IMPLEMENTING NEW BEAM LINE ELEMENTS INTO A MOMENT METHOD BEAM DYNAMICS CODE*

T. Roggen†, H. De Gersem, B. Masschaele
KU Leuven Kulak,
Wave Propagation and Signal Processing Research Group, Kortrijk, Belgium
W. Ackermann, S. Franke, T. Weiland
TU Darmstadt, Theorie Elektromagnetischer Felder (TEMF), Darmstadt, Germany

Abstract

Developing beam dynamics simulation tools using the moment method has advantages in terms of precision and efficiency when interests lie in average or rms dimensions of the beam, projected emittances or total energy. The moment method implemented in the V-Code solves the Vlasov equation by time integration, from an initial particle distribution represented by a discrete set of characteristic moments, accounting for all acting internal and external forces along the particle's path. The moment method delivers highly accurate beam dynamics results within a very small CPU time. This article proposes, illustrates and validates a new beam line element (BLE) for a radio frequency quadrupole (RFQ) for insertion in the V-Code. The focus will be on the RFQ cell structure, the electric field distribution and the insertion of the field distribution in the moment code.

INTRODUCTION

New particle accelerator projects rely heavily upon numerical simulations ranging from proof of concept to optimisation and fine tuning of the accelerator's individual components. Beam dynamics simulations are of vast importance to guarantee compatibility between all individual accelerator components in the optimisation cascade. Moment based algorithms have a major calculation time window advantage over full particle in cell (PIC) codes, while their accuracy is superior to macro-particle tracking algorithms. The V-Code is a sixth order moment Vlasov solver which takes into account space charge effects [1, 2]. An initial particle bunch is represented in a six dimensional phase space with longitudinal and transverse dynamics. For improved accuracy not only the average bunch coordinates and momenta, but also higher order correlation parameters (up to the sixth order) are accounted for in a phase distribution function $f(\tau, \vec{r}, \vec{p})$, $\tau = ct$ being the equivalent time, c the speed of light, t the time, $\vec{r}$ the space coordinates and $\vec{p}$ the normalized momentum ($\vec{p} = \vec{P}/mc$). V-Code allows an accelerator component's electromagnetic field distribution (exerting a force $\vec{F}$ on the particles) to be represented by a multipole expansion of the field data from finite element (FE) or finite difference time domain (FDTD) simulation results. Eventually, for $\gamma = E/mc^2$ the normalized energy, with energy E and mass m, all parameters of the Vlasov equation (Eq.1) are known, enabling to use time integration to find a solution.

$$\frac{\partial f}{\partial \tau} + \frac{\partial f}{\partial \vec{r}}\frac{\vec{p}}{\gamma} + \frac{\partial f}{\partial \vec{p}}\frac{\vec{F}}{mc^2} = 0 \qquad (1)$$

This contribution proposes, illustrates and validates a new BLE for a radio frequency quadrupole (RFQ) for insertion in the V-Code. The focus will be on the RFQ cell structure, the electric field distribution and the insertion of the field distribution in the moment code.

FIELD DATA FILES

The required BLE field component to reconstruct the electromagnetic fields in V-Code is calculated using 3D field information from the BLE model in CST Studio Suite 2012. BLE-specific data extraction and post-processing guarantee smooth field transitions at borders and inside a BLE. The field component is then stored in a *.dat field data file and read by V-Code to reconstruct a 3D field distribution using multipole expansion techniques.

RADIO FREQUENCY QUADRUPOLE

A radio frequency quadrupole (RFQ) is a low-velocity, high-current accelerator component that can accelerate hadrons from protons to uranium to energies of about $2q/A$ [MeV/n] (q being the ion charge and A the ion mass in amu). It allows for velocity independent electric focussing and adiabatic bunching, resulting in compact bunches and nearly 100 % capture and transmission efficiency. RFQs come in two flavours: four vane structures and four rod structures (Figure 1),with differences in required space, RF mode sensitivity and manufacturing techniques. The operating principle is analogue though.

The four electrodes in quadrupole configuration produce RF induced electric fields for focussing of the ion beam. With a longitudinal sinusoidal modulation of the electrodes, the vertical pair shifted out of phase by π, an accelerating longitudinal field component is introduced. Since the electric field between the rods finds its origin in the potential difference between the rods, and not in the RF electric field, a quasistatic approach is valid as long as the electrode gap is small compared to the RF wavelength [3]. The scalar potential function $U(r, \theta, z, t)$, as proposed in [4], is given in (2). r, θ, z are the polar coordinates, time t,

---

[*] This research is funded by grant "KUL 3E100118" "Electromagnetic Field Simulation for Future Particle Accelerators", project FP7-Euratom No. 269565 and the Belgian Nuclear Research Centre (SCK·CEN).

[†] toon.roggen@kuleuven-kulak.be

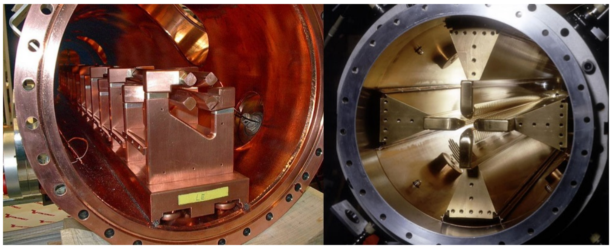

Figure 1: Left: four rod 217 MHz p$^+$ RFQ [5], right: four vane 101 MHz Pb^{27+} RFQ [6]

$\omega/2\pi$ the RF frequency, ϕ the initial phase shift, p, s and n indices with $n+s=2p+1$, I_{2s} the modified Bessel function, A_p and A_{ns} pole tip geometry dependent multipole coefficients in function of $m(z), a, k$ and V_0, with $m(z)$ the modulation parameter, minimal radius a, electrode potential difference V_0, $k=2\pi/2L$ with $2L=\beta_s\lambda$ the electrode modulation period, β_s the synchronous particle velocity and wave length λ (Figure 2).

$$U(r,\theta,z,t)=\sin(\omega t+\phi)\Big[\sum_{p=0}^{\infty} A_{0,2p+1} r^{2(2p+1)} \cos(2(2p+1)\theta) + \sum_{n=1}^{\infty}\sum_{s=0}^{\infty} A_{n,s} I_{2s}(knr)\cos(2s\theta)\cos(knz)\Big] \tag{2}$$

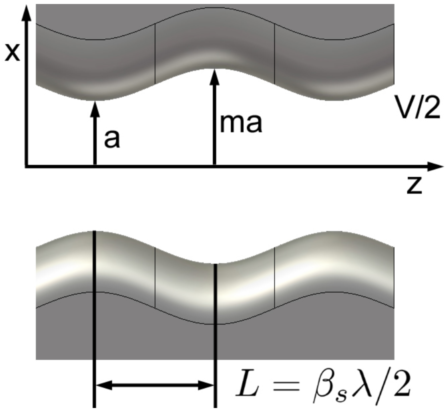

Figure 2: RFQ electrode tip geometry

TWO-TERM POTENTIAL FUNCTION

As a first approximation one may consider a hyperbolic rod geometry and only determine the quadrupole term $A_{0,1}$ of the first summation and the monopole term A_{10} of the second summation by solving the potential function on the rod tip for maximum and minimum vane modulation ($U=V_0/2$) at $z=0$. The electric fields are found by $\vec{E}=-\nabla U$.

$$A_{0,1}=\frac{V_0}{2a^2}\frac{I_0(ka)+I_0(kma)}{m^2 I_0(ka)+I_0(kma)}=\frac{V_0}{2a^2}X \tag{3}$$

$$A_{1,0}=\frac{V_0}{2}\frac{m^2-1}{m^2 I_0(ka)+I_0(kma)}=\frac{V_0}{2}B \tag{4}$$

$$E_r=\sin(\omega t+\phi)\frac{V_0}{2}\Big[X\frac{2r}{a^2}\cos(2\theta)+BkI_1(kr)\cos(kz)\Big] \tag{5}$$

$$E_\theta=\sin(\omega t+\phi)\frac{V_0}{2}\Big[-X\frac{r}{a^2}2\sin(2\theta)\Big] \tag{6}$$

$$E_z=\sin(\omega t+\phi)\frac{V_0}{2}\Big[-BkI_0(kr)\sin(kz)\Big] \tag{7}$$

IDENTIFICATION

A multipole expansion of $\vec{E}(r,\theta,z,t)$, replacing the Bessel function $I_n(kr)$ with its Taylor expansion, is integrated in V-Code to calculate a 3D field distribution for the volume occupied by the particle bunch. As a result of the quasistatic approach, time dependency of $\vec{E}$ can be understood as the product of $\vec{E}_{max}$ with $\sin(\omega t+\phi)$. $V_0, X, B, a,$ and k being constant for each accelerating cell, each electric field component of the 3D field can be calculated providing:

- E_θ: $\frac{dE_\theta}{dr}$ at $\pi/4$
- E_z: on axis E_z
- E_r: using E_θ, E_z and $\nabla.\vec{E}=0$

All data is normalised.

HIGHER ORDER TERM POTENTIAL FUNCTION

Limitations of the two-term hyperbolic rod geometry are related to manufacturing, RF breakdown and RF power consumption in a region far away from the beam axis. Therefore higher order potential functions are mandatory to allow manufacturing of RFQs not suffering these limitations. Higher order terms can be defined identifying higher order coefficients according to additional collocation points. Rods being milled with a half circular transversal cross section, give rise to a four-term potential function with terms $A_{0,1}$, $A_{0,3}$, $A_{1,0}$ and $A_{1,2}$ to be defined using the collocation points in table 1.

Table 1: Four-term Potential Collocation Points

	U	z	r	θ
1	$V_0/2$	0	a	0
2	$-V_0/2$	0	ma	$pi/2$
3	$V_0/2$	0	$\sqrt{(a+\rho)^2+\rho^2}$	$\cos^{-1}(\frac{a+\rho}{r})$
4	$-V_0/2$	0	$\sqrt{(ma+\rho)^2+\rho^2}$	$\cos^{-1}(\frac{ma+\rho}{r})$

The resulting electric field using the four-term potential function is again calculated using $\vec{E}=-\nabla U$.

$$U(r,\theta,z,t)=\sin(\omega t+\phi)\Big[A_{0,1}r^2\cos(2\theta) + A_{0,3}r^6\cos(6\theta)+A_{1,0}I_0(kr)\cos(kz) + A_{1,2}I_4(kr)\cos(kz)\cos(4\theta)\Big] \tag{8}$$

ISBN 978-3-95450-116-8

$$E_r = \sin(\omega t + \phi)\Big[2A_{0,1} r\cos(2\theta) + 6A_{0,3} r^5 \cos(6\theta) + kA_{1,0} I_1(kr)\cos(kz) + kA_{1,2} I_5(kr)\cos(kz)\cos(4\theta)\Big] \quad (9)$$

$$E_\theta = \sin(\omega t + \phi)\Big[-2A_{0,1} r\sin(2\theta) - 6A_{0,3} r^5 \sin(6\theta) - 4A_{1,2} I_4(kr)\cos(kz)\sin(4\theta)\Big] \quad (10)$$

$$E_z = \sin(\omega t + \phi)\Big[-kA_{1,0} I_0(kr)\sin(kz) - kA_{1,2} I_4(kr)\sin(kz)\cos(4\theta)\Big] \quad (11)$$

Implementation in V-Code of the four-term potential is foreseen for the near future.

3D FIELD MAPS OF RFQS

Typical RFQ lengths vary from 1 to several meters, which makes accurate 3D field map calculations of complete RFQs not recommendable. Additionally, a cell specific modulation parameter excludes the use of longitudinal symmetry conditions. As an alternative the minimum number of accelerating cells to be simulated to extract a stable 3D field in the middle cell n of interest was determined, independent of the cell length L. Modeling cells $n-6$ to $n+6$, one reaches a field accuracy of 10^{-4} %, equivalent to 1 V/m. RFQ beam dynamics are performed in a cell-per-cell cascade, enabling field parameter extraction from the numerical model using a denser mesh and convenient passing on of additional RFQ geometry parameters such as the aperture, modulation parameters and cell length.

BEAM DYNAMICS COMPARISON

To verify proper implementation of BLE and field file generation method, a beam dynamics comparison is set up between the V-Code and the CST particle tracker, and this for every BLE. Figure 3 shows good agreement for a Danfysik QSL040 quadrupole between V-Code and CST. A mesh of 20 million cells with symmetry settings in three dimensions was used to track 21000 computational particles representing individual electrons with an energy of 10 MeV (energy spread 0 %). The beam envelope is defined as 3σ, with σ the variance of the Gaussian bunch in the horizontal (x) or vertical (y) direction. The initial electron bunch has a transverse radius of 1 mm (3σ) and no transverse momentum. Both the horizontal and vertical beam envelopes of the bunch are plotted in function of the longitudinal propagation z of the particle in the BLE. The electron bunch enters the focussing quadrupole after a 166 mm drift tube. At 434 mm the bunch enters a second drift tube of 166 mm. The percental difference of the beam envelope size is less than 1 %, indicating good agreement between both particle trackers and confirming a proper implementation.

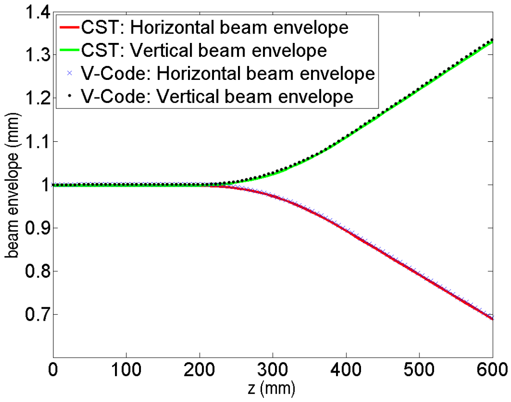

Figure 3: Beam envelope comparison between CST and V-Code for the Danfysik QSL040 quadrupole.

CONCLUSIONS

The RFQ as a new BLE of V-Code is implemented following a quasistatic approach. In a first phase the two-term potential function is incorporated, in a second phase the model will be replaced with the four-term potential function. The RFQ consists of a set of cell-by-cell components allowing accurate field parameters and convenient passing on of additional RFQ geometry parameters.

REFERENCES

[1] A. Novokhatski and T. Weiland, "Self-Consistent Model for the Beams in Accelerators", ICAP'98, Montery, September 1998, F-We21,

[2] S. Franke, W. Ackermann, T. Weiland, "A Fast and Universal Vlasov Solver for Beam Dynamics Simulations in 3D", ICAP'09, San Francisco, September 2009, `TH1IODN01`, pp. 208-211,

[3] T.P. Wangler, "Principles of RF Linear Accelerators", John Wiley & Sons Inc., NY, pp 225-257, ISBN: `978-0471168140`

[4] I.M. Kapchinskii, V.A. Teplyakov, "Linear ion accelerator with spatially homogenious strong focussing", Prib Tekh Eksp, vol. 119, No. 2, pp. 19-22, March-April 1970

[5] ntg, 222b4e3b29, Available at: `http://www.ntg.de/-typo3temp/pics/222b4e3b29.jpg` [Accessed: August 14 2012]

[6] CERN, Das Innere des RFQ, Available at: `http://www.lhc-facts.ch/img/rfq/rfq%20innenansicht.jpg` [Accessed: August 14 2012].

ISBN 978-3-95450-116-8