

RESONANT TE WAVE MEASUREMENT OF ELECTRON CLOUD DENSITY USING MULTIPLE SIDEBANDS*

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Abstract

A change in electron cloud (EC) density will change the resonant frequency of a section of beam-pipe. With a fixed drive frequency, the resulting dynamic phase shift across the resonant section will include the convolution of the frequency shift with the impulse response of the resonance. The effect of the convolution on the calculated modulation sidebands is in agreement with measured data, including the absolute value of the EC density obtained from ECLLOUD simulations. These measurements were made at the Cornell Electron Storage Ring (CESR) which has been reconfigured as a test accelerator (CESRTA) with positron or electron beam energies ranging from 2 GeV to 5 GeV.

INTRODUCTION

The electron cloud (EC) density in an accelerator can be measured by coupling microwaves in and out of the beam-pipe, typically using beam position monitor (BPM) buttons. In contrast to TE wave transmission measurements [1, 2], TE wave resonance measurements rely on the resonant response that is produced by changes in beam-pipe geometry [3, 4]. Changes in geometry such tapers and longitudinal slots generate reflections of the TE wave and result in resonant sections of pipe that can be less than a meter in length.

The presence of the electron cloud created by a train of bunches will shift the resonant frequency of the beam-pipe. The decay of the electron cloud is short compared to the 2562 ns revolution time of the beam at CESRTA, so a short train of bunches will produce a periodic modulation of the resonant frequency. In order to calculate the EC density, the effect of this resonant frequency modulation on measured signals needs to be understood.

A fixed excitation frequency is used that is close to one of the resonant frequencies of the beam-pipe. Figure 1 shows that a shift in the resonant frequency produces a shift in both the equilibrium amplitude and phase of the response signal. If the rate of change is slow compared to the damping time of the resonance, the phase shift will be proportional to the frequency shift. For more rapid changes in resonant frequency, the phase shift will not follow the frequency changes exactly, but is convolved with the impulse response of the resonance.

*This work is supported by the US National Science Foundation PHY-0734867, PHY-1002467 and the US Department of Energy DE-FC02-08ER41538, DE-SC0006505.

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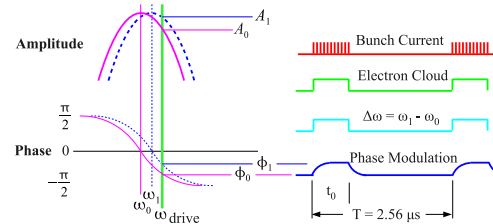


Figure 1: Equilibrium amplitude and phase shifts with a fixed drive frequency and a changing resonant frequency.

CALCULATING SPECTRA

The resonant frequency shift produced by a low density plasma in the absence of magnetic field is given by Eq. 1 where for a uniform plasma the (constant) EC density n_e can be brought outside of the integral over the resonant volume.

$$\begin{aligned} \frac{\Delta\omega}{\omega} &\approx \frac{e^2}{2\varepsilon_0 m_e \omega^2} \frac{\int_V n_e E^2 dV}{\int_V E^2 dV} \rightarrow \frac{e^2}{2\varepsilon_0 m_e \omega^2} n_e \\ &\approx \frac{1.59 \times 10^3}{\omega^2} n_e. \end{aligned} \quad (1)$$

Near the resonance ω_1 , the equilibrium phase shift ϕ_1 across a resonator driven at frequency ω is given by Eq. 2. For small shifts in the resonant frequency $\Delta\omega$ near resonance, the change in equilibrium phase is given by Eq. 3. The change in equilibrium amplitude should be small close to resonance unless the modulation is very large [5].

$$\phi_1 = \tan^{-1} \left[Q \frac{\omega_1^2 - \omega^2}{\omega_1 \omega} \right] \quad (2)$$

$$\Delta\phi \approx 2Q \frac{\Delta\omega}{\omega} \quad (3)$$

Combining equations 1 and 3 and values of physical constants, the equilibrium phase shift due to an EC density n_e is

$$\Delta\phi \approx 2Q \frac{1.59 \times 10^3}{\omega^2} n_e \quad (4)$$

If the rate of change in EC density $n_e(t)$ is rapid compared to the damping time of the resonance, the dynamic phase shift $\Delta\Phi(t)$ is obtained by convolving the equilibrium phase shift with the impulse response of the resonance, $\exp(-\alpha t)$, where α is the the damping rate of the resonance and $\tau = 1/\alpha$ its damping time.

$$\Delta\Phi(t) = 2Q \frac{1.59 \times 10^3}{\omega^2} \int_{-\infty}^t n_e(\xi) e^{(t-\xi)/\tau} d\xi \quad (5)$$

If $\Delta\Phi(t)$ is a periodic function with frequency $\omega_T = 2\pi/T$, it can be written as a complex Fourier series

$$\Delta\Phi(t) = \sum_{m=-\infty}^{+\infty} c_m \exp(-jm\omega_T t) \quad (6)$$

where

$$c_m = \frac{1}{T} \int_0^T \Delta\Phi(t) \exp(-jm\omega_T t) dt \quad (7)$$

A sine wave (carrier) of frequency ω that is phase modulated at a frequency ω_T with depth $M \ll 1$ can be expressed as

$$\begin{aligned} g(t) &= \sin[\omega t + M \cos(\omega_T t)] \\ &\approx \sin(\omega t) \\ &\quad + \frac{M}{2} [\cos(\omega + \omega_T)t + \cos(\omega - \omega_T)t]. \end{aligned} \quad (8)$$

This will be true for each of the frequencies in the Fourier series where C_m is the magnitude of $|c_m| + |c_{-m}|$.

$$\begin{aligned} g(t) &= \sin[\omega t + \Delta\Phi(t)] \\ &= \sin \left[\omega t + \sum_{m=1}^{+\infty} C_m \cos(m\omega_T t) \right] \\ &\approx \sin(\omega t) \\ &\quad + \sum_{m=1}^{+\infty} \frac{C_m}{2} [\cos(\omega + \omega_T)t + \cos(\omega - \omega_T)t] \end{aligned} \quad (9)$$

So the ratio of the amplitude of each of the sidebands to the amplitude of the carrier is $|C_m/2| = |c_m|$, expressed as decibels below the carrier: $dBc = 20 \log |C_m/2|$.

Example with Rectangular EC Density

If the EC density has a fixed value n_e from $0 \leq t \leq t_0$ and is zero otherwise (as sketched in Fig. 1), the dynamic phase shift (convolving $n_e(t)$ with $\exp(-\alpha t)$) would be

$$\begin{aligned} \Delta\Phi(t) &= 2Q \frac{1.59 \times 10^3}{\omega^2} n_e (1 - e^{-t/\tau}) \quad 0 \leq t \leq t_0 \\ &= 2Q \frac{1.59 \times 10^3}{\omega^2} n_e (1 - e^{-t_0/\tau}) e^{-t/\tau} \quad t \geq t_0. \end{aligned} \quad (10)$$

This function is periodic with frequency $\omega_T = 2\pi/T$ where T is the revolution period of the storage ring. The Fourier transform of $\Delta\Phi$ gives complex coefficients with the magnitudes given in Eq. 11 and shown in Fig. 2

$$\begin{aligned} c_m &= \sin\left(\frac{m\omega_T t_0}{2}\right) \frac{1}{\pi} \left[\frac{1}{m} - \frac{m\omega_T^2}{(\alpha^2 + (m\omega_T)^2)} \right. \\ &\quad \left. - j \frac{\alpha\omega_T}{(\alpha^2 + (m\omega_T)^2)} \right] \exp j \frac{m\omega_T t_0}{2}. \end{aligned} \quad (11)$$

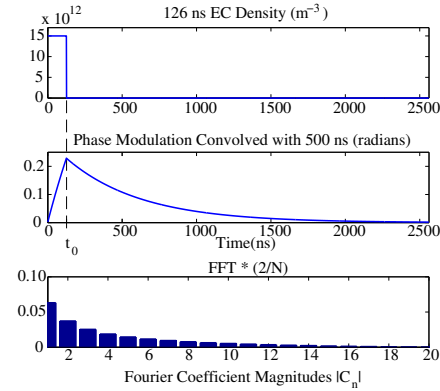


Figure 2: A 126 ns rectangular EC density $n_e(t)$ (top) is used to calculate the dynamic phase shift $\Delta\Phi(t)$ (middle) that is then Fourier transformed to obtain the sideband coefficients C_n (bottom) using a MATLAB script.

In the limit of a high damping rate α , Eq. 11 gives the coefficients that would be obtained by the Fourier transform of the rectangular EC density without convolution. We used this approximation in work published before 2013 where only the first upper and lower sidebands ($m = 1$) were used. The correction to the first sidebands that includes convolution would be an increased EC density of $\sqrt{2}$ times our previously published values in addition to any errors in choosing an EC density duration.

MEASUREMENTS AT CESRTA

The analysis that follows will focus on a roughly 3 meter long resonant section of aluminum chamber near the quadrupole at 15E in CESRTA. Longitudinal slots at the ion pump ports generate reflections for TE waves and a number of resonances as shown in Fig. 3. The first resonance at about 1.88 GHz has a Q of about 3000 ($\tau = 500$ ns). Phase modulation sidebands appear at multiples of the beam revolution frequency (390 kHz). Ten upper and lower sidebands of the first resonance were recorded.

10-Bunch Data

A 10-bunch train of 5.3 GeV positrons with 14 ns spacing was injected to 64 mA total current (about 10^{11} positrons/bunch). The sideband amplitudes are plotted in Fig. 4, where this data is compared to different calculations of the expected amplitudes. First, if the EC density is taken to be rectangular with an amplitude of $1.5 \times 10^{13} \text{ m}^{-3}$ having a duration equal to the length of the train (126 ns) and the phase modulation was also rectangular (without convolution), the calculated sideband amplitudes would follow

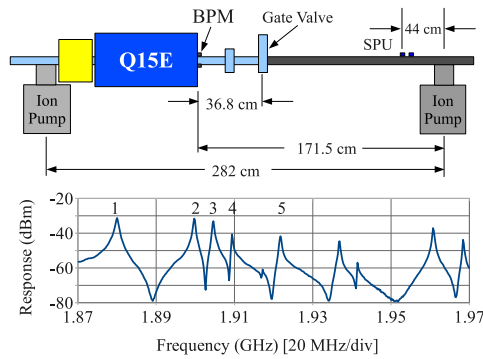


Figure 3: Resonances in the Al beam-pipe at 15E are generated between longitudinal slots that connect ion pumps to the beam vacuum space.

the envelope of the green dashed line of Fig. 4. This is a plot of the magnitude of Eq. 11 in the limit of large α . If the phase modulation calculation includes convolution as in Fig. 2, the result would be the cyan dashed line that appears just above the data in Fig. 4.

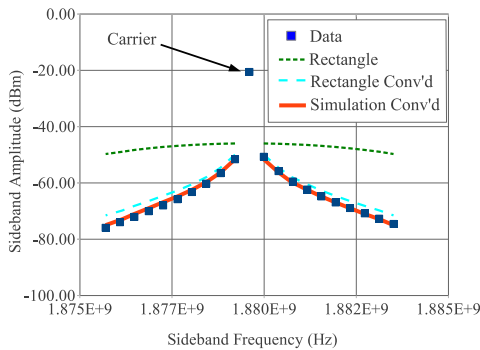


Figure 4: 10-bunch data (squares) are compared with calculated envelopes of sideband amplitudes. The calculation based on convolved rectangular density (cyan dashed line) is in fair agreement with the measurement. The envelope obtained using the ECLLOUD simulation is the solid orange line that (nearly) coincides with the data points.

One of the uncertainties in these calculations is the duration of the EC density. The recorded sidebands contain information about this duration, but the spectrum is dominated by the response time of the resonance, which is long compared to the changes in EC density. An alternative is to use independent measurements and simulations of this time evolution.

The simulation code ECLLOUD [6, 7] has been used extensively at CESR-TA in understanding signals from shielded pickups (SPU) that sample the flux of cloud electrons onto the inner surface of the beam-pipe [8, 9]. Simulation parameters as well as a model of the SPU are adjusted so that the simulation correctly predicts the signal recorded at the SPU. There is an SPU detector within the

resonant section of the TE wave measurement that can be seen on the right side of Fig. 3.

The upper plot of Fig. 5 shows an ECLLOUD simulation of a 10-bunch train at 60 mA total current (10^{11} positrons/bunch). This simulation along with the model for the SPU is in agreement with the measurement made with this SPU. This same simulation of the EC density was used to calculate the expected sidebands for the 10-bunch TE wave measurement. As shown in Fig. 5, the EC density is convolved with the impulse response of the resonance and the FFT gives the coefficients C_n using a MATLAB [10] script. The resulting sideband envelope is shown as the solid orange lines of Fig. 4 that nearly coincide with the data points.

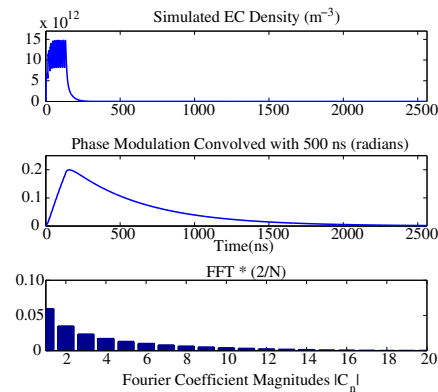


Figure 5: An ECLLOUD simulation (top) for a 10-bunch positron train with 10^{11} positrons/bunch is convolved with the impulse response of the beam-pipe resonance to calculate the expected sideband amplitudes $dBc = 20\log|C_n/2|$.

1-Bunch Data

Multiple sideband data was also taken with a single bunch of 8 mA 5.3 GeV positrons (1.28×10^{11} positrons/bunch). For calculation of the expected sidebands, an ECLLOUD simulation was used. In this case the matching simulation and SPU data were made at a current of 3 mA (4.8×10^{10} positrons/bunch). Previous measurements of sideband magnitude versus current for a single bunch have been reasonably linear, so we assumed a linear scaling of the sidebands. The result is shown in Fig. 6 and the comparison of this calculated sideband envelope with the measured data is shown in Fig. 7.

The calculated values are lower than the data by about 6 dB. This is probably explained by the following argument. For the 1-bunch data, the EC density will be more or less proportional to the amount of synchrotron light at each longitudinal position. The ECLLOUD simulation was made for the location of the SPU, which is near the end of the resonant TE wave section as shown in Fig. 3. The synchrotron light in this section decreases by a factor of 3, being smallest at the end nearest to the SPU. In contrast,

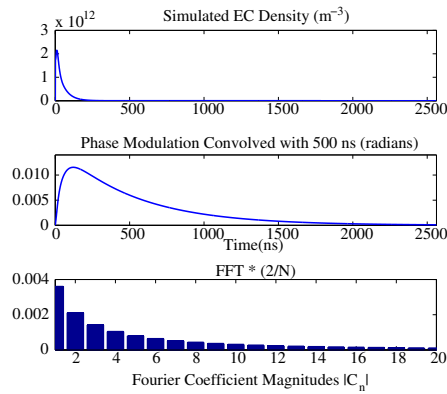


Figure 6: 1-bunch ELOUD simulation (top) is used to calculate sideband coefficients.

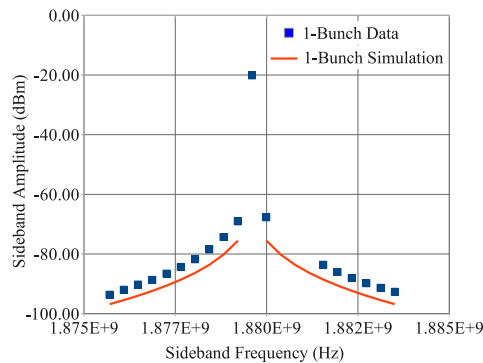


Figure 7: 1-bunch data at 8 mA (squares) with scaled simulated sideband amplitudes (solid lines).

for the 10-bunch data at 6 mA/bunch the EC density is saturated at the SPU and the increasing amount of synchrotron light in the other parts of the chamber do not result in a significantly higher EC density there. So for the 10-bunch data, the EC density should be more nearly uniform over the resonant section of beam-pipe.

SUMMARY

Our previous work used the approximation of a rectangular EC density without convolution and focused on the first sidebands of the carrier. Even if the EC duration were chosen correctly, this would result in an under-estimate of the EC density by $\sqrt{2}$. Use of a rectangular EC density that is convolved with the impulse response of the beam-pipe gives an envelope of multiple sidebands that is in fair agreement with measured data for a 10-bunch train. Use of the simulated EC density gives further improvement in the match of predicted to measured sidebands, including the absolute EC density.

Under the same conditions, the agreement of two completely independent measurements of EC density – from TE wave resonances and from shielded pickups – gives increased confidence in the measurement techniques and in

ISBN 978-3-95450-127-4

the validity of the ELOUD simulations and modeling.

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