted number of bunches. The luminosity entries in Table 2 have been obtained from this table.

E GeV	β^*_y m	ξ_{sat}	$\mathcal{L}_{actual}$ 10^{34}	N_{actual}	P_{rf} MW/beam
46	0.002	0.094	0.161	200	25
80	0.002	0.1	0.176	200	25
100	0.002	0.1	0.182	200	25
120	0.002	0.1	0.188	200	25
175	0.002	0.12	0.200	200	25
46	0.005	0.094	1.165	200	25
80	0.005	0.1	1.282	200	25
100	0.005	0.1	1.334	200	25
120	0.005	0.1	1.145	166	25
175	0.005	0.12	0.369	50	25
46	0.010	0.094	5.247	200	25
80	0.010	0.1	1.932	66.5	25
100	0.010	0.1	0.989	32.7	25
120	0.010	0.1	0.573	18.3	25
175	0.010	0.12	0.185	5.5	25

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