ESTABLISHING A CONSISTENT BASIS FOR BEAM PHYSICS, ACCELERATOR MAGNET DESIGN, AND MAGNETIC MEASUREMENTS

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Abstract

In mathematics, C^3 denotes 3-times continuous differentiable functions. We use this as shorthand for establishing a consistent basis for beam physics simulations and machine requirements, magnet design and manufacture, and magnetic measurements.

Magnetic field measurements serve for the quality assurance of the magnet production, both at an early stage in the production process and after the reception of the magnets in the laboratory. Measurements are also performed for validating numerical field simulation (FEM) tools and magnet design techniques. In order to study the performance of the accelerator and to allocate a budget for magnet misalignment in the tunnel, the beam physicists require the field maps and multipole field errors of the magnet system, either from validated numerical models or from magnetic measurements. Field measurements are also done for on-line monitoring of the magnet behavior and thus providing direct feedback to the accelerator control room for adjusting the magnet current cycles, and to tune the high-frequency accelerator cavities.

This paper describes the approaches and techniques in magnetic field measurements, discusses their limitations and attempts to make them consistent with the requirements from beam physics and magnet design.

INTRODUCTION

For the LHC magnet system [1], it was relatively easy to establish C^3. The aperture of the superconducting magnets is round and the beam is pencil shaped with respect to the dimensions of the magnet aperture. The LHC beam is ultra-relativistic and stiff, that is, the main dipole magnets exert only a small kick to the particle trajectory, even though their magnetic lengths is 14.3 meters. Moreover, the betatron amplitude changes only very slightly within the arc magnets (the beam is in good approximation paraxial with the magnetic axis of the magnets), so that the magnets can be represented as a set of thin lenses in the beam tracking codes.

A classical method for describing the magnetic flux density in accelerator magnets is to develop the eigenfunctions of the 2D Laplace equation (expressed in cylindrical coordinates) into the Fourier components of the magnetic flux density, calculated or measured at a given reference radius; usually 2/3 of the aperture, or 17 mm in case of the LHC [2]. This method works well for long, straight magnets and is perfectly in line with magnetic field measurements using rotating search coils. The Fourier coefficients of the field solution, known in the accelerator community as field harmonics, serve for the integrated field reconstruction and are the direct interface between the magnet "owners" and the beam physicists. Extensive beam tracking studies during the LHC design and construction phase, gave a good knowledge of the effect of multipole field errors and the role (and limitations) of a correction system to compensate their effect on the particle beam. The magnetic measurements were therefore based on rotating search coils assembled in long shafts and driven by a motor unit [3]. An angular encoder and a digital integrator [4] were used to re-parametrize the measured voltage (over time) into flux increments measured as a function of the coil's angular position. In this way the requirements on the uniform motion of the drive unit could be relaxed.

The design of the magnet system for the LHC could be based on the experience with the Tevatron and HERA machines [5]. In particular, the dynamic effects in superconducting magnets were intensively studied and brought under tight control during the magnet construction period. A strategy could therefore be developed to monitor the production process with field measurements at ambient temperature. Using a well established cold-warm relationship, the magnetic measurements at cryogenic conditions could be reduced to about 10% of the magnets [6]. Recently, a large number of magnetic measurement requests has arisen from new accelerator projects, such as SESAME, HIE-ISOLDE, ELENA, and Linac4, at CERN, as well as the magnet system for MedAustron and the FAIR superFRS [7] magnets that require more than just the measurement of the integrated bending strength. Limited resources and a narrow time slots impose optimized procedures and instrumentation. Standardization of measurement equipment becomes essential in order to increase efficiency in terms of installation time and workflow. Large efforts have been undertaken to optimize CERN measurement resources while keeping a stringent measurement quality. This resulted in a flexible control and acquisition software, a standard drive system, rotating-coil systems with standard assembly of tangential search coils, and multipurpose measurement benches. Moreover, an increasing number of measurement have been based on the stretched wire techniques, as these techniques are very universal and do not require magnet-specific probes and instrumentation.

However, the most important savings on material and resources are made on a different level: it is important to establish a consistent basis for beam physics, the magnet system, and the magnetic measurements in order limit the development of dedicated instrumentation and reduce the measurement time, while providing exactly the feedback needed. In this respect, the new projects are very challenging

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as magnets with small bending radii and large aspect ratios of the air gap will be built.

A well established measurement method consists of mapping the field with a 3D Hall probe, which is inaccurate compared to the rotating search coils and very time consuming. For fast-cycling magnets the field is often measured only on the mid-plane using a stationary fluxmeter which is expensive to fabricate in printed-circuit board (PCB) technology. While fluxmeter measurements yield a relatively fast feedback on the magnet-to-magnet reproducibility, the measurements are of limited relevance for beam simulations because they require ramping the magnet and deliver only the integrated field strength. However, in machines that are operated as storage rings, the highest precision of the magnet measurements is required at steady state, i.e., injection and/or high-energy plateau of the beam.

When the magnet is operated in steady state condition, the only meaningful measurement with a fluxmeter consists of moving the entire fluxmeter from a field-free region into a final position within the magnet, while integrating the induced voltage. This yields the total flux intercepted by the conductor, but requires a large support at the extremities of the magnet.

Components of an approach towards C^3 include the expansion of the field solution into bipolar (for curved magnets) and plane elliptic coordinates (for magnets with narrow gaps), the local description of field errors in the magnet ends, and the post-processing of measurement data using numerical field computation methods. For example, 3D hall probes can be calibrated by imposing that the measured magnetic flux density must be divergence free. The raw data (fluxes or fields) can be used as Dirichlet and Neumann data of the boundary element technique, and the positioning errors of the mapper can be corrected on boundary element mesh [8].

MAGNETIC MEASUREMENT TECHNIQUES

When magnets must be measured in fast-ramping conditions a flux meter at fixed position delivers a voltage signal in a pulsed field. Modern integrators with large bandwidth and time resolution connected to such a coil can give the full $B(t)$ curve and measure saturation effects of the iron yoke. Hysteresis and eddy current effects can be separated by measurements at different ramp rates. One precaution must be taken for the remanent field of the magnet, i.e., the field at zero current value. Possible measures are to use a bipolar power supply and perform symmetric sweeps from negative to positive maximum current. Other methods include demagnetizing the magnet with a bipolar power supply or measure the remanent field using flip coils or Hall probes.

Search coils are the standard method for measuring multipole field errors in accelerator magnets [9]. However, the measurement of magnets with small apertures of less than 20 mm in diameter is a challenging task [10]. Consider a rotating coil inside a bore of radius r, then the maximum number of coil turns that can be wound with a conductor of given diameter is proportional to r^2 and, consequently, the coil sensitivity factor scales with r^{-2}. The impact of mechanical tolerances and calibration errors increases drastically and the measurement uncertainty can be expected to be one to two orders of magnitude higher than in typical synchrotron magnets with an aperture radius of 50 to 100 mm. Moreover, experience has shown that the coils are affected by manufacturing tolerances resulting in a longitudinal non-uniformity of the width and radius of about 0.6%. This resolution must be compared to the beam dynamics requirements for the LHC, where both the absolute integrated magnet length as well as the field contribution from higher harmonics must be measured and controlled on a level of 10^{-5} of the main field, which is 8.3 T in the straight section.

When measuring a magnet that is shorter than the search coil, such variations imply that standard calibration of the average geometry becomes problematic and a in-situ calibration must be performed.

A method to circumvent these problems is based on displaced or vibrating wires [11–13]. A conducting wire is displaced inside the magnetic field by precision displacement stages at which the two end-points of the wire are fixed. Copper-Beryllium (CuBe) wires, 0.1 mm thick, are commonly used because of their high tensile strength and low martensitic contaminations. The return wire must be routed through a field-free region. The integrated voltage at the connection terminals of the wire is a measure for the flux linked with the surface that is traced out by the wire. This so-called single stretched wire method is commonly used to measure the magnetic field strength and magnetic axis. It can also be used in a vibrating mode when it is excited with an alternating current at resonance frequencies. The latter operation mode is very sensitive to determine the magnetic axis in solenoidal magnets [14].

The stretched wire method has recently been extended to measure multipole field errors in accelerator magnets by exciting the wire with an alternating current well below the natural resonance frequency. In this way we make use of the linear relationship between wire oscillation amplitude, integrated field, and current amplitude. To distinguish the wire methods we speak of vibrating (resonant) and oscillating (non-resonant) wires although these terms are often used synonymously [14, 15].

A classical method for measuring magnetic fields is the mapping with hall probes. The challenges here are the calibration of the probes, which are very sensitive to temperature variations. The accuracy of the measurement strongly depends on the mechanical stability of the displacement stage, as often a long shaft is required to enter into the aperture of the magnet. Typical specifications for such systems require ranges of about one meter in the transversal directions and up to five meters in longitudinal direction with an accuracy on the order of 0.1 mm. Mapping of fields is thus very time consuming even if the data acquisition can be done "on the fly".

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ISSUES IN ESTABLISHING C^3

To establish a consistent basis for accelerator physics, magnet design, and magnetic measurements for a large verity of accelerator magnets is no easy task. All parties involved must be aware about the risk of over-specification. Computational beam dynamics, based on tracking codes for simulating the effect of higher-order field harmonics on the stability of the stored particle beam, results in specifications of the field homogeneity on the order of one unit in 10000. This yields tight (and costly) tolerances on the coil manufacture, yoke-material, punching of laminations, and assembly of the magnets. This, in turn, leads to even higher demands on measurement probe calibration and data acquisition techniques for magnetic measurements. A classification of consistency issues can be done according to the main user of magnetic measurements. Measurements driven by beam physics requirements must deal with the appropriate definition of the good-field region and the field homogeneity. This will depend on the specific application of a magnetic element. For example, a curved, normal-conducting dipole magnet may be used as a bending magnet in a synchrotron, as a spectrometer, or as a bending magnet in a beam transfer line. Will it be sufficient to measure the longitudinally integrated field errors or will local field measurement be needed in the magnet extremities? We must also ask what are the systems variables of the beam tracking codes (multipole field errors, field maps, Taylor approximations, integrated strength values) and how can they be derived from the magnetic measurement raw data.

A large number of software packages have been developed to study the dynamics of charged particles in accelerators. For many applications it is sufficient to track an ensemble of single particles through the magnetic elements of the machine. Such codes include MADX [22], SixTrack [17], and PTC [18], among others. Self interactions between the particles in the beam (space charge) can cause emittance growth and beam loss. Special purpose codes to simulate space charge effects in linear and circular accelerators are pyORBIT [21] and Synergia [23], for example. In contrast to single particle codes, the numerical accuracy in space charge codes is limited by finite particle and grid effects.

MADX consists of different routines for (single) particle tracking, beam optics (twiss parameters and beam envelopes) as well as symplectic integrators. For tracking, thin and thick lens descriptions of dipoles and quadrupoles are used as elementary magnetic elements. PTC has been added to MADX in order to enable thick-element tracking with arbitrary order [19]. In general, MADX is used for the layout of the accelerator considering the integrated dipole field of the bending magnets and, eventually the feed-down [2] from misaligned quadrupole magnets, so defining the design orbit. With respect to this, the effect of all other elements, including higher order multipoles in the magnets, is computed. For the study of the beam envelope and long term stability in synchrotrons, as well as light sources employing wigglers and undulators, the higher order imperfections (or harmonics) in the magnetic field play an essential role. While the pattern of pure dipole and quadrupole fields establish a strictly linear particle oscillation, unwanted higher harmonics that are present in any real magnetic field create distortions on the particle dynamics and can lead to growing oscillation amplitudes and finally particle loss. Consequently, tolerance limits for these higher field harmonics must be established. They will depend on the beam size, the required storage time and the particle species (electrons, protons heavy ions). Tracking codes are used for this purpose, where the effect of a higher multipole field on the particle dynamics is described explicitly by the Lorentz-force acting on the particle.

The description of the magnet elements must be symplectic, that is, the integral operators acting on these elements must preserve the particle density in the phase space of all possible values of position and momentum variables. This is automatically the case as long as the magnetic field in a magnet can be described by a series of thin lenses with an equivalent kick. Only a few codes, such as PTC [18] and COSY INFINITY [20] can accurately handle a displacement of the design orbit in the magnet element itself. The field and its derivatives are required around the design orbit. The order is defined by the specific beam physics requirements, for example by the storage time or the maximum emittance of the beam. As long as the design orbit is not deviating too much from the central orbit, and the multipole field expansion is valid, the calculation of these fields is straightforward. This is, however, not the case in the magnet extremities, where highly non-linear field distributions are encountered and the multipole coefficients do not constitute a complete orthogonal function set for the field solution. This gives rise to so-called pseudo-multipoles which result from the magnet field variation in axial direction [16].

Especially in the case of solenoids a considerable influence on the dynamics of the particles is observed as they lead to coupling between the horizontal and vertical planes, which usually will have to be compensated either by compensating solenoids or an arrangement of skew quadrupoles. Measured or calculated fields on a grid are only appropriate to first and second order in the grid spacing because of the discretization errors. Often, the field description relies on measurements on the mid plane of the magnetic element. Out-of-plane field expansions therefore depend strongly on the accuracy of the field derivatives in the mid plane. This is not a problem for computed but indeed for measured fields. In this case it is advantageous to measure the field on a closed volumetric domain and to represent the field by integrals based on the Kirchhoff theorem. For this reason, the magnetic measurement section at CERN is developing a transversal-field transducers based on a short measurement coil, a non-magnetic piezoelectric motor and a longitudinal displacement mechanism. Because of its resemblance, this transducer is nicknamed as the "toy train".

When measurements are driven by modeling capabilities of the employed magnet design codes (usually based on structural and electro-magnetic finite-element FEM packages) it is important to check the necessary model reduction

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(omitting details like welding seams) and measure the effect of shims, chamfers, welds and other material parameters [2]. An iterative process is often required to distinguish manufacturing or modeling errors from measurement artifacts and to decide on corrective actions for the magnet design. It is here where a strong link between the magnet designer and the magnetic measurement engineer can yield improvements on both the measurement techniques and the modeling capabilities of the design codes. If the consistence is established, beam simulations can often be based on computed rather than measured data. It will then also be possible to simulate the effects of manufacturing tolerances and in this way arrive at a technical specification for the production of the magnets. When measurements are driven by (series) magnet manufacturing, the aim is to check the magnet-to-magnet reproducibility and to arrive at acceptance criteria for the delivered magnets. Often inverse field computations are needed in order to relate the measured field errors to manufacturing errors. During the series production of the magnets, a reduced set of magnetic measurements will be sufficient as long as there are no sudden changes in the built quality of the delivered magnets.

Accelerator Operation

When magnetic measurements are driven by the accelerator operation, the demands are much more challenging. In some cases the magnets exceed the model capabilities of the FEM codes. For some legacy equipment incomplete information on the design makes it impossible to set up such a precise model. Different machine cycles may result in a coupling of dynamic and hysteretic effects which are difficult to compute. Magnetic measurements employing search coils as well as NMR and ferromagnetic resonance probes are therefore required for on-line monitoring of the magnetic field in reference magnets that are excited at the same current as the magnets in the machine [24].

SOLENOID MAGNETIC MEASUREMENTS

We will now take the example of a solenoid to explain the above-mentioned issues in establishing C^3. Solenoids are used to focus charged particle beams in the low energy section of accelerators. Particles moving exactly along the magnetic axis of the solenoids do not experience any force, while off-axis particles are azimuthally accelerated by the radial field components in the fringe field region of the magnet. The resulting helical particle motion in the longitudinal field region of the magnet yields a focusing effect due to the radial Lorentz force. Solenoid magnets are not compatible with the standard measurement equipment optimized for accelerator magnets. Solenoids are therefore routinely tested with general-purpose instruments such as 3D Hall probe mappers. Field mapping makes no sense as long as the magnet is not perfectly aligned with the stages, because an apparent asymmetry results in the fringe field regions, which is due to misalignment and not due to magnet manufacturing errors or intrinsic limitations in the magnet design.

We have therefore developed magnetic measurement techniques for solenoids based on vibrating/oscillating wires as well as field transducers based on solenoidal search coils. Figure 1 defines the reference frames for the solenoid magnet and the stages of the stretched-wire system. The figure also indicates the position of the drive unit when rotating field probes are employed. Because of the radial symmetry of the field, we can assume, without lack of generality, that the two wire stages (Stages A and B) are perfectly aligned in the longitudinal direction such that the wire moves parallel to the z-axis.

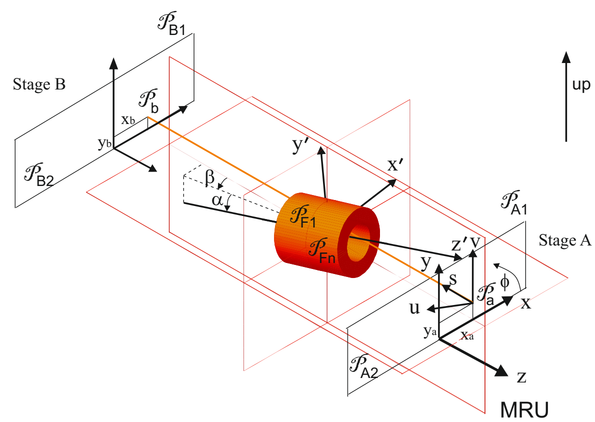

Figure 1: Coordinate systems of the magnet and the stages of the stretched-wire system. Definition of pitch (tilt) α and yaw (swing) β angles. Position of the motor unit (MRU) when rotating search coils are used.

A method for the alignment of solenoids, based on the oscillation wire method, has been developed at CERN [15]. Exciting the wire at its second resonance yields a wire motion in phase with the Lorentz forces due to the fringe fields on both sides of the magnetic center. When the wire is misaligned with respect to the magnet center, the fringe fields in the horizontal plane of the magnet give rise to Lorentz forces as shown schematically in Fig. 2.

The wire displacements are measured by means of phototransistors arranged in orthogonal directions at a longitudinal position z_0 close to Stage A [15]. When the wire is centered, the two fringe fields have the same direction and therefore the wire oscillation amplitude takes its minimum. But there may still be a swing and tilt misalignment with respect to the wire axis. In this case the fringe field components point into the same direction on either side of the magnet center. To align the stages with the magnetic axis, we must therefore switch to the first resonance frequency so that the wire motion will be in phase with the transversal component of the field. The magnet alignment consists of solving iteratively a minimization problem with two objectives (the oscillation amplitudes d_x and d_y in the horizontal and vertical planes) and two design variables (the x and y positions of the wire fixation points at both stages). The minimization method

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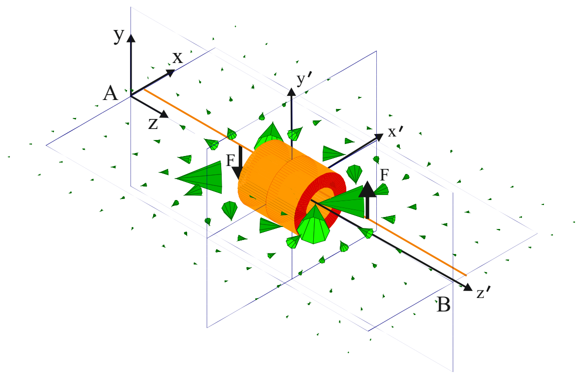

Figure 2: Stray field on the horizontal plane and direction of the Lorentz force acting on the off-centered stretched wire (no tilt and swing misalignment).

by Gauss-Seidel (also known as coordinate search) is used in combination with a linear regression of the measurement data in each coordinated direction. The linear regression is motivated by the fact that for small displacements from the magnetic center the Cartesian components of the fringe fields decrease linearly in the coordinate directions. After the magnet is aligned, the field homogeneity can be mapped in the coordinate system of the wire. To this end, we switch again to the second resonance, and map out the oscillation amplitudes for different angular positions on the same radius. A small azimuthal asymmetry is present because of the layer jumps of the conductor in multilayer solenoids.

The longitudinal field profile can then be measured with an instrument based on moving search coils. In thin lens approximation, the focusing effect in a solenoid is described by the following expressions of the rotation of the beam about the z-axis φ_L (the so called Larmor angle) and the focusing strength $1/f$ [26]:

$$\varphi_L = \frac{e}{2\gamma m v_z} \int B_z \mathrm{d}z := \frac{e}{2\gamma m v_z} F_1 \qquad (1)$$

$$\frac{1}{f} = \frac{e^2}{4\gamma^2 m^2 v_z^2} \int B_z^2 \mathrm{d}z := \frac{e^2}{4\gamma^2 m^2 v_z^2} F_2 \qquad (2)$$

The integrals, occurring in these expressions, are defined as the field integrals F_1 and F_2. It turns out that these field integrals are independent of the alignment. This can be understood by the theory of the magnetic double layer [2]. This double layer gives rise to a jump in the magnetic scalar potential, that is constant all across a plane coil loop.

As proposed in [27], the instrument for measuring the first and second field integrals consists of two coaxial search coils slightly spaced apart longitudinally. The flux variation in the search coils induces a voltage across the coil terminals, which will be integrated with a digital integrator. When the search coils are connected in series, the longitudinal field component can be calculated up to first order, deviding the flux by the calibrated coil surface. The coils can also be connected in series in opposite orientation. Because of flux preservation, the integrated voltage is a measure of the radial flux between the two coils, from which the radial field component can be calculated.

CONCLUSIONS

In an ideal world, a magnet is built to perfection, without any systematic manufacturing error, and without variations in the material parameters. The numerical field computation model is able to predict all static and dynamic phenomena, and the magnetic measurements, without systemic errors from calibration or any random errors, confirm the prediction of the magnet designer. In the real world, however, an iterative process is often required to distinguish manufacturing or modeling errors from measurement artifacts and to decide on corrective actions for the magnet design. In some cases, hysteresis and eddy current effects in the magnets require on-line monitoring of the field strength for the optimization of the accelerator performance.

In an accelerator project, the field measurements are often on the critical path, because delays in the performance or interpretation of these measurement lead to a late release of the series production or a late installation of the components in the accelerator tunnel. For the LHC, magnetic measurements at ambient temperatures were performed at the magnet manufacturers premises in order to identify manufacturing errors at a very early stage. A good knowledge on the correlation between measurements at ambient and cryogenic temperatures made it possible to limit the number of time-consuming measurements after the electrical tests at CERN.

For smaller accelerator projects, such an approach is hardly feasible; either because there is a considerably shorter R&D phase or because of the low number of magnet units. In these cases, the magnetic measurements must be well chosen and limited to the methods that yield the required feedback to the magnet designers and beam physicists at minimum cost of time and resources.

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