IMPLEMENTATION OF BENCHMARKS FOR PRECISION TRACKING IN STORAGE RINGS*

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Abstract

Currently, there is a much interest in making precision measurements utilizing storage rings [1]. For example, the Muon g-2 experiment at Fermilab [2] and the Electric Dipole Moment (EDM) program of the JEDI Collaboration [3] require measurements of sub-ppm precision. Of particular importance is the ability to treat all nonlinear effects arising from detailed field distributions, including fringe fields.

In this paper, we track existing and proposed storage ring lattices using multiple codes and compare the results. The storage rings in question are the COSY Jülich ring at Forschungzentrum Jülich [4] and the High Energy Storage Ring (HESR) [5] to be constructed as part of FAIR at GSI. We present new results of tracking of the HESR lattice with COSY INFINITY [6], including full fringe fields, and the resulting dynamic aperture is estimated.

Finally, a set of benchmarks has been proposed [7] to test the accuracy and speed of codes used for tracking orbital and spin dynamics in storage rings. We present a comparison of the results of such benchmarks with the codes COSY INFINITY, an eight-order Runge-Kutta Integrator, MADX, MAD8, and ZGOUBI tracking.

COMPARATIVE TRACKING

In a previous paper [8] we performed a comparative analysis of repetitive tracking for the storage ring Cosy Jülich with both COSY INFINITY and ZGOUBI [9], both of which can treat fringe fields. Despite the fact that ZGOUBI tracking is nonsymplectic, qualitative agreement between the tracking pictures was found. COSY INFINITY was an order of magnitude faster while producing comparable results.

HESR

Continuing work begun with COSY Jülich, we turned our attention to a future storage ring, the High Energy Storage Ring (HESR), part of the Facility for Antiproton and Ion Research (FAIR) at GSI in Darmstadt, Germany [5]. Following a similar line of analysis as for COSY Jülich, we perform comparative tracking experiments between COSY INFINITY, MADX [10] and MAD8 [11]. We present first results below.

General Layout of HESR

HESR is a racetrack storage ring similar to COSY Jülich, but of larger scale. The circumference of the ring is 575 m consisting of two 123m straight sections connecting two 157 m arcs. The momentum of the antiproton beam ranges from 1.5 GeV/c to 15 GeV/c. The number of elements is much larger than that of COSY Jülich; HESR consists of 44 bending elements and 84 focusing elements. This requires precise agreement amongst the transfer matrices of the various codes.

First Order Transfer Matrices

After implementing the given lattice in COSY INFINITY, the first check is to compare the first order transfer matrices with those of MADX and MAD8. Here are the matrix from MAD v8.51 Win32 and COSY.

$$\begin{bmatrix} -1.0456753 & -1.7327760 \\ 0.18033015 & -0.65749692 \end{bmatrix}$$

$$\begin{bmatrix} -1.046011 & -1.730853 \\ .1803385 & -.6576032 \end{bmatrix}$$

On the other hand, computing the transfer matrix with MAD-X 5.02.00 (64 bit, Linux), gives the following result:

$$\begin{bmatrix} 1.2664075 & 3.6824325 \\ -.077346854 & .56472775 \end{bmatrix}$$

The apparent difference is due to the fact that MAD-X uses kick approximation for the particle optical elements. Representing each of the elements by slicing into 128 sub-elements, we obtain good agreement with the matrices of MAD8 and COSY:

$$\begin{bmatrix} -1.0456753 & -1.7327760 \\ .18033015 & -.65749692 \end{bmatrix}$$

A further detailed study of the influence of the number of slices shows that the discrepancy follows an inverse square law (Figure 1), as expected from a kick approximation. In the

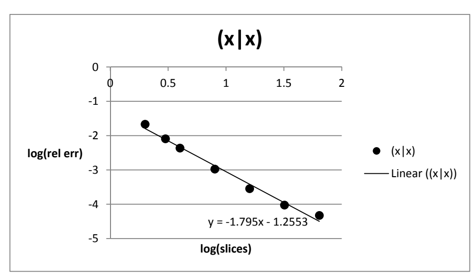

Figure 1: Relative matrix element discrepancy vs. number of slices.

following we therefore choose a large number of slices (128) for these simulations since CPU time was not the primary concern. The remaining discrepancy is likely attributable to slightly different values for natural constants and similar matters, the effect of which is amplified here due to large numerical cancellation effects arising particularly due to large drifts in the system.

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Having good baseline agreement in the transfer matrix, we can proceed to tracking with confidence. Here is the result of turning on fringe fields in COSY INFINITY:

$$\begin{bmatrix} -1.061066 & -1.462613 \\ .1675836 & -.7114451 \end{bmatrix}$$

Contrary to the case for COSY Jülich, in which the fringe fields rendered the lattice unstable, for HESR the lattice remains stable. We can thus proceed to tracking without re-fitting of the lattice. Figure 2 shows a tracking picture with fringe fields turned off. The orbits are at 1 cm increments, extending out to 6 cm, which is actually larger than the physical aperture of the beam elements. When we turn on fringe fields and repeat the tracking (Figure 3), we clearly see beam loss at radii greater than 4 cm. Confirmation of this is given by examining closely the region from 3–5 cm, Figure 4. These effects purely due to fringe fields are not apparent in tracking by MAD8 or MAD-X.

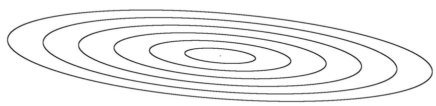

Figure 2: COSY INFINITY HESR tracking with no fringe fields.

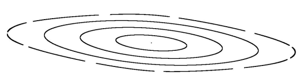

Figure 3: COSY INFINITY HESR tracking with full fringe fields.

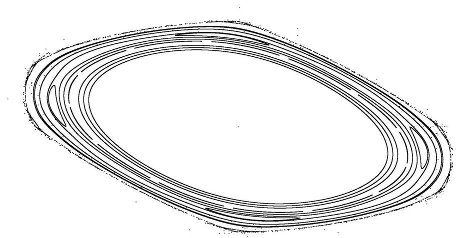

Figure 4: COSY INFINITY HESR tracking beam loss region.

BENCHMARKING

Recently a set of benchmarks for tracking software based on high precision analytical estimates for spin tracking has been proposed [7]. Our group at Michigan State University is actively engaged in implementing these benchmark tests in a variety of codes. Here we present some early results of this effort.

A straightforward test, easy to implement in any code, is off-reference motion through a dipole. The motion of a cosine-like ray through such a constant field is diagrammed in Figure 5. The solid circle is the reference orbit of radius R, and the location of the test particle is represented by the dashed circle and angle θ. The dashed circle is offset from the reference orbit by the quantity x_0. The 16 radial lines in the circle of the reference orbit denote the locations where the coordinates of the test particles will be plotted. A tracking code will calculate the position of the displaced particle by the particle optical coordinates x and a which can be derived from geometric considerations:

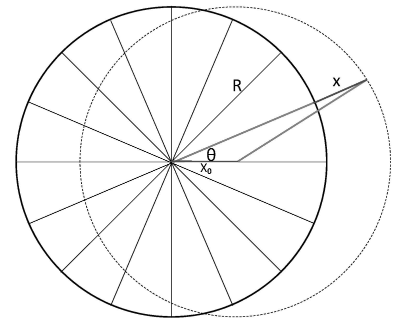

Figure 5: (x,a) coordinate of an off-axis cosine-like trajectory in a dipole.

$$\begin{aligned} x &= x_0 \cos\theta + \sqrt{(R^2 - x_0^2 \sin^2\theta} - R \\ x' &= x_0 \sin\theta - x_0^2 \sin\theta\cos\theta / \sqrt{R^2 - x_0^2 \sin^2\theta} \end{aligned}$$

Here x' is the derivative with respect to polar angle, which can be used to obtain the respective particle optical coordinate for slope or relative momentum.

The main appeal of this test lies in the fact that regardless of the launching position x_0, the orbit will return to exactly this point after one revolution through the lattice. Any deviation from this shows a limitation of the code, either due to numerics, or more importantly, through approximations in the dynamics.

The first test is a comparison of a Runge-Kutta integrator written in-house at MSU with COSY INFINITY at 19th order. Figure 6 shows the results of Runge-Kutta tracking (10000 turns @ $\approx$ 300 seconds CPU time). This is in excellent agreement with the analytic solution all the way out to $.9R$. Figure 7 is a plot of the same system tracked to 19th order by COSY INFINITY for 10000 turns. What is apparent is the loss of fidelity at approximately $.9R$. This is to be expected, as COSY INFINITY tracking is based on a Taylor

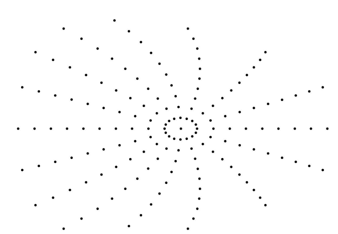

Figure 6: (x,a) coordinate tracking by Runge-Kutta of a cosine-like trajectory through a dipole at 7-70% radius (courtesy Eremey Valetov, MSU).

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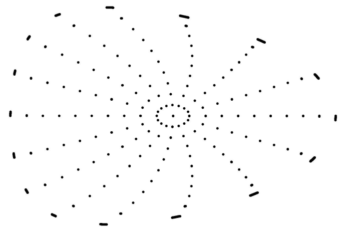

Figure 7: (x,a) coordinate tracking by COSY INFINITY of a cosine-like trajectory in a dipole from 7–70% radius (courtesy Eremey Valetov, MSU).

expansion about the reference orbit, and there will always be some radius beyond which a Taylor expansion will lose its accuracy. On the other hand, a Runge-Kutta integrator suffers no such limitation. We therefore expect the accuracy of an RK integrator to be independent of R. Nonetheless, even at $.9R$, agreement of COSY INFINITY with the analytic solution is on the order of 10^{-5}. Additionally, COSY INFINITY is an order of magnitude faster; this plot took only $\approx 30s$.

Similar benchmarking was performed using ZGOUBI (Figure 8). This tracking picture is substantially different from that of COSY INFINITY; it is worthwhile to investigate why. To see whether particles indeed return to their starting position, Figure 9 shows the result of eight single turns through the dipole. The initial locations of the particles are along the horizontal. The successive turns form branches which spiral away from the horizontal. The increasing curvature of the particle configuration causes the phase space orbits to fill out and become solid lines. Another interesting observation is the behavior of the first 5 particles from the center. These particles do not share in the spiraling motion; thus we suspect a software-related discrepancy between the handling of particles at $x < 5$ cm and those for which $x > 5$ cm. Figure 10 shows the same tracking test for MAD8. This tracking uses thick elements, which yield qualitatively similar orbits to ZGOUBI. The nonlinear features apparent in the ZGOUBI tracking picture of Figure 8 are not apparent in the MAD8 tracking.

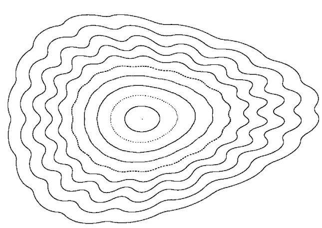

Figure 8: (x,a) ZGOUBI tracking of a cosine-like trajectory through a dipole at 7-70% radius

Finally, we repeat the same benchmark test with MADX, which does not use thick elements for tracking. Figure 11 shows the tracking picture. The asymmetrical nature of the orbits is lacking, which is due to approximations in the Hamiltonian MADX makes to allow the use of kicks. Again the orbits are filled, as we found in ZGOUBI. If we study the first few turns and plot the location of the particles after each turn, we observe a similar "spiraling" as present in ZGOUBI.

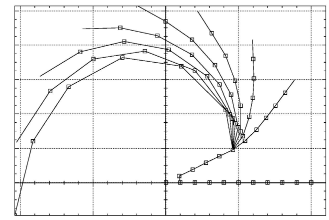

Figure 9: ZGOUBI locations of particles after first 8 turns. Each branch is a successive turn.

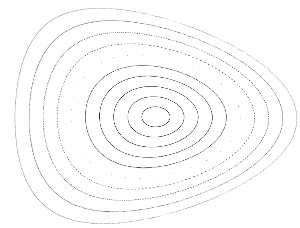

Figure 10: (x,a) MAD8 tracking of a cosine-like trajectory through a dipole at 7-70% radius

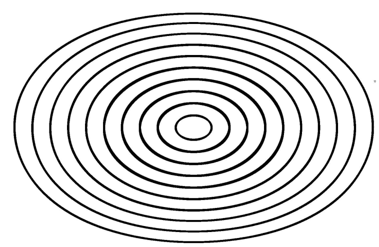

Figure 11: (x,a) MADX tracking of a cosine-like trajectory through a dipole at 7-70% radius

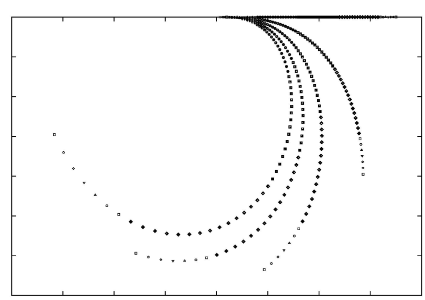

Figure 12: MADX locations of particles after first 4 turns. Each branch is a successive turn.

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