PARTICLE TRACKING IN ELECTROSTATIC FIELDS WITH ENERGY CONSERVATION

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Abstract

The key idea of the research is to consider spin dynamics in electrostatic fields. Due to the fact, that spin rotation frequency explicitly depends on velocity of the particle and its kinetic energy is changed in electrostatic fields it is important to use some technique that provides both conservation energy and simplicity condition. An appropriate mathematical model is described and the results of numerical calculation are shown. In conclusion, fringe fields influence is examined and compared with case of ideal fields.

INTRODUCTION

In the article particle dynamics is considered in 8-dimensional space. A state of dynamic system is described as $(x,\, x',\, y,\, y',\, S_x,\, S_y,\, S_s,\, t)$ vector, where x, x' and y, y' are transverse and vertical displacement and velocity respectively; $S_x,\, S_y,\, S_s$ are components of spin vector in curvilinear coordinate system (see Fig. 1); t is time variable. Note, that a state vector depends on arc length s, which is chosen as an independent variable.

The article consist of three parts. Firstly, mathematical models of the particle and spin dynamics are discussed. In the second part the numerical step-by-step integration approach is presented. And the last part is numerical experiment of fringe fields modeling, where the energy conservation is especially important condition.

MODEL DESCRIPTION

This section is devoted to the mathematical models of particle motion and spin dynamics. Both trajectory and spin equations are presented in generalized form along the design orbit. In case of straight orbit equations are similar to description in Cartesian coordinates. Equations along the arc of a circle are presented without derivation.

Particle dynamics in electrostatic fields is described by the Newton-Lorenz equation

$$\frac{dp}{dt} = qE, \tag{1}$$

where p is the momentum, q is the charged of the particle, E is the electric field.

For spin description BMT equation is used [1]

$$\begin{gathered}\frac{dS}{dt} = \omega \times S, \gamma = \frac{1}{\sqrt{1-v^2/c^2}},\\ \omega = \frac{Q}{m_0^2c^2}\frac{1}{\gamma}\left(G + \frac{1}{1+\gamma}\right) p \times E,\end{gathered} \tag{2}$$

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where $G = (g-2)/2$, g is the anomalous spin factor, γ is the Lorentz factor.

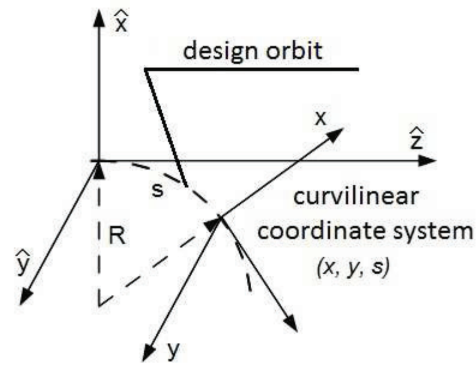

Figure 1: Curvilinear coordinate system.

Trajectory Equations

Derivation of the trajectory equations that describe the orbital motion uses generalized coordinates. The design orbit is chosen in accordance to symmetry of field distribution. For example, in quadrupole lenses it is a straight line, in cylindrical or spherical deflectors it is arc of a circle. Along the arc length the equations are following

$$\begin{aligned} x'' + (1-\frac{v^2}{c^2})^{1/2}\frac{HG}{v}x' - (1+\frac{x}{R})\frac{1}{R} &= \\ = \frac{QH}{m_0 v}&(1-\frac{v^2}{c^2})^{1/2}HE_x/v,\\ y'' + (1-\frac{v^2}{c^2})^{1/2}\frac{HG}{v}y' = \frac{QH}{m_0 v}&(1-\frac{v^2}{c^2})^{1/2}HE_y/v,\end{aligned} \tag{3}$$

where H, G is functions of variable x, x', y, y', R, R is a radius of curvature of the design orbit.

Spin Dynamics

The BMT equation in case of arc design orbit is presented as

$$\begin{gathered} S'_x = S_s/R + \frac{Q}{m_0c^2}\left(G+\frac{1}{1+\gamma}\right)\big((h_sE_x - x'E_s)S_s - \\ -(x'E_y - y'E_x)S_y\big),\\ S'_y = \frac{Q}{m_0c^2}\left(G+\frac{1}{1+\gamma}\right)\big((x'E_y - y'E_x)S_x - \\ -(y'E_s - h_sE_y)S_s\big),\end{gathered} \tag{4}$$

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$$S_s' = -S_x/R + \frac{Q}{m_0c^2}\left(G + \frac{1}{1+\gamma}\right)((y'E_s - h_sE_y)S_y - \\ - (h_sE_x - x'E_s)S_x).$$

Kinetic Energy Conservation

Note that right-hand sides of the Equations (4) and (3) contain velocity magnitude v. For the energy conservation satisfying we can calculate it as

$$v = c\sqrt{1 - \left((1 - v_0^2/c^2)^{-1/2} - \frac{Q}{m_0c^2}\Phi\right)^{-2}}, \qquad (5)$$

where v_0 is the velocity out of electric field, $\Phi = \Phi(x, y, s)$ is the potential.

NUMERICAL INTEGRATION

In ideal fields the Equation (5) provides the energy conservation itself. But due to the errors in numerical methods and artificial fields it is not enough. The next mathematical model allows to control the velocity and correct the particle motion.

The Equations (4) and (3) can be written as

$$\frac{d}{ds}X = F(s, X), \\ \frac{d}{ds}v_0 = 0, \qquad (6)$$

where $X = (x, x', y, y', S_x, S_y, S_s)$. Derivative of velocity is equal to zero, that indicates to the energy conservation.

This allows as to use classical step-by-step integration methods to solve this system. Article [2] provide both symplectic Runge-Kutta integration schemes, and the algorithm for it derivation up to the 12 order. For the current research a symplectic 2-stage Runge-Kutta scheme of 4 order was implemented as a basic approach. Moreover the set of methods was also implemented (symplectic Euler scheme, symplectic average point, etc.)

Table 1: 2-stage 4-order Implicit Runge-Kutta Scheme

$b_1 + \tilde{c}_1$	$b_1/2$	$b_1/2 + \tilde{c}_1$
$b_1 - \tilde{c}_1$	$b_1/2 - \tilde{c}_1$	$b_1/2$

$$b_1 = 1/2,\ 2b_1\tilde{c}_1{}^2 = 1/12$$

According to this scheme (Table 1), the solution of the Equations (6) can be presented in iterative form

$$\mathbf{X}_{n+1} = \mathbf{X}_n + h\sum_{j=1}^{2} b_j\mathbf{F}(s + hc_j, \mathbf{X}^{(i)}), \\ \mathbf{X}^{(i)} = \mathbf{X}_n + h\sum_{j=1}^{2} a_{ij}\mathbf{F}(s + hc_j, \mathbf{X}^{(i)}).$$

This integration method provide a symplectic solution by choosing the corresponding coefficients a_{ij}, b_j, c_j. Note that this symplectic scheme imposes the condition of constant integration step h. The simple software environment for designing and modeling was developed (see Fig. 2) and all these parameters can be set.

Moreover this scheme requires to solve of implicit equations and appropriate numerical methods can be used. In this research standard Newton method was used. So the precision of the numerical approach depends on step value and error tolerance for solution of implicit equations.

The scheme does not satisfy to energy conservation at all. For this purpose velocity derivative was added to the model. Though the velocity magnitude can be calculate directly by the x, x', y, y', s, t values, but the equation for it derivation let us introduce an addition control parameter.

For example, this control important to estimate influence of fringe fields. It is difficult task to introduce fringe fields that satisfy to Laplas equation. So the simple model for fringe fields is chosen, where such correction is necessary.

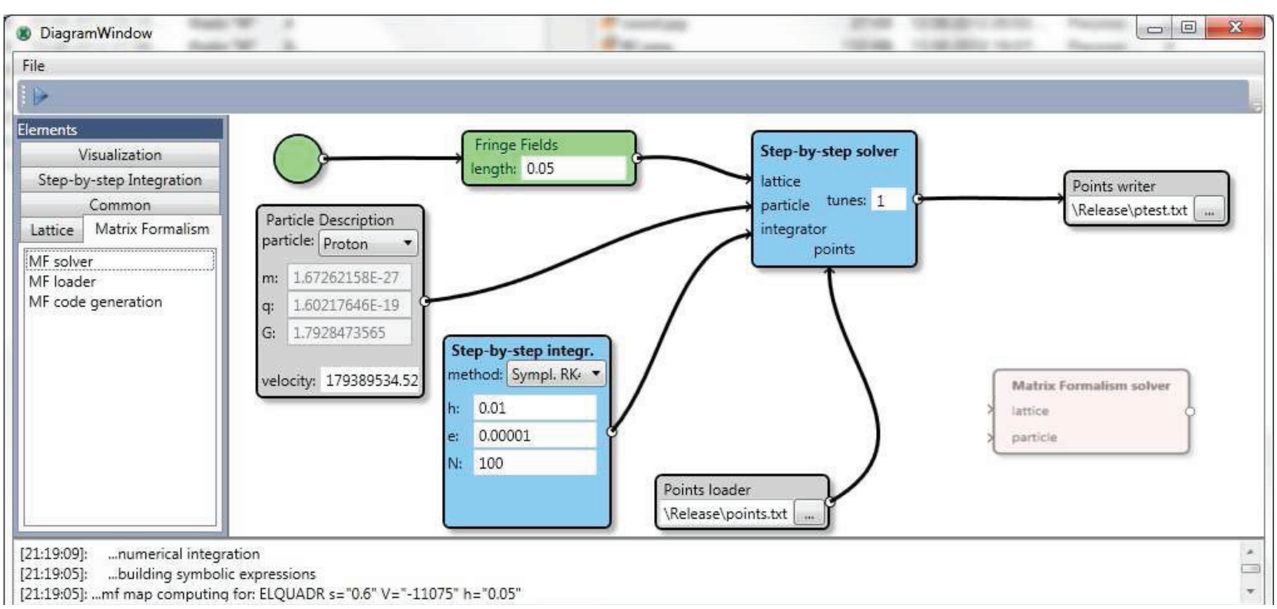

Figure 2: GUI for particle tracking.

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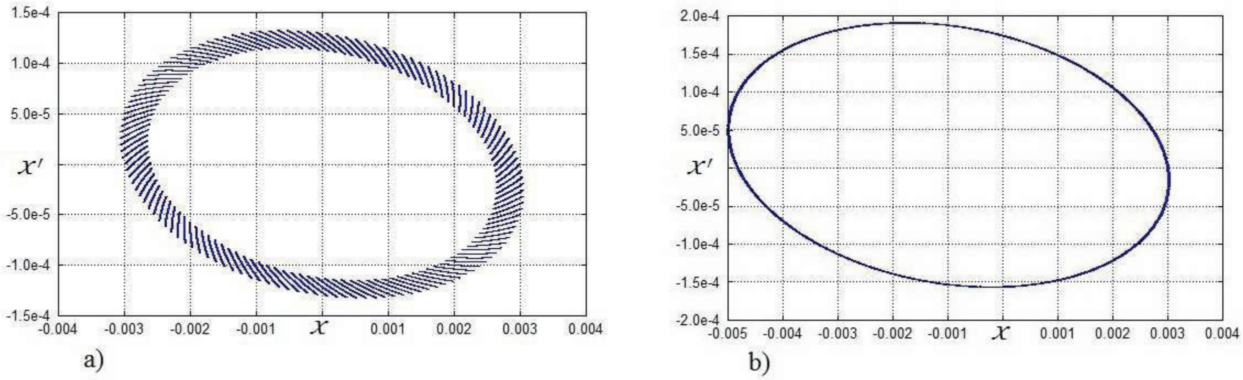

Figure 3: Fringe fields influence. a) Tracking without energy conservation. b) Energy conservation correction.

SIMULATION

The approach was used to examining particle dynamics in electrostatic storage ring. The lattice consist of hyperbolic quadrupoles, cylindrical deflectors and drifts.

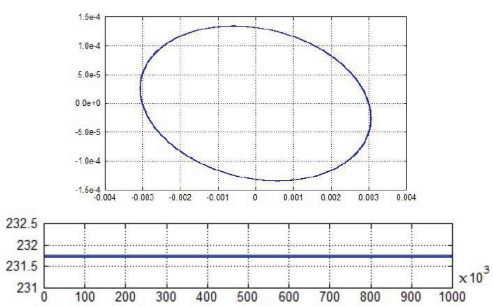

Figure 4: Symplectic phase plane and energy conservation.

Figure (4) shows the result of simulation of a particle. Ellips in $x - x'$ phase plane indicates to the symplectic mapping. The kinetic energy is also conserved.

Fringe Fields Influence

To introduce fringe fields simple model was used, when the potential in fringe fields change as

$$\tilde{\Phi}(x, y, s) = k(s) \cdot \Phi(x, y, s), \qquad (7)$$

where $\Phi(x, y, s)$ is the potential in ideal fields, $k(s)$ is a function of changes of the potential along the design orbit. For example, $k(s)$ can described by linear function or Enge function.

Although the electric fields which described by the Equation (7) do not satisfy to the physic laws, it can be use for modeling. Simulation in Cartesian coordinates indicates to the excessive acceleration of particles (see Fig. 3, a). This allows us to measure it and introduce to the Equation (6) a correction $v_0' = const$.In such way nonphysical acceleration is compensated by artificial chang of the mathematical model.

In Fig. (3, b) phase plane with fringe fields influence is shown. Center of the ellipse is shifted relative to the origin and symplectic condition is satisfied.

CONCLUSION

The approach described above is devoted to the high precession step-by-step integration. On the other hand there are exist mapping algorithms for beam dynamic simulation. Such methods allows to build map corresponded to the dynamic system. In the paper [3] matrix formalism for solving of ODE is presented. The same mathematical model is used for beam dynamics description.

Step-by-step integration allows us to obtain the correction of velocity derivation in fringe fields. This correction can be introduce to the mapping approach. So tracking algorithms aare used for precision estimation and examining of the particle dynamics, and mapping approaches are used for modeling of long-term evolution of beam.

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