DIGITAL PROCESSING OF PICK-UP SIGNALS FOR POSITION AND TUNE DETERMINATION

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Abstract

With the advent of fast high resolution Analog to Digital Converters (ADCs) and Field Programmable Gate Arrays (FPGAs), "all digital systems" for pick-up data processing to determine position and tune have become commonplace. This contribution compares the frequently used position estimators used in the digital systems in terms of measurement variance, bias and robustness to external interference. An analytical beam model, along with simulated pick-up signal and actual pick-up signal from the SIS-18 synchrotron are used for the comparison. The effect of precise position estimation on the tune spectra is discussed.

INTRODUCTION

High precision position estimation requires the optimization of the beam position measurement system in all the stages of development, which starts from EM simulations to optimize the pick-up design against unwanted resonances, establishing linearity while minimizing the cost of manufacturing [1]. The mechanical construction and installation of pick-ups with respect to the magnetic center of magnets within the specified tolerances is also a challenging task. Finally, the electronics required for acquisition and processing of the pick-up signals demand low noise and high dynamic range as well as periodic and precise calibration. The typical methods for signal processing and calibration are described in [2–4]. In the recent years, the signal processing have completely shifted to digital domain due to availability of fast high resolution ADCs and FPGAs and this contribution will focus on this aspect of position measurement system for circular accelerators.

The concepts of pick-up position sensitivity and offset are presented along with the typical signal spectra for bunched and coasting beams in the next couple of sections. Following that, the frequently used digital position estimation methods are discussed and a new approach to position estimation based on "linear regression model" is introduced. All the presented methods are compared with an analytical beam model, simulated beam data and the pick-up signal from the SIS-18 synchrotron in terms of estimated position bias and variance. The effect of position estimation methods on tune spectra calculated from the turn-by-turn position is discussed.

POSITION SENSITIVITY AND OFFSET

The pick-up position sensitivity and offset estimates are obtained from the EM simulations [5] or on-bench wire based measurements [6, 7]. The uncertainty in the position sensitivity measurement is given by the precision of the measurement equipment used for bench measurements and

simulation time/resources which are often < 0.1% of absolute values as shown in [5, 8]. In careful pick-up designs, the pick-up sensitivity is found to be constant within 0.1% of the absolute sensitivity value in the frequency region of interest [5]. Once the pick-up sensitivity and offset are known, the beam center-of-mass can be determined from the difference of the signal induced on opposite pick-up plates. There are two important features of pick-up signal which are relevant for digital position estimation a) Most of the pick-up types are "capacitive" or AC coupled, which leads to rejection of the DC component of the beam signal and b) The signal is sampled with fast ADCs such that many samples are acquired in each time interval for the position measurement. Thus the problem of position estimation is that of an overdetermined system whose low frequency components are significantly suppressed. The lower cut-off is given by the termination impedance of the pick-up [2].

PICK-UP SIGNAL SPECTRUM

The pick-up signal spectrum of a ring accelerator is unique due to the periodic crossing of beam particles through the pick-up. A beam of particles traversing the synchrotron or
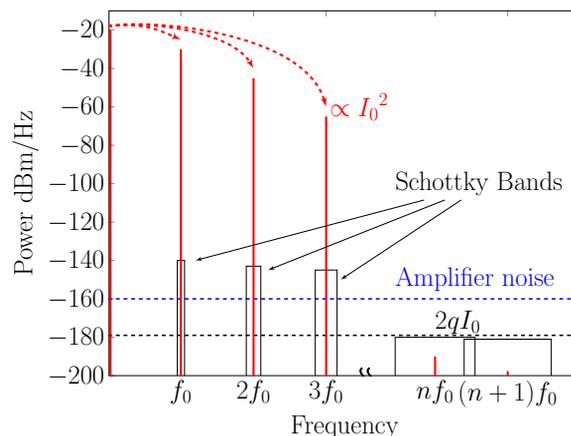

Figure 1: A bunched beam power spectrum of a U^{28+} bunched beam with 10^9 particles at injection energy in SIS-18. $2qI_0$ is the shot noise level while blue dashed line represents the electronics noise. The Schottky bands are shown for reference.

storage ring with a constant energy is referred to as coasting beam. The beam is said to have no coherent longitudinal structure due to absence of any longitudinal focusing. However, due to finite momentum spread, finite number of particles and periodic traversal of particles through the BPM, signals proportional to the square root of number of particles are induced at the revolution frequencies. The power in each revolution band is given by $2qI_0$, where q is the charge

state and I_0 is the beam current. They are called Schottky signals due to their origin in shot noise [9]. When an external field imposes a longitudinal structure on the beam, power is transferred from the DC component of the beam to higher harmonics at the revolution frequency. Any harmonic with sufficient power in the difference signal spectrum can be used to obtain the position information. This coherent power is proportional to the number of particles and is usually large enough for calculation of bunch-by-bunch beam position measurement. Figure 1 shows the estimated power spectra of a U^{28+} bunched beam with 10^9 particles at injection energy. In comparison, the power in the Schottky bands of the unbunched beam with the same current is also shown. Though the beam position measurement usually implies bunched beam position measurement; with high beam intensities and long measurement times, Schottky signals can be utilized for beam position measurements of a coasting beam. Detailed introduction to pick-up signal spectra can be found in [10].

POSITION ESTIMATORS

Each pick-up electrode signal is either individually sent through an amplifier/attenuator chain or passive "hybrids" are utilized to create sum and difference signals before amplifying the signal. Hybrids are preferred when the pick-up signals are large and require to be attenuated before digitization. The electronics chain is designed to match the pick-up signal to the ADC input range, while the ADCs are selected such that the effective number of bits ENOB is matched to the amplifier noise. The sampling rate is kept as high as possible while satisfying the criterion mentioned above. The ADC selected for SIS-100 has 12 effective number of bits and sampling rate of 250 MSa/s. A calibration scheme is used to periodically correct the amplifier gain and offset drifts as well as the ADC offset drift. This is performed to eliminate the systematic biases in the measurement. The N digitized signal samples from opposite electrodes are denoted by $U_{l,i}$ and $U_{r,i}$, where $i \in \{1, N\}$ denotes the sample index.

There are two classical approaches which are frequently used for calculation of beam position. The first and most frequent approach referred to as integral method resurrects the lost DC or baseline, which is lost due to AC coupling of the pick-up [11]. The baseline of the signal from each capacitive plate is individually determined from the samples in between the bunches (See Fig. 4 and 7). In frequency domain, it translates to the reconstruction of the DC signal from the higher harmonics (See Fig. 1). If the baseline of each electrode is denoted by B_l and B_r, the beam position of the whole bunch $<x>$ can be calculated from the difference-over-sum ratio:

$$\frac{<x>}{k} = \frac{\sum\limits_{i=1}^{N}(U_{l,i}+B_l) - \sum\limits_{i=1}^{N}(U_{r,i}+B_r)}{\sum\limits_{i=1}^{N}(U_{l,i}+B_l) + \sum\limits_{i=1}^{N}(U_{r,i}+B_r)} \qquad (1)$$

Here k is the pick-up sensitivity in *mm* units. Obviously, this method is very prone to biases resulting from baseline calculation.

The second position estimator is based on the power of individual signals and uses the root-sum-square (RSS):

$$\frac{<x>}{k} = \frac{\sqrt{\sum\limits_{i=1}^{N} U_{l,i}^2} - \sqrt{\sum\limits_{i=1}^{N} U_{r,i}^2}}{\sqrt{\sum\limits_{i=1}^{N} U_{l,i}^2} + \sqrt{\sum\limits_{i=1}^{N} U_{r,i}^2}} \qquad (2)$$

This approach is a reasonable substitute to the earlier approach since the baseline restoration is not required. In the frequency domain, this estimator weights each harmonic with its magnitude, and thus the noise characteristics are significantly better than integral method.

The third and the newer approach represents the position estimation problem as the case of simple linear regression and solves it with the ordinary least square (OLS) approach which takes a simple closed form [12]. For the conciseness of the formulation, the electrode signals are represented in difference $U_{d,i} = U_{l,i} - U_{r,i}$ and sum forms $U_{s,i} = U_{l,i} + U_{r,i}$ and the estimator can be given as,

$$\frac{<x>}{k} = \frac{N \cdot \sum\limits_{i=1}^{N}(U_{d,i}U_{s,i}) - (\sum\limits_{i=1}^{N} U_{d,i})(\sum\limits_{i=1}^{N} U_{s,i})}{N \cdot \sum\limits_{i=1}^{N} U_{s,i}^2 - (\sum\limits_{i=1}^{N} U_{s,i})^2} \qquad (3)$$

This estimator has two important advantages over the difference-over-sum approach. The first is the minimization of the residuals in the least square sense. The second is immunity to any slow offset errors which might occur due to ADC offsets, external interferences or any amplifier offsets. This is especially advantageous for operating in asynchronous mode or closed orbit mode, when position is calculated from long data sets.

Bias and variance

The bias and variance characteristics for each estimator are evaluated using a simple triangular beam model analytically. The triangular model is chosen for the simplicity in performing the error propagation calculations while capturing many relevant features of a real beam. Figure 2 shows the triangular beam model, where the dashed line with blue dots represents the AC coupled signal from pick-up plates, while the solid line with red dots represent the baseline restored signal. σ_V is the std. deviation of the individual data samples. V_{FS} is the full scale voltage while A is the relative amplitude of the signal with respect to the full scale voltage. N_S is the number of "signal samples" inside the triangular bunch while N_B is the number of "baseline samples" outside the bunch. It should be noted that for a raw signal, the distinction between signal samples and baseline samples is not clear since, baseline samples also carry position information. Since the analytical calculations rely on this distinction, perfect baseline restoration is assumed.
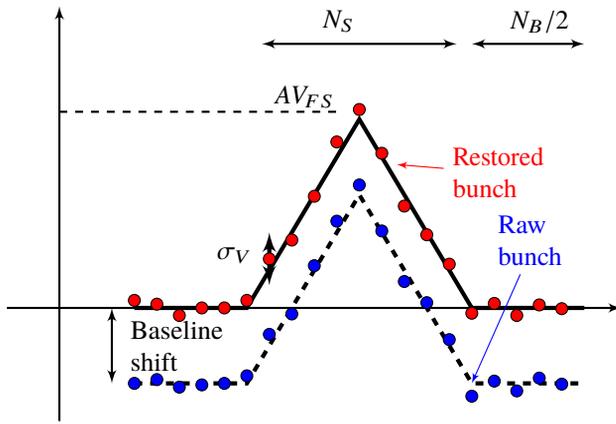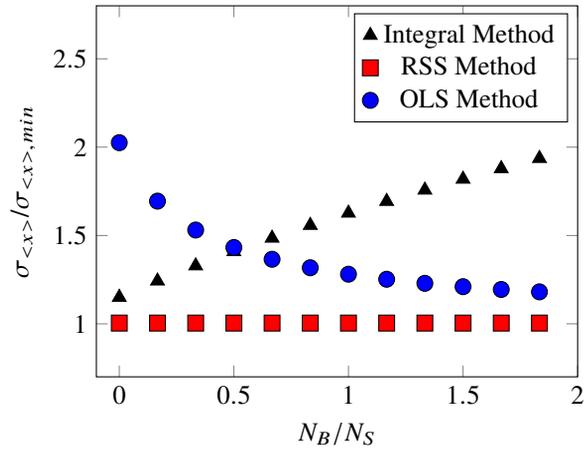
Figure 2: Triangular beam model.

Figure 3: Predictions of the std. deviation of positions calculated for the analytical beam model against the ratio of number of baseline samples to number of signal samples ($N_S = 80$). Each std. deviation value is normalized to the smallest value.

Using error propagation of independent samples, it can be shown that for a perfectly restored baseline, the std. deviation of the position calculated for a centered beam by the estimators in Eq. 1, Eq. 2 and Eq. 3 are given by,

$$\frac{\sigma_{<x>}}{k} = \sqrt{2} \cdot \left(\frac{\sigma_V}{A \cdot V_{FS}}\right) \cdot \frac{\sqrt{N_S + N_B}}{N_S + 2} \qquad (4)$$

$$\frac{\sigma_{<x>}}{k} = \sqrt{3/4} \cdot \sqrt{2} \cdot \left(\frac{\sigma_V}{A \cdot V_{FS}}\right) \cdot \frac{1}{\sqrt{N_S + 3 + 2/N_S}} \qquad (5)$$

$$\frac{\sigma_{<x>}}{k} = \frac{\sqrt{3}}{\sqrt{2}} \cdot \left(\frac{\sigma_V}{A \cdot V_{FS}}\right) \cdot \sqrt{\frac{N_S(N_S + N_B)}{(N_S + 2)(\frac{1}{4}N_S^2 + N_S N_B)}} \qquad (6)$$

The detailed calculations can be found in [13]. Figure 3 shows the std. deviation as a function of ratio of number of baseline samples N_B to the number of signal samples for each method. The number of signal samples within the triangular beam is $N_S = 80$, and the number of baseline samples is varied. It gives an interesting insight on how each estimator treats the baseline samples. The std. deviation of the integral method increases with the addition of baseline samples since the samples do not contain any position information. The std. deviation of RSS method is shown to be independent of inclusion of baseline samples. The OLS method, calculates the slope by fitting the difference signal to the sum signal, and each point improves the estimate, including the baseline samples. This is evident in reduction of the std. deviation as more baseline points are added.

The outcome of the triangular analytical beam model is compared with a beam generated with MADX particle tracking [14] through the SIS-18 lattice. The generated beam traverses the SPICE model of the capacitive pick-up and acquisition electronics. The simulated pick-up data has the same characteristics as the SIS-18 injection pick-up signals. Figure 4 shows the bunches along with the processing windows, in order to select the different number of baseline samples for the position calculation. The positions are calculated
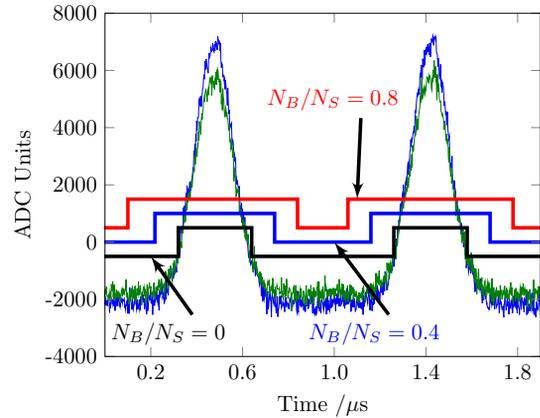

Figure 4: The simulated beam and different processing windows with selected samples for position calculation.

for both restored and non-restored data for 800 consecutive bunches. Figure 5 shows std. deviation of the calculated positions as a function of the baseline samples and signal samples for comparison with Fig. 3. The restored beam case can be directly compared with the analytical model. The negative values of N_B/N_S depict the cutting into the bunches or removing some signal samples. The behavior is similar to the predictions from the analytical model, the OLS estimate improves with inclusion of the baseline samples, while the performance of integral method worsens with inclusion of more samples. The crossing point where OLS produces better results compared to the integral method is at $N_B/N_S = 0$ in comparison to $N_B/N_S = 0.5$ in case of analytical model. The position values obtained from RSS has the smallest std. deviation and is independent of the number of samples included in the calculation. For $N_B/N_S > 0.6$, the OLS closes into RSS performance. Figure 6 shows the mean of the calculated position with restored and non restored beam.
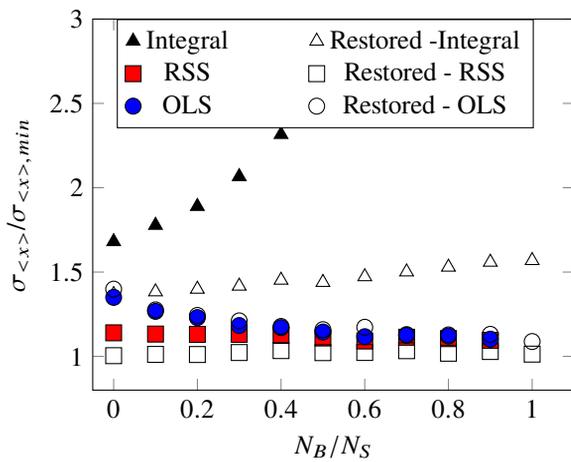
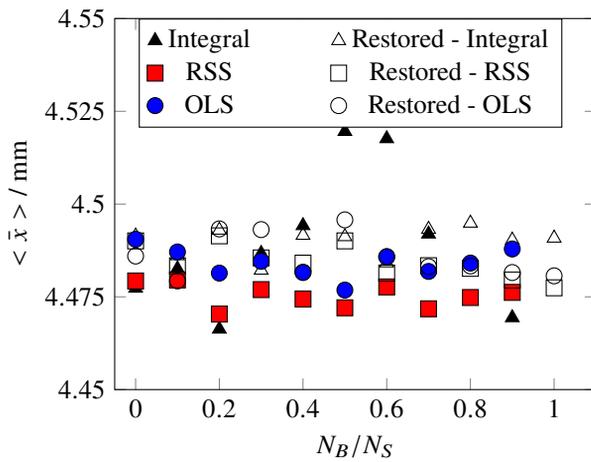Figure 5: Std. deviation of the position calculated from the simulated pick-up data.

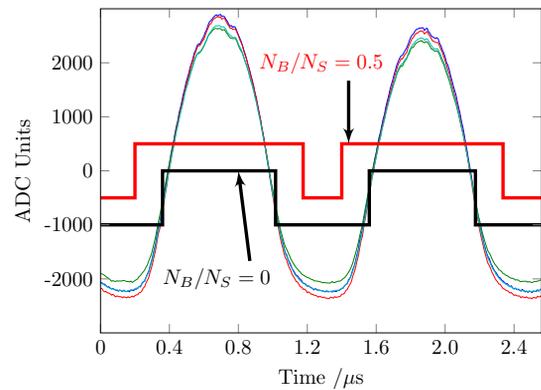Figure 6: Mean of the position calculated from the simulated pick-up data.

Figure 7: The pick-up data from SIS-18 injection beam along with the processing windows with samples used for position calculation.

vestigation, a slow 5KHz common mode interference (CMI) on the pick-up signal seems to cause the difference. OLS method strongly suppresses any common mode signal while the RSS output is disturbed by external interferences. In the
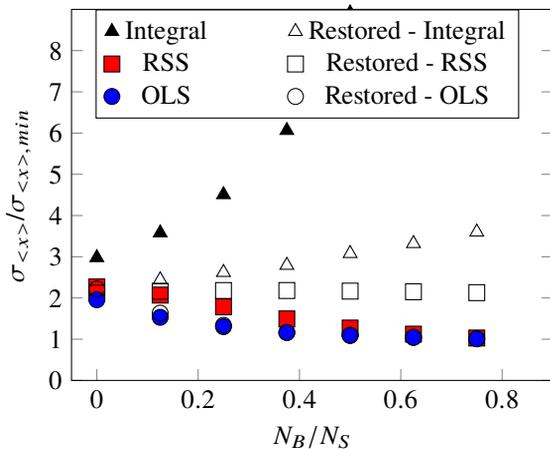

Figure 8: Std. deviation of the position calculated from the SIS-18 injection beam pick-up signal.

All the algorithms predict the true position value correctly in the scanned range.

Finally, the actual beam data is chosen for comparison with the predictions of the analytical model and results from the simulated beam. The beam data is carefully chosen at the injection plateau of SIS-18 in the vertical plane where no transverse oscillations and beam drifts were visible. This is important to distinguish the variance due to the electronics noise from actual beam oscillations or beam drifts. Figure 7 shows the longitudinal bunch structure along with the processing windows. 1000 consecutive bunches were used to calculate the positions. The std. deviation and mean for the positions calculated by each algorithm are shown in Figures 8 and 9 respectively. The integral algorithm expectantly diverges in the non-restored beam case. Even in the case of baseline restoration, it has the largest std. deviation, and thus offers the least position resolution of all methods. The surprising outcome is that the performance of the OLS method has bettered the RSS method in the whole range of N_B/N_S in contradiction to the simulated beam data. On further in-

mean calculation, the baseline restoration seems to produce a bias in the estimated position for integral and RSS methods, while OLS estimates are independent of the baseline restoration. It is clear from the results in this section that the integral method is not suited for position calculation. In addition, the baseline restoration is a non-linear latent procedure which seem to cause bias under external interferences, and is actually not required by both RSS and OLS procedures. One can thus conclude, that the baseline restoration and the integral method should be avoided for position calculation altogether. Rest of the document will only deliberate on the RSS and OLS methods.

Robustness

The robustness of the position estimators in this context is defined as its ability to cope with electronic baseline drifts e.g. ADC or amplifier offset drifts, external electromagnetic
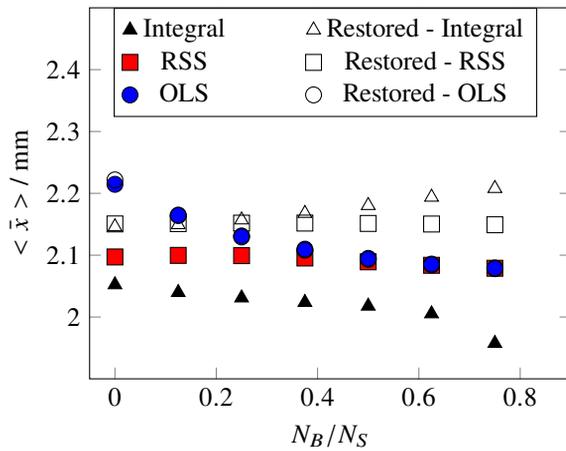
Figure 9: Mean of the position calculated from the SIS-18 injection beam pick-up signal.
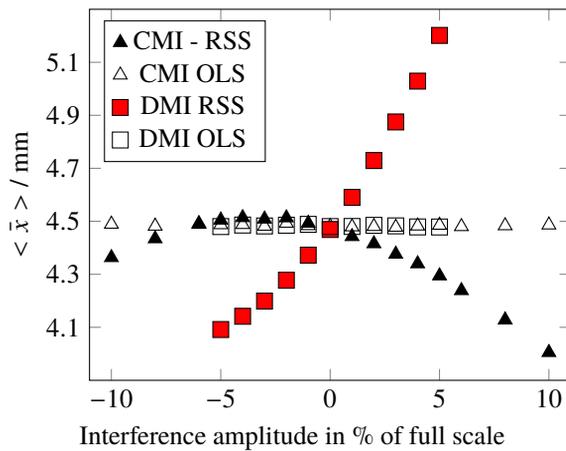

Figure 10: The effect of common mode and differential mode interferences on the position estimate by RSS and OLS fit.

interference, bunch detection errors and dependency on the external signals. The performance of RSS method with the SIS-18 beam data in presence of common mode interference shown to be poor in the last section. Here we illustrate the performance of RSS vs OLS method to common mode interference (CMI) and differential mode interference (DMI). Non-beam related constant offsets are added to the simulated beam signal shown in Fig. 5 in common mode and differential mode. First the common mode signal in the units of % of full scale voltage are added and mean of estimated position is plotted in Fig. 10. The OLS method is immune to the CMI while the RSS has an asymmetric response depending on the the sign of CMI. Similarly, the effect on positions calculated by OLS to DMI is negligible while the gradient for the position calculation error for the RSS method to DMI is large. In real beam scenario, these interferences are unavoidable, and the ability of OLS to suppress them is a big advantage over the RSS method. The next section emphasizes this point further.

TUNE CALCULATION

Tune is the measure of transverse phase advance of the beam after one turn around the synchrotron. The discussions on the robustness of algorithms to non-beam related interferences become extremely important when tune from bunch by bunch position data since any arbitrary external interference can significantly modify the tune spectrum as clearly suggested by Fig 10. Figure 11 shows the tune spectra cal-
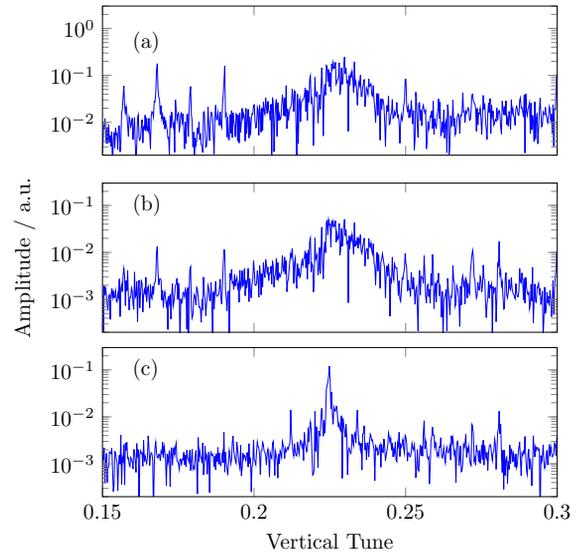

Figure 11: The vertical tune spectrum calculated for an excited beam at SIS-18 injection from the positions calculated from (a) Integral method (b) RSS method (c) OLS method.

culated from the positions calculated by (a) integral method, (b) RSS and (c) OLS on the baseline restored pick-up data of U^{28+} beam which is excited with a wideband noise exciter. The spectra from integral method and power method look similar except for the noise floor, which is higher for integral method. However, the spectra from OLS fit method is strikingly different the other methods, and demonstrates its ability to suppress external disturbances and improve the tune spectrum estimate.

SUMMARY

A new regression based approach to position calculation is presented. It is compared with traditionally utilized estimators with the help of triangular beam analytical model, simulated pick-up signals and SIS-18 injection beam pick-up signals. The performance of the new approach is superior to the traditional algorithms. The robustness of the new OLS based position estimator to external interference is of special significance for bunch-by-bunch position calculations and betatron tune measurements.

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