EFFICIENT PLASMA WAKEFIELD ACCELERATION SIMULATIONS VIA KINETIC-HYDRO CODE ARCHITECT

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Abstract

Start-to-end simulations are needed for sensitivity studies and online analysis of experimental data of the Plasma Wakefield Acceleration experiment COMB at SPARC_LAB facility, Frascati (Italy). *Ad hoc* tools are needed for the plasma section modeling. Particle in cell codes are the most widely used tools for this purpose, but they suffer from the considerable amount of computational resources they require. We seek for a simple, portable, quick-to-run approach. For this purpose we introduce a time-explicit cylindrical hybrid fluid-kinetic code: *Architect*. The beam particles are treated with PIC-like kinetic approach, while the plasma wake is treated as a fluid. Since the number of computational particles used by the hybrid model is significantly reduced with respect of full PIC codes with the same number of dimensions, the time required for a simulation is reduced as well.

INTRODUCTION

The use of accelerating electromagnetic fields in the wake of a perturbation of the neutrality of a plasma channel appears as one of the most promising ways to overcome the conventional accelerators' limits [1]. In particular in the Plasma Wakefield Acceleration (PWFA) scheme high energy driving particle beams created in a conventional beam line are injected into a plasma channel, to excite wakefields used to accelerate a witness bunch injected in the proper phase of the wake [2]. The most widely used technique to study the underlying physics and to guide the experimental efforts in PWFA is the Particle in Cell method [3]. The amount of computational resources (i.e. number of cores and simulation time) required for 3D full PIC simulations of typical experimental setups of plasma acceleration is still too high to use PIC codes for systematic scans. For this reason much interest has been devoted to reduced models, to perform faster simulations. In particular the codes using the quasi-static (QSA) approximation [4–6] proved to significantly reduce the computational time required to simulate plasma acceleration when the characteristic timescales of the beam evolution are significantly greater than the characteristic timescales of the plasma evolution.

An alternative efficient technique is the hybrid kinetic-fluid approach, which treats the relativistic beams kinetically and the background plasma electrons as a fluid. The reduced number of computational particles needed in this technique drastically reduces the simulation time with respect to a PIC code of the same dimensions. This approach has been proven successful for the Laser Wakefield Acceleration scheme (e.g. INF&RNO [7]) and for the PWFA with QSA (LCODE [8]). We present the 2D time-explicit hybrid kinetic-fluid code *Architect*, developed to aid the preliminary parameter skimming for the PWFA experiments planned at at SPARC_LAB facility in Frascati, Italy [9]. The time-explicit formulation of the code allows to initialize the bunch electromagnetic fields in vacuum and to investigate the transient phase of the beam entrance in the plasma channel.

IMPLEMENTATION OF A HYBRID MODEL FOR PWFA

Architect simulates PWFA treating the relativistic electron beams as an ensemble of macroparticles as in a PIC code and the background plasma electrons as a relativistic cold fluid. The plasma ions are treated as a uniform immobile background. The current densities of the two species are projected on the grid, using PIC techniques [3] and computing the fluid current on the grid respectively. The sum of the current densities acts as a source for the evolution equations of the electromagnetic fields, i.e. Faraday's Law and Ampere-Maxwell equation. The macroparticle positions and momenta in the 6D space evolve using the updated electromagnetic fields as source terms. The plasma background fluid density and momentum on the grid evolve using the updated electromagnetic fields as source terms.

For each particle of the kinetic bunch(es) we identify a position, $\mathbf{x}_{\text{particle}}$, a velocity, $\boldsymbol{\beta}_{\text{particle}}c$ and a relativistic momentum, $\mathbf{p}_{\text{particle}} = m_e\boldsymbol{\beta}_{\text{particle}}c/\sqrt{1-|\boldsymbol{\beta}_{\text{particle}}|^2}$ (m_e is the electron mass).

Thus, the the macroparticle positions and momenta evolve following the equations of motion:

$$\begin{aligned} d_t\mathbf{x}_{\text{particle}} &= \boldsymbol{\beta}_{\text{particle}}c \\ d_t\mathbf{p}_{\text{particle}} &= q\left(\mathbf{E} + c\boldsymbol{\beta}_{\text{particle}} \times \mathbf{B}\right) \end{aligned} \quad (1)$$

where q is the electron charge, $\mathbf{E}$ is the electric field, $\mathbf{B}$ the magnetic field, c the speed of light.

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The background plasma electron density n_e and momentum $\mathbf{p}_e$ evolve according to [10]:

$$\begin{aligned} \partial_t n_e + \nabla \cdot (\boldsymbol{\beta}_e c\, n_e) &= 0 \\ \partial_t \mathbf{p}_e + c\boldsymbol{\beta}_e \cdot \nabla \mathbf{p}_e &= q(\mathbf{E} + c\boldsymbol{\beta}_e \times \mathbf{B}), \\ \boldsymbol{\beta}_e &= \frac{\mathbf{p}_e}{m_e c\sqrt{1+|\mathbf{p}_e/m_e c|^2}}. \end{aligned} \quad (2)$$

The first equation is the mass conservation equation; the second equation is the momentum conservation equation.

The electromagnetic fields, induced by both the beam particles and the fluid background, evolve according to Faraday's Law and Ampere-Maxwell's equation:

$$\begin{aligned} \nabla \times \mathbf{E} + \partial_t \mathbf{B} &= 0 \\ \nabla \times \mathbf{B} - c^{-2}\partial_t \mathbf{E} &= q\mu_0 c\,(n_e\boldsymbol{\beta}_e + n_\mathrm{b}\boldsymbol{\beta}_\mathrm{b}), \end{aligned} \quad (3)$$

where $\boldsymbol{\beta}_b c$ the velocity for the electron bunch and n_b the bunch density.

The fluid Eqs. (2) and the evolution equations for the electromagnetic fields, i.e. Eqs. (3) are integrated on a cartesian $r-z$ grid assuming cylindrical symmetry, i.e. the partial derivatives along the poloidal direction are all zero. To save memory and reduce the simulation time, the fluid and electromagnetic quantities are only integrated in a window around the beam (depicted in Fig. 1), which moves with the beam center of mass. The direction of propagation for the beam is z. Cylindrically symmetric boundary conditions are assumed on the window boundary corresponding to the z axis; free flux conditions are assumed on the other window edges. The code loop for each timestep is composed as follows; the

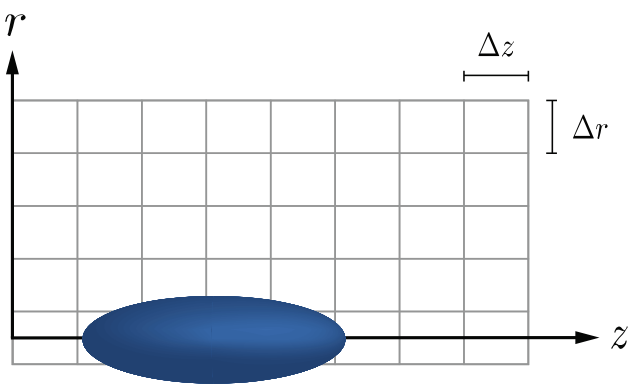

Figure 1: Architect $r-z$ grid. In blue a gaussian electron bunch, which moves in the z direction.

beam particles' position $\mathbf{x}_\mathrm{particle}$ and momentum $\mathbf{p}_\mathrm{particle}$ are used to project the beam current $\mathbf{J}_\mathrm{b} = n_\mathrm{b}\boldsymbol{\beta}_\mathrm{b}c$ by means of PIC techniques [3]. The fluid current $\mathbf{J}_\mathrm{e} = n_e\boldsymbol{\beta}_e c$ is then computed on the grid. The sum of the two currents is then used as a source term to integrate Eqs. (3) through the Finite Difference Time Domain (FDTD) method [3]. The updated fields are then used as source terms to integrate the equations of motion for every macroparticle, i.e. Eqs. (3), with a second order leap-frog with Boris rotation around the magnetic field [3]. The fields acting on each macroparticle are extrapolated from the grid through PIC techniques [3]. The updated fields are used also as source terms to integrate the fluid Equations (3). Such integration is performed through the Flux Corrected Transport (FCT) scheme [11, 12], a simplified shock-capturing scheme to catch the shock-like features of the ion bubble closing-up region. Once the fluid quantities and the macroparticle positions and momenta are updated the loop of the code timestep, summarized in in Fig. 2, is iterated. The self-consistent simulation of the beam-plasma

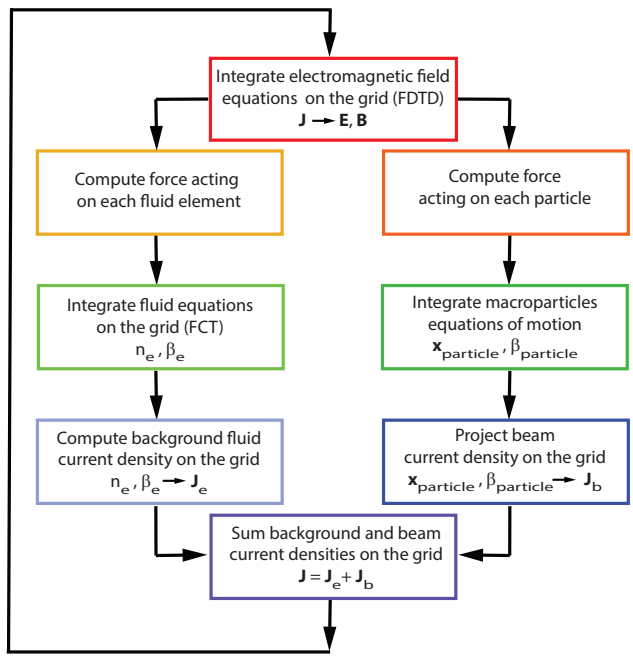

Figure 2: Architect Loop. On the left branch are the steps which involve the fluid quantities; on the right branch are the steps involving the beam macroparticles, identical to the steps of a standard PIC code.

interaction requires the correct initialization of the beam electromagnetic field in vacuum before it enters the plasma channel, which is initially at rest. Typical COMB simulations have an initial beam energy spread of 0.1%, thus all the beam particles move initially with a nearly equal velocity. The electromagnetic fields induced by the electron beam can thus be computed solving Poisson's equation in the beam rest frame and transforming the fields back in the laboratory frame. Poisson's equation is solved in Architect through finite differences, and using a Successive Over Relaxation (SOR) method to accurately solve the Poisson associated problem. Appropriate optimizations reduces the required computation time down to seconds even with very refined meshes.

RUNNING TIME

To highlight the speed of the hybrid approach, we report the time required for a 1 cm-simulation of an electron beam injected in a uniform plasma channel of density $n_0 = 10^6\,\mathrm{cm}^{-3}$. The initial beam parameters, chosen from a realistic SPARC_LAB scenario, are: charge $Q = 113\,\mathrm{pC}$, rms-lengh $\sigma_z = 50\,\mu\mathrm{m}$, rms-transverse size $\sigma_x = \sigma_y = 8\,\mu\mathrm{m}$,

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energy $E_0 = 100$ MeV, energy spread $\Delta\gamma/\gamma = 0.1\%$, transverse normalized emittance $\varepsilon_x = \varepsilon_y = 1$ mm-mrad. Such benchmark is in the weakly nonlinear regime [13, 14]. The simulation parameters are: integration timestep $\Delta t = 0.88$ fs, mesh cell size $\Delta z = 2\,\mu$m in the z direction and $\Delta z = 0.8\,\mu$m in the r direction. The colormap of the beam and background electron density after 0.1 cm of propagation are shown in Fig. 3. The simulation reported in the figure used 50000 particles to model the electron beam.

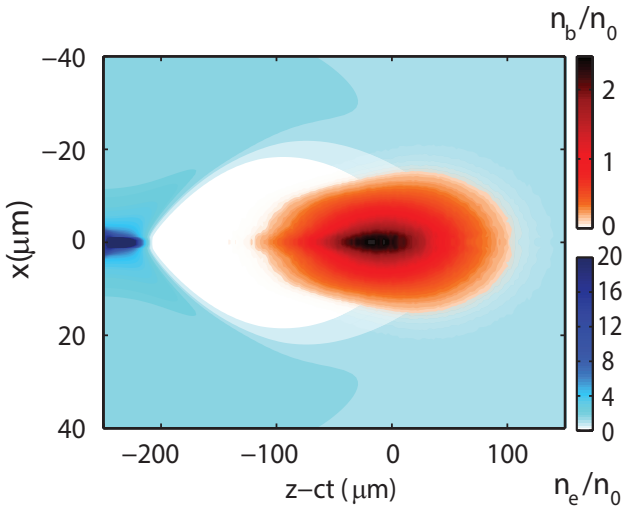

Figure 3: Beam and background electron density for the considered benchmark, after a 0.1 cm beam propagation.

The time scaling of the described benchmark, varying the number of beam particles is reported in Table 1. The reported running times are referred to a single cpu, since the code does not need parallelization. From the table we note that a preliminary run of 1 cm in the plasma channel, with the same number of beam particles of Fig. 3, needs less than one hour on a single cpu.

Table 1: Benchmark Simulation Time, Referred to an 1 cm-Run on a Single CPU

Number of beam particles	Running time (hours/cm)
30000	0.25
50000	0.38
70000	0.51
100000	0.71
200000	1.37
500000	3.34
1000000	6.63

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