ALGORITHM TO IMPROVE THE BETA-FUNCTION MEASUREMENT AND ITS EVALUATION IN STORAGE RINGS LATTICES

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Abstract

In any beam-line, one of the basic measurements in the beam-diagnostics is the measurement of the Beta-Function. This can be achieved, in Storage Rings, by taking the tune change obtained when varying the intensity of quadrupoles, or by using the matrix response to fit the corresponding parameters, or by shaking the beam to obtain a betatron motion. In accelerators like the LHC, the Beta-Function measurement is done from the Phase Advance Measurement using the Transfer Matrix. In this paper, a study of a new algorithm or numerically approximation for this measurement is presented, as well as the results of simulations on LHC and CLIC lattices. The deduced and implemented algorithm takes into account a fraction of the both transverse planes measurements. A random (uniform) deviation of the MAD-X phase values is taken to obtain the measured values and then used to study the Beta Function measurement for a different amount of orbits. There are observed cases where the improvement is close to 30% and 50% compare to a traditional method.

INTRODUCTION

The measurement of the Beta Function in accelerators is an important task during the commissioning, because all the properties of the focusing structure are described and calculated using the Twiss functions or Courant-Snyder parameters. For any beam-line, in colliders and transport lines of high energy particles, the horizontal and vertical Beta functions determine the transverse beam sizes that change around the storage ring. During the beam-diagnostics, to know the twiss parameters implies, in general, to be able to determine all the dynamic beam parameters. [1],[2],[3].

One of the techniques used to measure the beta-function in colliders or storage rings, is by using the tune shift induced by quadrupole excitation, this consists in *to detect the shift in the betatron tune as the strength of an individual quadrupole magnet is varied*, pag. 17 on [4]. The theoretical expression is obtained from the trace of the corresponding transport matrix for the entire ring multiplied by the perturbation matrix, which represents the effect of the gradient change. In the simplified final expression, each transverse beta function depends on the tune change in the corresponding plane, the gradient change, and constants. A specific variation of this method, using two symmetric placed quadrupoles, allows measuring the beta-function at the interaction point in colliders [5].

A second method is given by shaking the beam to obtain a betatron motion. The betatron oscillations are measured with multi-turn beam position monitors (BPMs) and *the beta function is calculated from the betatron phase advance between three adjacent BPMs*, pag. 21 on [4]. Theoretically, from the Transfer Matrix on element to element, one can obtain a set of two independent equations with the information of three BPMs. The final expression involves the matrix elements depending of the designed optics and the $\tan$ value of the measured phase advances. The α measure can also be obtained with this procedure.

Additionally, having a betatron motion in the beam-line allows measuring the Beta Function using an interpolation of the twiss functions between the BPMs. In this way, the matrix response is used to fit the corresponding parameters assuming that the magnetic gradients in the transfer line model between the monitors 1 and 3 are perfect. The theoretical expressions are similar of what was discussed for the previous method. Computers are used to calculate the fit, where the measured variables are normalized to create a symmetric covariance matrix to be solved by least squares. The final expression depends on the measured phase advances and Beta Function in the BPMs. [1], [5].

Another simple method is to measure the orbit change when a steering corrector magnet is excited at different values. This method use a BPM nearby the corrector, and it is where the beta function value is obtained. The theory involves the expression of the closed-orbit distortion in the presence of a single dipole kick. The final beta function measurement if obtained from the $\tan$ value of the tune, the closed-orbit distortion, the steering error value and constants. [4], [2].

For instance, in the LHC the measurement of the Beta Function for the relativistic beams is performed by using the second method described above, eq. (1) in [6].

THEORY

During the measurement of any optical quantity in an accelerator, it is expected to have a correspondence between the model scenario and what is measured at the machine. Using the transfer matrix for the beta function measurement, it is found that the following should be fullfilment

$$\cot \Delta\Phi_{1,2}^{ide} - \cot \Delta\Phi_{1,3}^{ide} = \cot \Delta\Phi_{1,2}^{meas} - \cot \Delta\Phi_{1,3}^{meas} \quad (1)$$

where, Φ are the phase advance in the transverse plane, at the three different longitudinal positions 1, 2 and 3; the labels *ide* and *meas* stand for the 'ideal' and 'measured' scenarios, respectively. The discrepancies among the model phase advances, or 'ideal' values, and the ob-

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served or 'measured' values, are then propagated to the beta function measurement.

The proposed new algorithm is to correlate both planes to a common factor, which as before it is expected to be true. It implies the following:

$$\frac{\cot \Delta\Phi^{x,ide}_{1,2} - \cot \Delta\Phi^{x,ide}_{1,3}}{\cot \Delta\Phi^{y,ide}_{1,2} - \cot \Delta\Phi^{y,ide}_{1,3}} = \frac{\cot \Delta\Phi^{x,mea}_{1,2} - \cot \Delta\Phi^{x,mea}_{1,3}}{\cot \Delta\Phi^{y,mea}_{1,2} - \cot \Delta\Phi^{y,mea}_{1,3}} \quad (2)$$

This allows counting on the coupling that could interfered in the Beta Function measurement. In this way, the improvement algorithm consist of take into account the information of both planes. Therefore, there exists an optical function that would be called ρ from where the β function can be measured.

$$\beta^{x,measure} = \rho_z[\cot \Delta\Phi^{y,ide}_{1,2} - \cot \Delta\Phi^{y,ide}_{1,3}] \quad (3)$$
$$\beta^{y,measure} = \rho_z[\cot \Delta\Phi^{x,ide}_{1,2} - \cot \Delta\Phi^{x,ide}_{1,3}] \quad (4)$$

Additionally an optimization based on the possibility of small deviations of the expected values, turns into a problem of mathematical geometry which I solved to be

$$\rho_z = \begin{cases} \beta^{z,ide}\frac{\tau(x,2)\pm\cot \Delta\Phi^{x,mea}_{1,2}}{\tau(y,2)\pm\cot \Delta\Phi^{y,mea}_{1,2}} & \text{or} \\ \beta^{z,ide}\frac{\tau(x,2)\pm\cot \Delta\Phi^{x,mea}_{1,3}}{\tau(y,2)\pm\cot \Delta\Phi^{y,mea}_{1,3}} \end{cases} \quad (5)$$

where $\tau(z,m) = \cot \Delta\Phi^{z,mea}_{m-1,m} - \cot \Delta\Phi^{z,mea}_{m-1,m+1}$ and its depends on how close the measure fraction is close to the ideal fraction. These differences, although small quantities, are also what may improve the measurement when high noise (meaning a high deviation of the phase advanced) is presented.

The approach described in eq.(5) is not obtained from a statistical quantity, or based in them, as most of the traditional methods does. Although, the combinations of the above equations imply that the fitting is done to obtained the number 1.

TRANSFER MATRIX REMARK

The Transfer Matrix that allows measuring the Beta-Function in a collider, is given by:

$$\begin{pmatrix} \sqrt{\frac{\beta_f}{\beta_i}}(\cos\phi_{fi} + \alpha_i \sin\phi_{fi}) & \sqrt{\beta_f\beta_i}\sin\phi_{fi} \\ -\frac{1+\alpha_i\alpha_f}{\sqrt{\beta_f\beta_i}}\sin\phi_{fi} + \frac{\alpha_i-\alpha_f}{\sqrt{\beta_f\beta_i}}\cos\phi_{fi} & \sqrt{\frac{\beta_f}{\beta_f}}(\cos\phi_{fi} - \alpha_f\sin\phi_{fi}) \end{pmatrix} \quad (6)$$

When applying to three adjacent BPMs, the value of the beta function at the first lattice element can be obtained as a function of the matrix elements, on the first row of the matrices that transfer from the element 1 to 2, and from 1 to 3, usually called m_{11}, m_{12} ,n_{11}, and n_{12}, respectively, [7]. See the details of the theory development for LEP [1] and the application at LHC in [6].

RESULTS ON LATTICES

The proposed new algorimth is tested on the lattices examples for two storage rings, using the standard software MAD-X [8] by a comparison with the traditional way to obtain the beta function measurement. The lattice source code is obtained from the MAD-X examples at [9].

The storage rings optics choose for this paper are the LHC and CLIC. In the optics corresponding to the CLIC lattice, a ring of 357.46 m of length is simulated, to a tune correspondence of 72.69 2π rad in the horizontal plane and 35.42 2π rad in the vertical plane. The maximum beta function at those planes are 30.00 m and 9.21 m, respectively. The beam consists of 3.1×10^9 positrons. For the LHC case, the simulated sequence is the Beam1, which is composed by 1.15×10^{11} protons in a ring of 26658.88 m length. The protons have an energy of 450 GeV, reaching a horizontal tune of 64.28 2π rad with a maximum beta function of 592.8 m, and a vertical tune of 59.31 2π rad with a maximum beta function of 611.97 m.

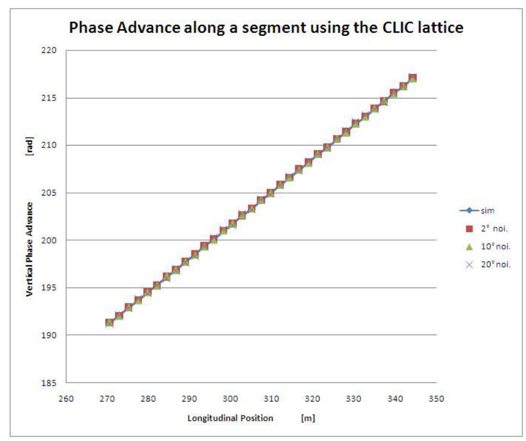

Figure 1: Phase Advances for a sector of the CLIC lattice, without and with noise for 2, 10 and 20 degrees of deviation, for the Y plane.

To obtain the measurement for the comparison, just a sector of each ring is used. In the cases for the CLIC lattice, 35 BPMS are taken into account, while for the LHC, 39 BPMs located at the arcs 4 and 5 are used. This is to have a difference in the phase advance in between the adjacent BPMs, and therefore having a better analysis on the Beta Function measurement.

The 'ideal' values during this test corresponds to the quantities obtained using the *twiss* function of MAD-X. These values are obtained for the phase advances and the Beta Fuction in both transverse planes.

The 'measured' values are interpreted as the total value of each 'ideal' phase advance plus or less a deviation according with a random uniform distribution. Three maximum deviations are studied, those values are 2.0, 10.0 and 20.0 degrees. The deviation value is half equally distributed around the 'ideal' values, to give as positive as negative deviations. These values are obtained using the

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random library of PYTHON [10].

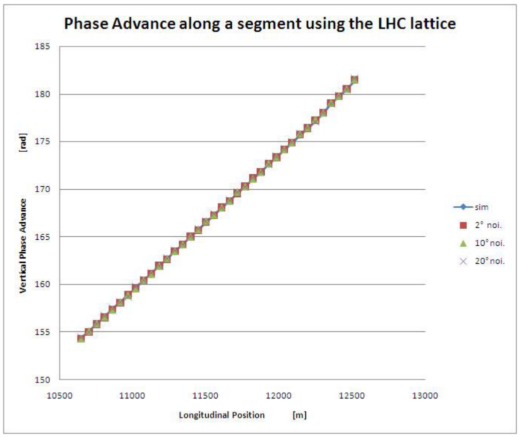

Figure 2: Phase Advances for a sector of LHC lattice , without noise and with noise of 2, 10 and 20 degrees.

Figures 1 and 2 shows the corresponding 'ideal' and 'measured' phases advances, for the studied cases in the plane Y using the CLIC and LHC optics, respectively. The *ide* values are denoted by the label "sim" and linked with a line. The *meas* values are presented using points of different colors for the studied deviations and denoted by adding the label "noi".

The plots for the two storage rings lattices are straight lines with a positive slope as expected. Similar plots were obtained in the horizontal plane, for both lattices. Evenmore the noise is almost not perceptive to the eye.

Using the noised phase advance values at three adjacent locations the equations (3) and (4) are applied to the corresponding transverse plane to obtain the Beta Function, as well as the traditional equations given by eq.(6) to obtain the same quantity, along the segment of the ring.

The measured value of the beta function is affected by the noised phase advances as presented in figures 3 and 4, for the different lattices. In the plots the noise is now perceptible. The notation used is the label "sim" for the ideal beta, label "t" for the traditional β- measurement, label "a" for the new algorithm, and together with the numbers 2, 10 and 20 according with the distribution used for the noise.

For each beta measurement a relative error to the ideal beta is obtained. And for the entire segment a global relative error, denoted by $Err.$, is associated to be the average of the errors at the different locations. Each particular noised segment is going to be called an orbit, and with different random numbers used each time, different noised segments are obtained.

Taking different orbits Tables 1 and 2 are constructed, for each lattice independently. The first colummn is the studied case label, it corresponds with the amount of simulated noise and the type of theory used for the β measurement; the notation is the same as explained for Figures 3 and 4. The second and third column are the average of the global error, in each transverse plane with its corresponding uncertainty. The forth a final column contains the number of orbits used to obtained the average of the global error.

For the 2 degrees cases the new algorithm has a small increment of the global error % compare to the traditional way, in both cases the LHC and CLIC, this could be explain by the fact than a very small amount of noise presented does not favors the use of the both noise planes to do the Beta measurement. Specifically for the LHC cases, the results show that both -t and -a approaches are on the same order to make the measurement.

Table 1: Beta Fuction Err. Using CLIC Lattice

Case.	$Err. < \beta_x >$	$Err. < \beta_y >$	Num.
	[%]	[%]	Orb.
20-t	14.0064 ± 0.47	21.5973 ± 1.1	10
20-a	11.1688 ± 0.46	11.3702 ± 0.54	10
20-t	14.0898 ± 0.20	21.0753 ± 0.37	100
20-a	10.4034 ± 0.14	10.4941 ± 0.14	100
20-t	14.4705 ± 0.063	20.8487 ± 0.11	1000
20-a	10.5313 ± 0.046	10.6460 ± 0.046	1000
10-t	6.6708 ± 0.2369	10.1476 ± 0.35	10
10-a	8.6858 ± 0.34	8.9647 ± 0.35	10
10-t	6.9188 ± 0.093	9.9884 ± 0.14	100
10-a	8.8234 ± 0.10	8.9866 ± 0.10	100
10-t	6.8664 ± 0.029	10.0190 ± 0.050	1000
10-a	8.7920 ± 0.031	8.9517 ± 0.031	1000
2-t	1.4010 ± 0.062	1.9980 ± 0.11	10
2-a	2.4296 ± 0.097	2.4272 ± 0.098	10
2-t	1.3594 ± 0.018	2.0168 ± 0.031	100
2-a	2.4009 ± 0.040	2.4002 ± 0.040	100
2-t	1.3584 ± 0.0056	1.9917 ± 0.010	1000
2-a	2.3827 ± 0.012	2.3823 ± 0.012	1000

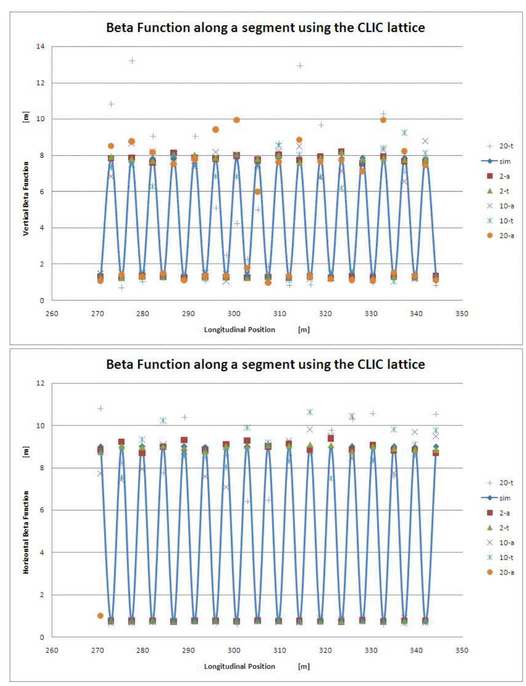

Figure 3: Beta Functions for a sector of the CLIC lattice, without and with noise for 2, 10 and 20 degrees of deviation.

With 10 degrees of noise, for the CLIC lattice the behaivor of the global error for the vertical plane is oppossed

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with respect to the horizontal plane. Err always increases by an amount of 20% of the traditional method by using the new algorithm for the horizontal plane, but it always decreases for the vertical plane by an amount of 10% of the traditional one.

Using the LHC lattice, for all the cases with 10 degrees noise, the new algorithm (-a) decreases the global error Err, to value close to 30% each time, compare with the traditional way to do the measurement for both planes.

With a noise of 20 degrees, for both lattices, it is found that the average of the global error is decreasing when using the new algorithm compare to the traditional one. This is expected, if the new algorithm is over stabilized values for the fraction of noise, and without favored to a particular plane. The reported values imply that percents of 26 and 49 for the horizontal and vertical planes for CLIC, and 29 to 27 for the horizontal and vertical planes for LHC, are reduced in the global error, when the new algorithm is applied compare with the traditional one.

Taking the average of the β function at each location before taking the global error, it is not studied in here to have the completed effect of the noise. Taking that average will reduced the noise in advance.

Table 2: Beta Fuction Err. Using LHC Lattice

Case.	$< Err.\beta_x >$ [%]	$Err.\beta_y >$ [%]	Num. Orb.
20-t	23.9845 ± 1.6	21.0685± 0.53	10
20-a	16.0208 ± 0.94	15.6167± 0.70	10
20-t	22.8245± 0.53	22.7674 ±0.37	100
20-a	16.7904± 0.45	16.3095± 0.37	100
20-t	22.3585± 0.16	22.7038± 0.12	1000
20-a	16.9809± 0.15	16.5626± 0.11	1000
10-t	10.4422± 0.43	11.2105± 0.65	10
10-a	7.1806± 0.54	7.1872± 0.57	10
10-t	10.3884± 0.16	11.0943± 0.17	100
10-a	7.2590± 0.15	7.2989± 0.15	100
10-t	10.4041± 0.055	10.9358± 0.054	1000
10-a	7.5054± 0.053	7.4269± 0.051	1000
2-t	1.9998± 0.075	2.1757± 0.12	10
2-a	2.2319± 0.051	2.2607± 0.058	10
2-t	2.0484± 0.033	2.1714± 0.033	100
2-a	2.2017± 0.027	2.2133± 0.027	100
2-t	2.0329± 0.010	2.1555± 0.010	1000
2-a	2.2014± 0.0092	2.2127± 0.0090	1000

In general, in all cases, the observed uncertainty of the global error is reduced or keeping in the same level as using the traditional way to do the measurements.

Nevertheless it is possible to run the new algorithm to calculate the beta function for the entire ring. The observed advantage is that there are few times when the measurements can be done with the new algorithm, in comparison with the old case. This could means that the difference in the phase advance is less restricted in new algorithm case.

Further studies could determine at which conditions the new algorithm favors, and to stablished the differences when using the traditional fitting processes, and or the fact that both plane measurements are involved. This would depend in the tunes of the accelerator used and the distance between the elements from whre the measurement is done.

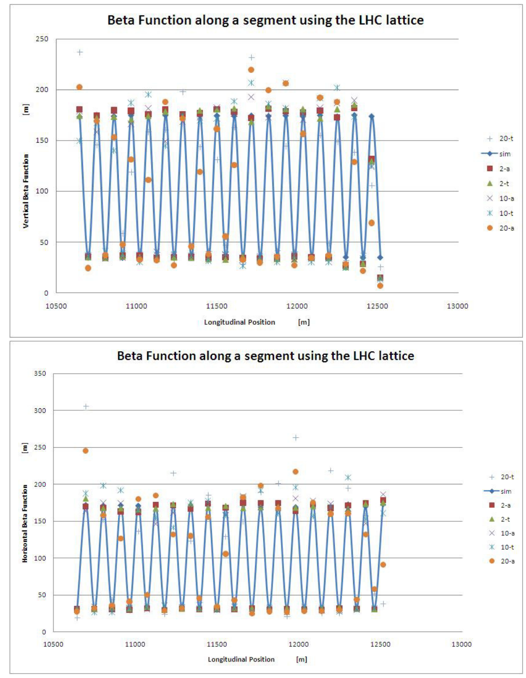

Figure 4: Beta Functions for a sector of the LHC lattice , without and with noise for 2, 10 and 20 degrees of deviation.

CONCLUSION

The algorithm introduced in this paper allows reducing the noise presented when performing the Beta Function measurement. In applications to the LHC Mad-X lattice, it is found cases where the improvement is close to 30% compare to the traditional one, when the noise is 10^o or 20^o; using the CLIC Mad-X lattice a reduction close to 25% and 50% are observed with a noise of 20^o . Althought further studies are needed to establish the ideal conditions for its application in real machines, this new algorithm could serve as a complement and/or improvement to the traditional technique.

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