A NEW MODEL-INDEPENDENT METHOD FOR OPTIMIZATION OF MACHINE SETTINGS AND ELECTRON BEAM PARAMETERS*

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Abstract

Nonlinear programs are widely employed in particle accelerators and storage rings to compute machine settings for optimal model-predictive control of beam parameters. Conventional iterative methods today suffer from problems with finding the global optimal solution when the start solutions are outside the basin-of-attraction for a given objective function to be minimized. A new iterative matrix inversion global optimization (IMIGO) method [1] has been developed to overcome this limitation. IMIGO unlike the existing iterative nonlinear solvers, it calculates only the Jacobian vector of the objection function and not the Hessian matrix at each iteration-this unique feature has led to a new application of IMIGO for optimization of electron beam parameters for cases when a model is unavailable or only an inaccurate model is available. Some possible applications of this IMIGO-based model-independent optimization method will be presented in the paper.

INTRODUCTION

A nonlinear program is a solver typically employed to find the global minimum of a given objective function subjected to certain conditions known as constraints. For optimization of beam parameters, $(b_1, b_2, \cdots, b_n)$, the control variables are the strengths or settings of a group of accelerator elements, $(a_1, a_2, \cdots, a_m)$, that are used to control these beam parameter values. In general, an objective function is defined as a function of the beam parameters. Since each of the beam parameters is a function of the control variables, the value of a given objective function is determined by the values of the control variables, $f_{\rm obj}(a_1, a_2, \cdots, a_m)$. In beam parameter optimization, the start values of the control variables $(a_1^{\rm start}, a_2^{\rm start}, \cdots, a_m^{\rm start})$ are known. Nonlinear programs are used to find the lowest value of a given objective or 'cost' function for the values of the control variables within given bounds: $\Delta_k > (a_k - a_k^{\rm start}) > -\Delta_k$ for $k = 1, 2, \cdots, m$. When the absolute minimum value of the objective function is found, $f_{\rm obj} \Rightarrow f_{\rm obj}^{\rm min}$, the set of variable values is commonly referred to as the global minimum solution: $(a_1^{\rm sol}, a_2^{\rm sol}, \cdots, a_m^{\rm sol})$.

The inherent difficulty of using an iterative method to find the global-minimum solution is well known. In general, an iterative method requires an initial guess solution. If this start solution is too far from the global-minimum solution, the program will find only a local-minimum solution. This problem is known as the basin-of-attraction limit (BOA). A BOA is defined to be the biggest region around a given minimum solution. The problem with the existing iterative nonlinear programs is that they will only find the actual solutions for a special case in which the start solution is inside the BOA corresponding to the global-minimum solution. The new nonlinear programming method IMIGO provides a mitigation to this limitation.

Existing nonlinear programs can be classified into two basic types: One uses an analytical iterative method and the other relies on a stochastic search method such as a genetic algorithm. The inherent difficulty of using an iterative method to find the global-minimum solution is well known. In general, an iterative method requires an initial guess solution. If this start solution is too far from the global-minimum solution, the program will find only a local-minimum solution. As an illustration, a surface plot of the objective function for a minimization problem with two variables is shown in Fig. 1. This figure shows the locations of local-minimum points and the global-minimum point.

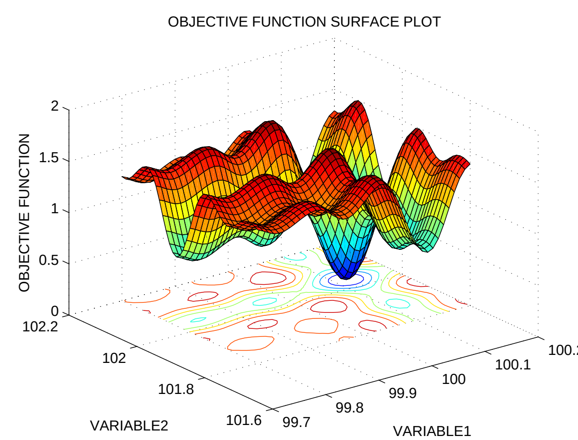

Figure 1: Object function of two variables.

AN OVER VIEW OF IMIGO

IMIGO, like conventional solvers, finds the solution by solving the following set of equations iteratively starting from a given start solution:

$$f_k(a_1, \cdots, a_m) = \partial f_{\rm obj}/\partial a_k = 0 \qquad (1)$$

for $k = 1, 2, \cdots, m$.

One unique feature of IMIGO is that it solves these equations without the using the values of the derivatives:

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$\partial f_j / \partial a_k = 0$ for $j, k = 1, 2, \cdots, m$, *i.e.*, Hessian matrix [1]. In other words, IMIGO is a non-derivative based solver. The iterative process in OASIS is shown in Fig. 2. When the values of the variables converge, the set of values of the variables at the end point is a solution that corresponds to a minimum, maximum, or saddle point of the objective function. This unique feature has led to a simple two parameter (s, p) search method to find the global minimum of the objective function.

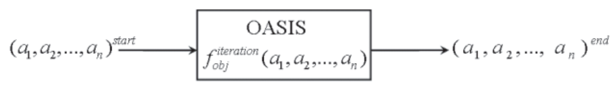

Figure 2: Block diagram of the iterative process to find a solution that minimizes the value of a given objective function.

A main advantage of IMIGO is that it can find the global-minimum solution even when the start solution is not within the BOA corresponding to the global-minimum solution. Another salient feature is that it can search for a path that ends at the global-minimum solution independent of the size and complexity of a given problem, *i.e.*, the problem and the objective function can include many variables and the problem may be very non-linear. To use IMIGO, the user makes a guess on the variable values at the start point for a given objective function to be minimized. The user also imposes specific upper and lower bounds on each of the variables. IMIGO first uses an exhaustive search method to find the start values of the two convergence control parameters.

A Two-Variable Problem

As an illustration of how OASIS works, the results obtained for a typical small-scale minimization problem with two variables a_1 and a_2 are presented [2]. In this example, the same bounds, $\Delta = 0.2$, are imposed on the values of both bounded variables: $\Delta > (a_k - a_k^{\text{start}}) > -\Delta$ for $k = 1, 2$ with $a_1^{\text{start}} = 100$ and $a_2^{\text{start}} = 102$. Figure 3 shows a plot of the objection function for a solution path starting at a given point and ending at the global minimum point. It can be seen from this plot that, because IMIGO is a non-Hessian-based algorithm, the objective function values it finds for points on the solution paths first rise above the objective function value at the start point before falling toward zero at a minimum point: $a_1^{\text{end}} = 100.13$ and $a_2^{\text{end}} = 102.04$. Figure 4 also shows another special feature of OASIS Pathfinder-Its unique ability to find the global-minimum solution when the start point is outside of the BOA of the global-minimum point.

An Eight-Variable Problem

As an application of IMIGO to a real accelerator project, we have used IMIGO to optimize the two bunch compressor setting for the LCLS at SLAC [3]. The objective function is formed to set the final electron rms bunch length,

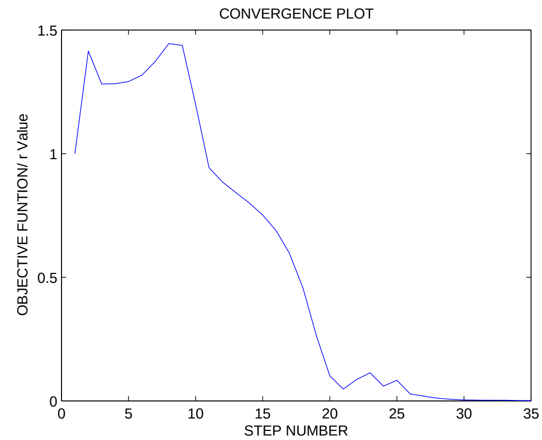

Figure 3: Variation of objective function on solution paths to the global minimum.

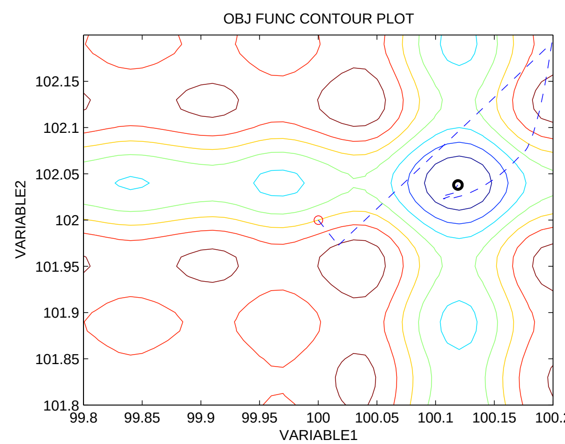

Figure 4: Plot of a solution path that ends at the global minimum point.

the final centroid energy, and the final energy chirp along the electron bunch; and simultaneously minimize the rms bunch length fluctuation and the final energy chirp fluctuation. The objective function is a function of eight variables: the LINAC acceleration phase and total acceleration voltage in the three linac section, and the R_{56} of the two bunch compressors. As an example, the objective function as a function of the two LINAC section (L1 and L2) phases is shown in Fig. 5. However, as described about, even though this is an eight-dimensional optimization, IMIGO in fact does the search in two-dimension, namely in the (s, p) 2-dimensional space. The IMIGO was able to find minimum solution. Yet, the model in Ref. [3] does not include the space charge effect or the coherent synchrotron radiation (CSR) effect. Going into more detailed study with space charge and CSR effects are time consuming with numerical simulation, and is not easy to get a closed analytical expression for the objective function. Hence in the following we will discuss the possibility of using IMIGO to do a model independent beam optimization. The machine is in

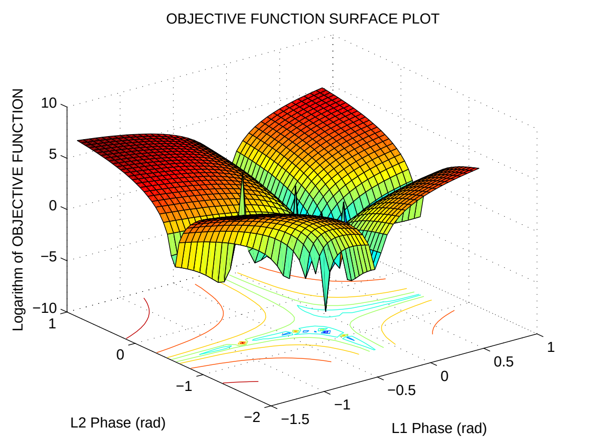

Figure 5: Objective function varies with L1 and L2 phases.

fact the 'model', and the measurement data is the output of the 'model'. Directly working with the machine measurement data, one can also optimize the system by running IMIGO to tell how to set the machine parameters. Hence this is the model-independent approach.

MODEL-INDEPENDENT BEAM OPTIMIZATION

In reality, as described above, normally the model is either too simplified compared to real situation, if an analytical expression for the objective function is needed; or the model can be time-dependent, which will need the control system to be self-adaptive or at least to have a dynamics response function. Therefore, staying with an over-simplified model, or using a static model for a dynamic system will lead to the malfunction of the accelerator system. A model-independent analysis is therefore needed.

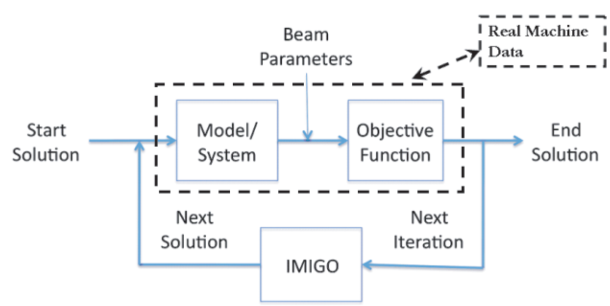

Figure 6: Illustration of the flowchart for model-independent beam optimization.

Shown in Fig. 6, we show the difference between a model-independent optimization and a conventional model-based optimization. In the model-based case, the model is first validated and then the objective function is either constructed analytically or numerically. While in the model-independent case, the real machine measurement data are used to form the objective function.

As emphasized above, since IMIGO is a non-Hessian algorithm, the real measurement data even though will error, can provide objective function accurate enough for IMIGO to find the global minimum. As the follows, we describe a Gedanken experiment to do model-independent beam optimization for a space charge dominated beam. We assume that the machine will deviate from the single particle model significantly, *i.e.*, the space charge is not negligible.

Since the space charge effects are small at low beam current, the objective function predicted from the single-particle model is approximately equal to the value measured on the real accelerator. As the beam current increases, the measured value of the objective function changes. Thus, the size and shape of the BOA of the objective function are different for different beam currents. Since the global-minimum solution is at the 'bottom' of the BOA, its value is also different for different beam currents. By replacing the model-predicted objective function with the measured objective function, IMIGO can be used to find the global minimum solution for any desired value of beam current without using the model as shown in Fig. 6. In practice, the optimization process can be carried out incrementally by repeating it over many min-step current changes. As long as the global minimum solution for the previous step is within the BOA of the objective function of the next step, the global minimum solution for the desired beam current will be found.

DISCUSSION

In this paper, we introduce a new nonlinear optimization package, called IMIGO. We demonstrate the power of IMIGO by both working out a detailed challenge example normally associated with the famous Levenberg-Marquardt method. It is shown that IMIGO can find the global minimum even if the start value is out of the BOA of the global minimum. We also demonstrate that IMIGO can solve real accelerator optimization problem as in Ref. [3]. Furthermore, with IMIGO a model-independent beam optimization is possible. Additional research is being conducted to find the global optimal solution for other objective functions such as luminosity and beam lifetime in colliders, and Free Electron Laser peak power and brightness for light sources.

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