# OPTIMISATION OF THE SVD TREATMENT IN THE FAST ORBIT CORRECTION OF THE ESRF STORAGE RING 

E. Plouviez, F. Epaud, L. Farvacque, J.M. Koch, ESRF, Grenoble, France

## Abstract

The ESRF fast orbit correction system has been in operation since May 2012 [1]. The orbit correction scheme relies classically on the calculation of an orbit correction based on the SVD analysis of the response matrix of our 224 BPMs to each of our 96 correctors. The rate of the calculation of the corrections is 10 KHz ; we use a PI loop achieving a bandwidth of 150 Hz completed with a narrow band pass filter with extra gain at 50 Hz . In order to make the best use of the correctors dynamic range and of the resolution of the calculation, it can be useful to limit the bandwidth of loop for the highest order vectors of the SVD, or even to totally remove some of these vectors from the correction down to DC. Removing some of the eigen vectors while avoiding that the loop becomes unstable usually increases a lot the complexity of the matrix calculations: we have developed an algorithm which overcomes this problem; The test of this algorithm is presented. We present also the beneficial effect at high frequency of the limitation of the gain of the correction of the highest SVD eigen vectors on the demand of the peak strength of the correctors and on the resolution of the correction calculation.

## The ESRF Orbit Control System

The 224 BPM pick up sets of our storage ring are equipped with Libera Brilliance electronics interconnected by the so called Communication Controller [2] network broadcasting the position data of the 224 Libera at a rate of 10 KHz . The orbit correction is applied by a set of 96 corrector magnets; the bandwith of these correctors goes from DC to 500 Hz . The dynamic range of the correctors is $75 \mu \mathrm{rad}$ up to 50 Hz but decreases down to only $7.5 \mu \mathrm{rad}$ up to 500 Hz . We are using the corrector magnets embedded in the sextupole magnets cores to steer the beam; the control of the magnet power supplies and the orbit correction calculation are performed by 8 separate processor boards as shown on figure 1. For this processing, we selected PMC boards embedding a Xilinx Virtex-5 FPGA.

## SVD BASED ORBIT CORRECTION

Since the number of the BPMs is not the same as the number of the correctors we derive the orbit correction from the BPM measurements using a correction matrix obtained by the classical Singular Value Decomposition (SVD) method [2]; we measure the orbit and refresh the corrector settings at a rate of 10 KHz . We have used for this a 10 KHz iteration of the values of the correction currents with a PID algorithm combined with an additional 50 Hz notch filter aimed to improve the
damping of the perturbation at the AC main power supply frequency.


Figure 1: Layout of the new orbit correction system.

## SVD Principle

SVD of the response matrix R of the BPMs to the correctors turns the initial Mr matrix into a set of three matrixes $\mathrm{U}, \mathrm{S}, \mathrm{V}$ with $\mathrm{R}=\mathrm{U}^{*} \mathrm{~S}^{*} \mathrm{~V}^{\prime}$; U performs the projection of the effect of the 96 correctors on the orbit, as measured by the 224 BPMs on a set of 96 unitary eigen vectors; V performs the projection of an orbit measured by the 224 BPMs on the same set of 96 unitary eigen vectors; the effect of the correctors is given by the $S$ diagonal matrix, then the combination of eigen vectors produced by the action of the correctors is converted into BPMs outputs by the matrix V as shown on figure 2 .


Figure 2: SVD of the response matrix.
S coefficients are positive in decreasing order. The C correction matrix, inverse of the R response matrix will then be given by: $\mathrm{C}=\mathrm{V}^{*} \mathrm{~S}^{-1} * \mathrm{U}$. We can see on figure 3
that the correction of the higher order eigen vectors contribution to the orbit distortion puts a very high demand on the correctors for a small effect, compared to the contribution of the lower order vectors.


Figure 3: Pattern of the 96 eigen values of the response matrixes and correction matrixes; dashed lines: correction eigen values limitation used for the tests of the eigen vectors weighting.

## Iteration

The result of the multiplication of the orbit error by the correction matrix is performed after every period of the 10 KHz ; however this result is not applied all the way to the correctors input but by a loop combining a PI corrector and a selective 50 Hz damping filter, resulting for each of the eight feedback processor in the data flow shown below


Figure 4: Loop iteration step.

## Effect of the Eigen Values Manipulations

When generating the correction matrix, it is possible, instead of using the original $\mathrm{S}^{-1}$ coefficients given on figure 2 , to apply a ponderation on these eigen values, or even to set to zero the coefficients corresponding to somed of the eigen vectors. If a weighting is applied to the S correction matrix coefficients, the final DC orbit will be not be modified by the weighting, due to the effect of the integrator in the PI corrector, but the frequency response of the loop will be different for each eigen vector, depending on the value of the weighting factor. If we want to limit the effect of the correction on the lower eigen vectors down to DC, we must then set to zero the ponderation coefficients; the orbit correction will not be performed at all on these vectors and the the final orbit will be slightly different; this can be beneficial if the vectors ignored are mostly sensitive to imperfections of the orbit correction system like response matrix measurement errors or BPM misalignment rather than real beam orbit distortion.

## Reduction of the Loop Gain for the Highest Order Vectors

As shown on the figure 2, in order to obtain the same frequency response on all the eigen vectors, the gain of the iteration loop, must be much higher on the upper vectors; it means that if the correction matrix is slightly incorrect, the noise applied on the beam due to this slightly wrong correction will come mostly from the highest order eigen vectors. At DLS this is avoided with the application of a weighting based on the so-called Tikhonoff regularisation on the eigen values of the correction matrix [2], resulting in a significant reduction of the spurious beam orbit distortion at high frequency.
We tested the application of a simple weighting limiting the value of the coefficients of the $\mathrm{S}^{-1}$ matrix to a maximum value (dashed line on the plots of the figure 3). We found that the effect of such a normalisation on the final stability is visible (at least in the vertical plane) but not dramatic on the ESRF storage ring beam; the average vertical beam motion integrated from 0 to 1 KHz is $1.2 \mu \mathrm{~m}$ without feedback, $.59 \mu \mathrm{~m}$ with feedback and no eigen values weighting, and $.55 \mu \mathrm{~m}$ with weighting; the horizontal beam motion is $3 \mu \mathrm{~m}$ without feedback, $1.04 \mu \mathrm{~m}$ with feedback and without weighting, and $1.08 \mu \mathrm{~m}$ with feedback and weighting; we think it is because, compared to the effect on the DLS beam, the original ESRF beam stability is worst, and the Libera Brillance used at ESRF have a lower noise than the Libera Electron used at DLS, so the relative effect of the noise reduction due to the matrix regularisation is then small compared to the rest of the residual beam motion. However such a weighting have other beneficial effects for us: the FPGA used for the correction calculation are performing fixed point calculation, which limits the resolution of the calculation; we choose to set our regularisation parameters in order to achieve a reduction by a factor 2 of the range of the absolute value of the coefficients of the correction matrix, resulting in a more accurate correction calculation for a given number of bits used in the calculation.


Figure 5: Plot of the column 10 of the horizontal correction matrix, without (left plot) and with (right plot) weighting applied to the $\mathrm{S}^{-1}$ coefficients.

Another beneficial effect of this weighting is to reduce the peak amplitude of the correction applied in response to some fast and localized orbit distortion; on our storage ring the injection bump is not perfectly closed, and the parasitic kick applied to the stored beam at each injection results in a large correction bump applied by the orbit correctors in the kickers vicinity. The maximum
amplitude of the short current pulses delivered to these correctors reaches half their fast signal dynamic range. Reducing the loop gain on the highest eigen vectors also reduces the demand on the correctors in the region of the injection kickers without spoiling the quality of the orbit correction for the beam delivered to the users between the injections.


Figure 6: Plot of the horizontal correction applied during an injection kick (left: no weighting, right: weighting applied

## Total Suppression of Higher Order Vectors

It can also be useful to totally suppress some eigen vectors in the correction matrix; If we suspect that the BPM alignment will always be slightly imperfect and that the static orbit reading projection on the highest eigen vectors is mostly due to these misalignment, we will get an orbit closer to the optimal orbit by not using them at all. A simple way to do this is to set to zero in the $\mathrm{S}^{-1}$ matrix the coefficients corresponding to these vectors:


Figure 7: Simplest but unstable method of suppression of the higher orders vectors.

However, in this case, if the result of the multiplication of the BPM data by the correction matrix generates some spurious signals at the iteration loop input (and it will!), the part of this spurious signals generating an orbit distortion on these highest eigen vectors will not be any longer cancelled by the loop, now limited to the lower order vectors, and will accumulate since the integrator is the last stage of the loop, and eventually will drive the correctors to saturation, even if the effect on the beam is actually small.

We have tested the effect of our orbit correction with the correction matrix shown on figure 7 with the $67^{\text {th }}$ to $96^{\text {th }}$ upper vectors suppressed; we observed that just after the loop closure, the initial damping of the fast beam motion was normal but after a few seconds the correctors settings reached values higher than their dynamic range, leading eventually to a loss of the orbit control. Looking
at records of the BPM and correctors data, we noticed a continuous increase of the correctors setting; the SVD analysis of these settings, showed a normal and small level on the first 66 vectors, the correction level increase is present only on the upper vectors, as shown on figure 8.


Figure 8: Drift of the correctors setting when method of the figure 7 is used for the higher vectors suppression (left pattern of the plot current drift on a set of three of the 96 correctors, right SVD analysis of the 96 currents during the drift)

A clean way to overcome this problem would be to split the correction calculation as shown on figure 9:


96X12 matrix
Figure 9: Stable method of suppression of the higher orders vectors.

In this way there is not any more the possibility of an accumulation of spurious correctors settings on the highest eigen vectors. The drawback is that this scheme would now require at least one 96*224 matrix multiplication followed by the iteration step and a $96 * 96$ matrix multiplication instead of one $96 * 224$ matrix multiplication followed by the iteration step if the calculation is performed on one single processor. This drawback is even worse in the case of the ESRF system where the normal correction calculation, is split in eight $96 / 8 * 224$ matrix multiplications, since it is performed on eight FPGA, and would then require eight time a $96 * 224$ matrix multiplication followed by the iteration loop step and a $96 * 96 / 8$ matrix multiplication; it is not possible to implement it in the FPGA used on our system. The solution that we found was to keep all the eigen vectors active for the fast calculation of the orbit correction but to prevent the DC damping on the higher order eigen vector with a trim of the input BPM data: the low average settings of the correctors is multiplied by $\mathrm{R}_{\text {HOEV }}$, the system response matrix limited to the higher order eigen vectors that we want to suppress; the result of this calculation is then subtracted from the BPM data at the input of the loop every 30 seconds; this prevents the loop from applying any DC correction on these higher order eigen vectors; all this data treatment being made at a slow
rate, it puts no extra demand on the FPGA processing power.


Figure 10: Principle of the DC suppression of the orbit correction on the highest order SVD eigen vectors.

## TESTS

We have tested the effect of the cancellation of upper SVD eigen vectors on the orbit correction of our storage ring; the parameters of the additional loop acting on the position offsets added to the main loop input have been set in order to cancel the vertical DC correction of the vectors 67 to 96 . We have then tested the orbit correction just after a full refill of the SR, in order to see how the system will behave during the significant changes in the orbit correction pattern caused by the thermal stabilisation of the ring. The figures 11 and 12 shows the evolution of these position offsets and the evolution of the average value of the correctors settings over one hour, while the fast orbit correction was active. The figure 13 shows the evolution of the BPM readings, with the offsets subtracted; the orbit drift is low and the level of the higher order vectors contribution to this drift is one hundred time lower than the offsets applied on the BPM readings (figure 11), which shows the effectiveness of the algorithm. The left plots shows BPM and correctors settings and the right plot shows the projection of this settings on the eigen vectors space. The effect of the additional loop is obvious: while the BPM offsets addition is done only on the upper vectors, the change in the setting of the correctors is limited to the $66^{\text {th }}$ lower vectors of the SVD.


Figure 11: Plot of the offsets applied to the input data (BPM) to cancel the correction on the $67^{\text {th }}$ to $96^{\text {th }}$ upper eigen vectors (right plot: SVD analysis).


Figure 12: Plot of the drift over one hour of the correctors settings with the upper vectors correction suppressed (right plot: SVD analysis).


Figure 13: Plot of the drift over one hour of the BPM readings (real position minus figure 4 offsets) with the upper vectors correction suppression algorithm active (right plot: SVD analysis).

## CONCLUSION

We have tested the effect of the application of different gains on the different eigen vectors of the SVD of our orbit correction system; if a total cancellation of the gain is needed, when we want to suppress the contribution of some higher order eigen vectors down to the DC orbit correction, we also found an algorithm which allows a stable operation of the loop, without increasing the complexity of the part of the correction calculation which must be performed at the full 10 KHz iteration rate of the loop.

## REFERENCES

[1] Eric Plouviez, Kees Scheidt, Jean Marc Koch, Francis Epaud, "Fast orbit correction for the ESRF storage ring", IPAC2011, San Sebastian, Sept 20011
[2] J. Rowland, M. G. Abbott, J. A. Dobbing, M.T. Heron, I. Martin, G. Rehm, I. Uzun, S. R. Duncan, "Status of the Diamond Fast Orbit Feedback System", ICALEPS 07, Knoxville, Oct 2007.

