BEAM BASED CALIBRATION FOR BEAM POSITION MONITORS

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Abstract

Beam position monitoring is one of the most fundamental diagnostic tools in an accelerator. To get good performance of the BPM system, the beam-based alignment method has been developed and used for more precise BPM alignment and maintaining the performance. The signal from a BPM is transferred by coaxial cable, and processed by signal processing circuit. The beam position is calculated from the relative ratios between the 4 outputs of the BPM head. The circuit gain is calibrated in the beginning on a test bench. But this calibration changes with each passing year. To escape from this problem, a method for calibration of the gain similar to beam-based alignment is a key issue to maintain the good performance of the BPM system. For this propose, a beam-based gain calibration method has been developed and used at KEK. Both beam-based alignment and beam based gain calibration methods are presented using concrete examples.

INTRODUCTION

For high energy accelerators, the measurement of the beam position is one of the basic diagnostics along with the beam intensity and the betatron oscillation frequency. Stability of the closed orbit is very important for stable operations to maintain good performance in an accelerator. Therefore we have prepared a BPM at each quadrupole magnet. For example, there were 186 BPMs in the J-PARC Main Ring. The BPM system requires a high accuracy measurement. In order to satisfy the requirement, we have done careful calibration of the BPM system in three steps before the commissioning. But, in KEKB, we found noticeable errors larger than 0.1 mm in almost all BPM readings.

These errors come from the alignment error of a BPM to its adjacent quadrupole magnet, and the imbalance among 4 output data of the BPM. Beam-based alignment (BBA) is a method for correcting the offset of a BPM head based on beam measurement [1]. The center position of each BPM should be known in terms of offset from the magnetic center of the adjacent quadrupole magnet. The relative gain of the output data may drift due to unpredictable imbalance among output signals from the pickup electrodes, because the output signals must travel through separate paths, such as cables, connectors, attenuators, switches, and then are measured by the signal detectors. For this reason, the gains of every BPM of KEKB have been calibrated by a non-linear least-square method [2]. The same process of gain calibration used in KEKB has been tried with the BPM system in J-PARC Main Ring, however the fitting result gave indefinite solutions.

A new beam-based method to calibrate the gains of BPMs at the J-PARC Main Ring has been developed using the Total Least Square method (TLS) [3].

CALIBRATION DURING INSTALLATION

The output data from a BPM system was usually calibrated in the following three steps on the test bench at KEKB [4].

- 1. Mapping measurement of BPM system
 - The BPM heads were fabricated to within $a \pm 0.1$ mm tolerance. However, variations of frequency response between button electrodes cannot be ignored considering the accuracy requirements. All BPMs were mapped at a test bench with a movable antenna to identify the electrical zero position of each BPM.
- 2. Alignment of geometrical offset
 - Most BPMs (~97%) were aligned in relation to their nearest quadrupole magnet. After installation of BPM heads in the ring, we measured the geometrical offsets of the BPM heads relative to the quadrupole magnet. But the measured offsets were not the offset from the field center of quadrupole magnet.
- Attenuation ratio of transmission line We employed 4 twisted coaxial cables with foamed Polyethylene insulation between BPMs in the tunnel and electronics at a local control room above ground. To measure signal attenuation at the detection frequency, the cables together with the electronics were also calibrated to 50 μm accuracy.

BEAM BASED ALIGNMENT

In order to align a BPM to the field center of a quadrupole magnet, the BPM offset is calibrated by finding the position of the closed orbit at that BPM which is insensitive to a change of the field strength of the adjacent quadrupole magnet. Calibration data are taken for different beam orbits and different field strengths of the quadrupole magnet. The orbit change due to the field gradient change Δk of the quadrupole magnet is proportional to the closed orbit displacement Δx from the magnetic center of the quadrupole magnet. Figure 1 shows an example of BPM offset measurement by BBA in the main ring at J-PARC [5]. A correction coil wound on each pole of a quadrupole magnet was used to change the field strength. The current on the correction coil, I_Q was changed from -4 A to 4 A nominally. To change the

orbit, a bump orbit Δx was set to three different orbits of -8 mm, 0 mm and 8 mm. When the beam orbit is in the vicinity of the center of the quadrupole magnet, even if the current of the correction coil is changed, the orbit does not change significantly (Fig. 1-(a) ~ (c)). In the measurement, three $\Delta x / \Delta I_Q$ values were obtained for the 3 bump orbits as shown Fig.1-(d). The beam position which gives $\Delta x / \Delta I_Q = 0$ is the offset of the BPM.



Figure 1: x-COD vs. QM current I_Q . (a) bump ~ -8 mm, (b) bump ~ 0 mm, (c) bump ~ +8 mm.

The orbital change due to ΔI_Q can be monitored not only the BPM but by any other BPMs in the ring. Figure 2 shows the offset positions observed by all BPMs when the field strength of quadrupole magnet is changed.



Figure 2: Offset values calculated with the all BPM response. y_0 at BPM#130 at J-PARC.

The vertical and horizontal offsets were obtained for almost all BPMs in the J-PARC MR by this BBA method.

Figure 3 shows the offset distributions for MR BPMs. The BPM Offsets measured by this method were installed in the data base.



Figure 3: Distribution of BPM offsets obtained with BBA in J-PARC MR. (a) x-plane, (b) y-plane.

The effects of the BPM offset correction can be seen in the beam orbit. Figs. 4(a) and (b) show the closed orbit distortion (COD) with and without the offset correction for the MR. The closed orbit was corrected better than it was without the correction, especially the vertical closed orbit.



Figure 4: CODs along the MR, corrected without / with the BBA offset data (red / blue lines, respectively). (a) x-COD, (b) y-COD.

Figure 5 show the offset distribution obtained by BBA in the LER and the HER at KEKB. We have also set the offset data in the data base of BPM system.



Figure 5: BPM offsets. Blue bars and red bars show horizontal offsets and vertical offsets, respectively.

The effects of the BPM offset correction can be seen in the beam orbit. Figures 6 upper and lower show the COD before and after the beam based alignment for the LER. The orbit is smoother, especially in the arc sections, after the offset correction is included.



Figure 6: COD change by BBA in the LER at KEKB.

BEAM BASED GAIN CALIBRATION

The beam based gain calibration (BBGC) is a very effective method for achieving BPM accuracy. We introduce the two gain analysis methods that have been developed at KEKB and J-PARC.

BBGC for BPM with Four Buttons

The BPM model assumes the configuration with four electrodes as illustrated in Fig. 7. The output voltage of the *i*-th electrode for the beam position (x, y) against the BPM center is expressed as:

$$V_i = qg_i F_i(x, y)$$

where g_i is the relative gain factor, $F_i(x,y)$ is the response function normalized to $F_i(0,0)=1$, and q is the proportional factor to the beam current. The response function depends only on the geometrical structure of the BPM head.

The beam positions are measured m times with a pickup head, by changing the orbit at the monitor each time, the signal from the i-th electrode at the j-th measurement is given by,

$$V_{i,j} = g_i q_j F_i(x_j, y_j), \quad i = 1, ., 4, \ j = 1, ., m$$



Figure 7: BPM model with 4 electrodes.

Since we can set g_1 to 1 with a proper scaling factor for the beam charge, there exist only 3 unknown gains, g_2 , g_3 and g_4 . We measure $V_{1,j}$, $V_{2,j}$, $V_{3,j}$ and $V_{4,j}$ at each measurement. Since g_i will not change at each measurement, q_j , x_j and y_j are unknown parameters. After the m-th measurement the number of the unknown parameters is 3+3m. The known parameters are 4m. When m is larger than 4, then 4m exceeds 3+3m, and the unknown parameters, including the gains, can be calibrated using a non-linear least-square method,

$$J(a) = \sum_{i=1}^{4} \sum_{j=1}^{m} \left[V_{i,j} - g_i q_j F_i(x_j, y_j) \right]^2$$
$$a = \left(g_2, g_3, g_4, q_1, x_1, y_1, \cdots, q_m, x_m, y_m \right)$$

where a denotes the array of fitting parameters. The fitting analysis has been performed using the Marquardt method [6] which is able to obtain the optimum value with sufficient accuracy. This BPM model has the nice symmetry that all of the response functions can be expressed with only one function,

$$F_{1}(x, y) = 1 + a_{1}x + b_{1}y + a_{2}(x^{2} - y^{2}) + b_{2}(2xy) + a_{3}(x^{3} - 3x^{2}y) + b_{3}(3xy^{2} - y^{3}) + a_{4}(x^{4} - 6x^{2}y^{2} + y^{4}) + b_{4}(x^{3}y - xy^{3}) F_{2}(x, y) = F_{1}(-x, y), \quad F_{3}(x, y) = F_{1}(-x, -y), F_{4}(x, y) = F_{1}(x, -y)$$

The expansion coefficients (a_1, a_4) and (b_1, b_4) are determined by fitting the measured mapping at the calibration stand or the calculated mapping by the finite boundary element method. Figure 8 shows the example of the relative gains of g_2/g_1 , g_3/g_1 and g_4/g_1 of all BPM pickups in the ring which were obtained by BBGC at KEKB [6].



Figure 8: The Relative gains obtained by BBGC in LER (upper) and HER (lower) at KEKB. Here, horizontal scale is BPM No.

The results of the figure are the first measurement for BBGC at KEKB, where the spread of these gains is $-8\% \sim +8\%$. The gain data are used into the software to calculate the beam position, to correct the gains of all BPMs.

We converted the measured value of BPM gain into BPM offset value. We tried to compare the offset with the offset from BBA. Figure 9 shows the correlation of both offsets in the LER and the HER at KEKB. From these correlations we believe that BPM gain drift has caused the offset change.



Figure 9: The correlation of both offsets in the LER and the HER at KEKB.

BBGC for BPM with Diagonal Cut

In the J-PARC MR, we adopted an electrostatic pickup with a diagonal-cut cylinder type duct as shown in Fig. 10, where the horizontal and vertical beam positions are independently detected by two pairs of pickup electrodes. The simulation was performed by using the method in the previous section to estimate gains of the MR BPMs. For these linear response pickups, the above-mentioned least squares (LS) method minimizing the sum of the square of the difference between each electrode output and the model response function is not applicable. The simulation showed the result that these gains were changed depending on the given initial values for q_j , x_j and y_j in the fitting process. The non-linear fitting method was not able to be used for the gain analysis of such diagonal cut electrodes.



Figure 10: BPM heads with diagonal cut electrodes in J-PARC MR.

The outputs of diagonal cut electrodes for the beam position (x, y) are given by

$$V_L = \lambda \left(1 + \frac{x}{a}\right), \quad V_R = g_R \lambda \left(1 - \frac{x}{a}\right), \quad (1)$$

$$V_D = g_D \lambda \left(1 - \frac{y}{a} \right), \quad V_U = g_U \lambda \left(1 - \frac{y}{a} \right), \quad (2)$$

where λ is the proper normalization factor proportional to the beam current, g_R , g_U and g_D are the relative gains to the electrode L and g_L is normalized to 1, and a is the radius of the diagonal cut electrode. By eliminating λ , x, y and *a* in the above formula, we obtain the equation

$$V_L = -\frac{V_R}{g_R} + \frac{V_U}{g_D} + \frac{V_D}{g_D}$$

This linear equation express three gains in terms of four outputs. When beam positions are measured m times, the simultaneous linear equations are expressed in a matrix representation of

$$A\mathbf{x} = \mathbf{b}$$
,

where

$$\boldsymbol{A} = \begin{pmatrix} -V_{R,1} & V_{U,1} & V_{D,1} \\ \vdots & \vdots \\ -V_{R,j} & V_{U,1} & V_{D,j} \\ \vdots \\ -V_{R,m} & V_{U,m} & V_{D,m} \end{pmatrix}, \\ \boldsymbol{x} = \begin{pmatrix} 1/\\ g_R \\ 1/\\ g_U \\ 1/\\ g_D \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} V_{L,1} \\ \vdots \\ V_{L,j} \\ \vdots \\ V_{L,m} \end{pmatrix},$$

Then $V_{L,j}$, $V_{R,j}$, $V_{U,j}$ and $V_{D,j}$ denotes the measured output at the j-th measurement. The approximate solution by least squares (LS) of the linear system Ax = b is given by

$$\boldsymbol{x}_{LS} = \left(\boldsymbol{A}^T \boldsymbol{A}\right)^{-1} \boldsymbol{A}^T \boldsymbol{b}$$

when the components of matrix A have no errors. On the other hand, when A has errors, the best approximated solution is given by total least squares (TLS) method [7]. The solution of TLS is given by

$$\boldsymbol{x}_{TLS} = \left(\boldsymbol{A}^{T}\boldsymbol{A} - \boldsymbol{\sigma}_{n+1}^{2}\boldsymbol{I}\right)^{-1}\boldsymbol{A}^{T}\boldsymbol{b},$$

where n is the rank of A and σ_{n+1} is the smallest singular value of the matrix [A b].

We compare the TLS method with the LS method by using simulations. In this simulations, the mapping data were generated from model outputs with the defined Eq. (1)-(2), 12500 points at 25 displaced positions with 0.2% Gaussian noise, as shown in Fig. 8. The gains were given reasonable values, set $g_R = 1$, $g_L = 1.01$, $g_U = 1.005$, $g_D = 0.975$.

| Table 1: Simulation Resul | lt |
|---------------------------|----|
|---------------------------|----|

| | g _L | g u | g d |
|-----------|----------------|------------|-------|
| LS | 1.034 | 1.015 | 0.988 |
| Variation | 0.024 | 0.01 | 0.013 |
| TLS | 1.012 | 1.005 | 0.977 |
| Variation | 0.002 | 0.0 | 0.002 |

The results of values given to relative gains and variation from true gains in both TLS and LS simulations are summarized in Table 1. The TLS gives smaller variations than LS. Corrected positions by obtained gains are shown as black points in Fig. 11.



Figure 11: Reconstructed mapping data. Red: (x, y) without correction, Black: (x, y) with TLS.

By using real beam, we tested both the TLS and the LS method for diagonal cut BPMs in the J-PARC MR. The position measurements were done in nine displacements of beam positions at the BPM as shown in Fig.12. The results of gain calibrations are summarized in Table 2. We can see differences in the relative gains depend on the fitting method. The beam positions corrected by the new gain are overlapped on Fig. 14.



Figure 12: Measured mapping data and corrected mapping data. Red points are the measured positions without gain correction. Black points are the corrected positions with TLS method.

Table 2: Corrected Relative Gains by TLS and LS Methods

| BPM001 | g _L | g u | g d |
|--------|----------------|-------------|----------------|
| TLS | 1.0062 | 1.0024 | 0.9873 |
| LS | 1.0103 | 1.0045 | 0.9892 |
| BPM002 | gL | $g_{\rm U}$ | G _D |
| TLS | 0.9568 | 0.9811 | 0.9463 |
| LS | 0.9617 | 0.9838 | 0.9487 |

We analyzed the gains using the data obtained with actual beam. In order to obtain the mapping data of beam positions the beam orbit was kicked by a steering magnet.

The gains g_R , g_U and g_D are plotted in Fig. 13(a),(b) and (c), respectively, as functions of the address number along MR. The gains for two cases of beam intensities (low and high) are plotted as blue and red solid circles, respectively. The beam amounts of "Low" and "High" intensities are 10^{13} and 10^{14} -order protons per pulse, respectively. The gains are different by a maximum of 2–3% between the cases of "Low" and "High" intensity. The accompanied error bars are calculated as follows.



Figure 13: Relative gains calculated by TLS method. g_R , g_U and g_D are plotted. The gains for low intensity (blue line) and high intensity (red line) are plotted Here, horizontal scale is BPM No.

EVALUATION OF GAIN CORRECTION

Examination of Four Button Pickups at KEKB

Usually the beam position is calculated from the output of four electrodes as Fig.1. We obtain the normalization of the signals (X, Y) as

$$X = \frac{V_1 - V_2 - V_3 + V_4}{V_1 + V_2 + V_3 + V_4}, \quad Y = \frac{V_1 + V_2 - V_3 - V_4}{V_1 + V_2 + V_3 + V_4}$$

Mapping measurement was made at many mesh points in the central area. We fitted third order polynominals (F_x, F_y) of two variables (X, Y) for these mesh data to describe the relation between (x, y) and (X, Y) for each BPM as follows

$$x = F_{X}(X, Y), y = F_{Y}(X, Y),$$

where

$$F_{X}(X,Y) = a_{0} + a_{1}X + a_{2}Y + a_{3}X^{2} + a_{4}XY + a_{5}Y^{2} + a_{6}X^{3} + a_{7}X^{2}Y + a_{8}XY^{2} + a_{9}Y^{3}$$

$$F_{Y}(X,Y) = b_{0} + b_{1}X + b_{2}Y + b_{3}X^{2} + b_{4}XY + b_{5}Y^{2} + b_{6}X^{3} + b_{7}X^{2}Y + b_{8}XY^{2} + b_{0}Y^{3}$$

where the coefficients $(a_n, b_n \cdots n = 1, 2, ., 9)$ are obtained by fitting of the mapping data.

The beam position is also obtainable from the output voltage of any three electrodes chosen out of four electrodes. Using the same data, we also obtained the normalizations of two electrodes as

Then it gives the four relations between the beam position and the normalization as follows:

$$\begin{cases} (x_1, y_1) = (F_X^{ABC}(X_1, Y_1), F_Y^{ABC}(X_1, Y_1)) \\ (x_2, y_2) = (F_X^{BCD}(X_2, Y_1), F_Y^{BCD}(X_2, Y_1)), \\ (x_3, y_3) = (F_X^{ACD}(X_2, Y_2), F_Y^{ACD}(X_2, Y_2)) \\ (x_4, y_4) = (F_X^{ABD}(X_1, Y_2), F_Y^{ABD}(X_1, Y_2)) \end{cases}$$

where functions (F_x, F_y) are third order polynomials for three electrodes of BPM.

If the four outputs have ideal correlation, these four beam positions $((x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4))$ should coincide with each other. The software procedure for the BPM system performs calculation of not only beam positions using four electrodes, but also four beam positions using three electrodes to examine consistency among these positions. The deviations among the four beam positions are represented by the standard deviation formula as follows:

$$\begin{cases} \sigma_{X} = \sqrt{\frac{1}{4} \sum_{i=1}^{4} \left(X_{i} - \overline{X}\right)^{2}} & \text{with} \quad \overline{X} = \frac{1}{4} \sum_{i=1}^{4} \left(X_{i}\right), \\ \sigma_{Y} = \sqrt{\frac{1}{4} \sum_{i=1}^{4} \left(Y_{i} - \overline{Y}\right)^{2}} & \text{with} \quad \overline{Y} = \frac{1}{4} \sum_{i=1}^{4} \left(Y_{i}\right) \end{cases}$$
(3)

where σ_x and σ_y are the standard deviations among four beam positions called the consistency error of beam position measurement.

Figure 14 shows an example of the distributions of the consistency error at KEKB.

As a result of having corrected the output voltage with new gains, the consistency error became very small.

BPMs and Beam Stability



Figure 14: Consistency before (top) and after (bottom) gain calibration in LER at KEKB, horizontal scale is BPM No.

Examination of BPM Gain at J-PARC MR

To evaluate the analyzed gains, we checked the consistencies of four positions calculated from Eqs. (1) and (2) as following,

$$x_{1} = \frac{V_{L} - V_{R}/g_{R}}{V_{L} + V_{R}/g_{R}}a, \quad y_{1} = \frac{V_{U}/g_{U} - V_{D}/g_{D}}{V_{U}/g_{U} + V_{D}/g_{D}}a$$

$$x_{2} = \frac{V_{L} - V_{R}/g_{R}}{V_{U}/g_{U} + V_{D}/g_{D}}a, \quad y_{2} = \frac{V_{U}/g_{U} - V_{D}/g_{D}}{V_{L} + V_{R}/g_{R}}a$$

$$x_{3} = \left(\frac{2V_{L}}{V_{U}/g_{U} + V_{D}/g_{D}} - 1\right)a, \quad y_{3} = \left(\frac{2V_{U}/g_{U}}{V_{L} + V_{R}/g_{R}} - 1\right)a$$

$$x_{4} = \left(\frac{-2V_{R}/g_{R}}{V_{U}/g_{U} + V_{D}/g_{D}} + 1\right)a, \quad y_{4} = \left(\frac{-2V_{D}/g_{D}}{V_{L} + V_{R}/g_{R}} + 1\right)a$$

where (x_1, y_1) is the position using two electrodes in the horizontal or vertical direction, (x_2, y_2) is the position using four electrodes, and (x_3, y_3) and (x_4, y_4) are obtained using three electrodes. We also defined the consistency of x and y as σ_x and σ_y in Eq. (3).

Table 3 shows improvement of consistency error by using BBGC.

Table 3: Consistency Before and after Gain Calibration of BPM at J-PARC MR

| | Before | | After | |
|--------|-----------------------|------------------|-----------------------|------------------|
| | $\sigma_X[\text{mm}]$ | $\sigma_{Y}[mm]$ | $\sigma_X[\text{mm}]$ | $\sigma_{Y}[mm]$ |
| BPM001 | 0.524 | 0.518 | 0.018 | 0.018 |
| BPM002 | 0.964 | 0.954 | 0.025 | 0.025 |

CONCLUSION

We should pay some special attention to guarantee precise measurement of beam positions over a long time. The BBA measurement is useful for correction of the BPM offset error. The gain balance among four outputs of a BPM changes gradually over a long period. The imbalance among the gains gives offset errors to beam position. The most probable source of the gain drift is the change in the electrical characteristics of the transmission line of the signal by temperature drift, because we found seasonal variation in the gain drift at KEKB. We have achieved a high-accuracy BPM system by monitoring the consistency error and applying beam-based gain calibration [8].

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