

# TOOLS FOR ANALYSIS AND IMPROVEMENT OF LINAC OPTICS DESIGN FOR HIGH BRIGHTNESS ELECTRON BEAMS

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## Abstract

The optics design of single pass high brightness electron linacs usually aims at the preservation of the transverse emittance. Collective effects mainly impose constraints to the optics design such as at the low-beta interaction points in colliders and magnetic compressors in free electron lasers (FELs). Other constraints are from the trajectory correction scheme, performance of diagnostics, collimation systems and physical space limitations. Strong focusing is typically prescribed for all the aforementioned cases, although it may hamper the main goal of emittance preservation through the excitation of optical aberrations. Strong focusing also potentially leads, through focusing errors, to large beam optics mismatch. Based on these sometimes conflicting requirements, we have developed tools for the analysis and improvement of electron linac optics. They are based on the Elegant code [1] and allow the user to identify: i) local sources of phase space distortions and emittance dilution, ii) lattice areas particularly sensitive to focusing errors, iii) poor trajectory steering. The analysis does not require massive particle tracking since it deals with the single particle motion in the normalized phase space.

## ELEGANT ON-LINE

In order to use Elegant as an on-line machine model [2], we have developed a set of utilities for interfacing the simulator with the accelerator. We have exploited the Elegant capability to read and write a set of element parameters from a file in SDDS format via the `load_parameters` and `write_parameters` commands. The SDDS file is filled in with the actual parameters of the running accelerator by means of a dedicated utility which examines the SDDS files and maps the requested parameters to the appropriate control system variables by means of database tables. The requested variables are then acquired from the control system and scaled if necessary, e.g. converting beam position monitors (BPM) readings from millimeter to meter). The reverse path is followed for setting parameters: Elegant writes the new values to the SDDS files, another dedicated utility reads the new parameters from the SDDS file, maps them to control variables and set the values via standard control system calls. Since Elegant works with normalized machine physics quantities, we have developed a set of specialized control system servers, called Tango devices [3], which perform the conversion from engineering quantities (e.g. current) to machine physics quantities (e.g. quadrupole strength) by means of calibration tables. Such tables are directly handled by the Tango server.

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## OPTICS MATCHING

The goal of optics matching is to impose the design values of the Twiss functions to the electron beam. This is typically done with at least four quadrupole magnets. For beam energies lower than  $\sim 100$  MeV, the beam optics cannot be predicted with sufficient accuracy in Elegant since the particles move in the space-charge dominated regime. For this reason, it is very important to measure the beam optics at the *end* of the injector, where the electron spatial distribution is frozen to any practical purpose. The matching loop is illustrated in Fig. 1.

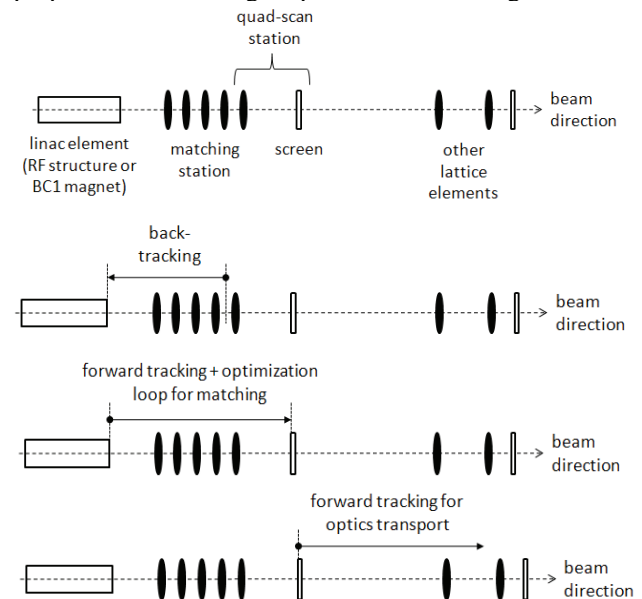


Figure 1: Illustration of the optics matching loop [2]. From top to bottom: i) the beam Twiss parameters are measured with the quadrupole scan technique [4] at the entrance of the last quadrupole magnet of the matching station; ii) the present machine configuration is read by Elegant and the measured Twiss parameters are back-tracked to a point upstream of the matching station; iii) starting from the present machine configuration, Elegant starts optimizing the quadrupole strengths to match the beam Twiss parameters to the design values; iv) once the matching has been performed, the beam is transported through the downstream lattice.

The matching loop has been coded in MATLAB [5] and a Graphical User Interface (GUI) is available as a standard control room application. A theoretical betatron mismatch parameter is defined as follows [6]:

$$\xi = \frac{1}{2} (\bar{\beta}\gamma - 2\bar{\alpha}\alpha + \bar{\gamma}\beta) \quad (1)$$

where  $\bar{\beta}, \bar{\alpha}, \bar{\gamma}$  are the design Twiss parameters, for each transverse plane.  $\beta, \alpha, \gamma$  are the Twiss parameters computed by Elegant at the end of the optimization process. By definition,  $\xi \geq 1$ ; the closer this value is to 1, the closer the Elegant solution is to the design optics. A graphical output is displayed that shows the betatron functions as they are computed in the back-tracking mode and in the forward tracking mode. All intermediate data and results of the matching procedure are exchanged via SDDS format files and can be plotted with standard SDDS based tools.

### OPTICS SENSITIVITY

The capability of the perturbed magnetic focusing to generate optics mismatch is here investigated. The focusing error may be due to a large energy, a mean energy mismatch or a gradient calibration error. We initially refer to [7] and assume that: i) the radiofrequency focusing can be neglected compared to the magnetic focusing; ii) the beam optics mismatch at the end of the line is dominated by the focusing errors along the lattice, *not* by the optics mismatch at the injection point. Unlike [7], however, we assume an identical relative focusing error  $k\delta$  for all the quadrupoles, so that the final mismatch parameter is computed in each plane as follows:

$$m_{\delta}^f \equiv 1 + \frac{1}{2} \left[ \left( \sum_{i=1}^N k_i \beta_i L_i \delta_i \cos(2\Delta\mu_i) \right)^2 + \left( \sum_{i=1}^N k_i \beta_i L_i \delta_i \sin(2\Delta\mu_i) \right)^2 \right] \quad (2)$$

The sum in Eq.2 is over  $N$  quadrupoles,  $k$  is the nominal quadrupole strength,  $L$  is the quadrupole magnetic length,  $\delta$  is the fractional strength error and  $\Delta\mu$  is the betatron phase advance. We now consider Eq.2 for one local source of mismatch at the time (one quadrupole), while all the other magnets have the nominal strength ( $\delta = 0$ ). We thus obtain the optics mismatch induced by each individual quadrupole magnet:

$$m_{\delta}^q - 1 \equiv \frac{1}{2} (k\beta L \delta)_q^2 \equiv \zeta_q \quad (3)$$

In the case of perturbations that may lead to (small) emittance dilution, it can be shown through the beam matrix formalism that Eq.3 also describes the relative emittance growth induced by the error kick. We define  $\zeta_q$  as the optics sensitivity to focusing error. If we assume that the emittance dilution or mismatch parameter at the end of the line  $m_{\delta}^f = 1 + C$ , is the result of many identical, uncorrelated and small perturbations  $\zeta_q = \zeta \ll 1$ , the maximum sensitivity allowed to each quadrupole (tolerance) is of the order of  $m_{\delta}^q - 1 = \zeta_q \approx C/\sqrt{N}$ . For typical numbers  $C = 5\%$  and  $N = 100$ , the lattice regions characterized by  $\zeta_q \geq 0.5\%$  (see Fig. 2) should be reviewed either by imposing tighter tolerances on the quadrupole setting or by adopting a weaker focusing.

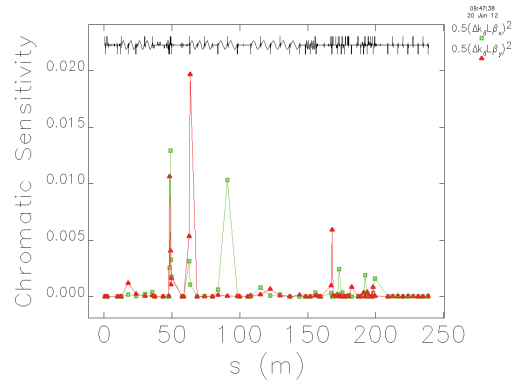


Figure 2: Optics sensitivity to the energy spread in the FERMI FEL [8].

### CHROMATICITY

The chromatic properties of the design optics are evaluated by varying the beam mean energy by  $\delta$  (*i.e.*,  $\pm 1.0\%$ ) at the injection point, in small steps. The Twiss function,  $f(s)$ , is computed at each step along the entire lattice (see Fig. 3). A linear fit provides the coefficient  $df/d\delta$ , at any location along the lattice. The amount of local mismatch can be estimated as  $df(s)/d\delta$  times  $\delta$ , this being the energy deviation relative to the initial energy.

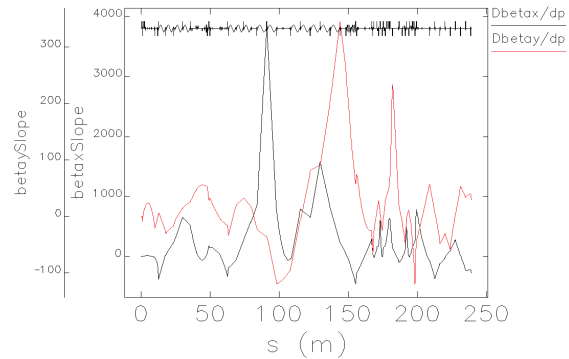


Figure 3:  $d\beta/d\delta$  for the FERMI baseline optics.

### COURANT-SNYDER INVARIANT

To evaluate the impact of chromatic and geometric aberrations on the distortion of the transverse particle distribution, we mapped the Courant-Snyder (C-S) invariant of the bunch centroid along the lattice. To do this, transport matrices up to the second order in the particle coordinates are used in Elegant. To move the centroid on a nonzero amplitude trajectory we excite a betatron oscillation through an initial angular kick (typically as large as 1 mrad or so) and compute the invariant for the design optics (see Figure 4). We experienced that tracking only  $10^3$  particles or so is enough to obtain reliable indications of the phase space distortion.

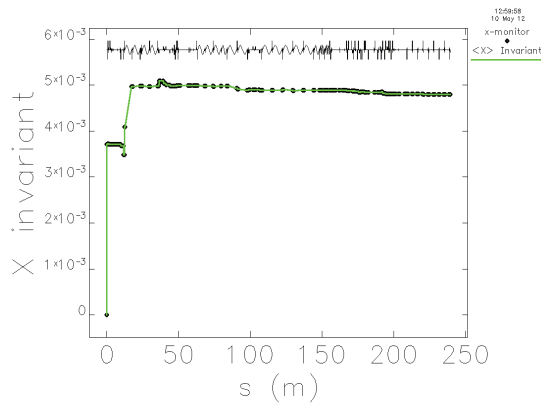


Figure 4: The horizontal C-S invariant of the bunch centroid along the FERMI FEL.

### H-FUNCTION

The design H-function,  $H = \beta\eta^2 + 2\alpha\eta\eta' + \gamma\eta'^2$  plays a role for the collective effects. On the one hand, it should be made as small as possible to minimize the emittance growth induced by the emission of coherent synchrotron radiation (CSR) [9]:

$$\gamma\epsilon = \gamma\epsilon_0 \sqrt{1 + \frac{H}{\epsilon_0} \sigma_\delta^2} \quad (4)$$

A computational approach to the dependence of the CSR instability on the Twiss functions can be found in [10]. However, Eq.4 is considerably cheaper in terms of CPU time and the H-function can be used as a constraint in the Elegant optimization loop. On the other hand, a large H function determines a large path length difference of particles with different energies:

$$\sqrt{\langle \Delta l^2 \rangle} = \sqrt{H\epsilon}, \quad (5)$$

where the integral is computed over a generic dispersive path and the final RMS path length is averaged over the beam particle ensemble (see Fig.5).

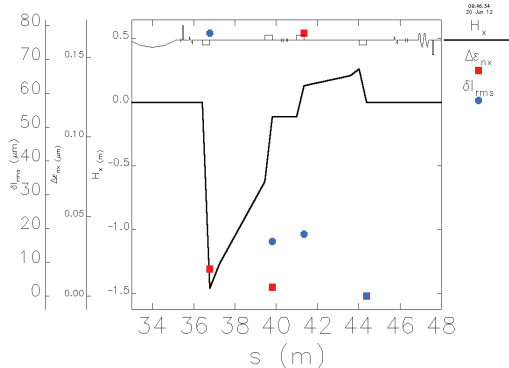


Figure 5: The design H-function in the first magnetic compressor of FERMI, together with the emittance growth (Eq. 4) and the RMS phase mixing (Eq. 5) for each dipole magnet.

The spread of the path length translates into the particles longitudinal phase mixing that washes out energy and density modulations such as those due to the microbunching instability [11]. Because of the conflicting

requirements on the design of H, an optics design should include from the very beginning a proper tuning of the Twiss functions in the dispersive regions traversed by a short, high charge beam or characterized by a high gain of the microbunching instability. This tuning would allow the manipulation of the H function and eventually reach a compromise between the aforementioned effects.

### FLOQUET SPACE

The amount and the phase advance separation of steering magnets and BPMs determine the beam trajectory sampling and control. At least 4 BPMs per betatron period ensure an accurate reconstruction of the beam trajectory. The trajectory control is then very efficient if the steering magnets are close to local maxima of the betatron function and if each consecutive steerer and BPM are separated by  $\pi/2$  phase advance. To analyze the efficiency of the beam steering we plot in Fig.6 the bunch centroid position normalized to the local  $\sqrt{\beta}$  versus the betatron phase advance.

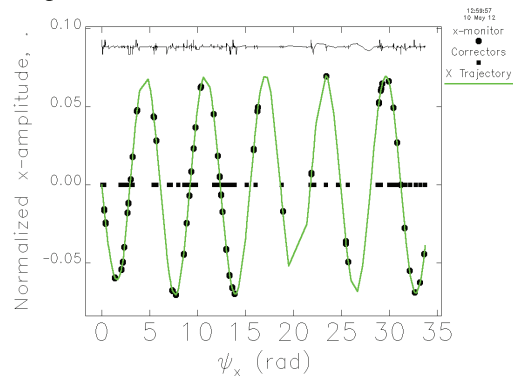


Figure 6: The horizontal bunch centroid trajectory in the Floquet coordinates along the FERMI FEL. BPMs (on the curve) and steering magnets (on the central straight line) are superimposed to the trajectory.

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