NUMERICAL CALCULATION OF EIGENMODES IN PETRA 7-CELL CAVITY UNDER PRECISE CONSIDERATION OF COUPLER STRUCTURES*

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Abstract

The PETRA accelerator [1], which served originally as a pre-accelerator for the large HERA facility, is nowadays the world's best storage-ring-based X-ray radiation source. It provides the scientists experimental opportunities with X-rays radiation. The PETRA accelerator is based on a 7cell normal conducting copper cavity operating at 500 MHz and delivers a particle beam with energy up to 6 GeV. To enable proper beam dynamics simulations, it is important to determine the eigenmodes in the accelerating cavities with high precision. In the real installation, the input coupler is required to transfer the energy from the sources to the particle beam. For this reason, a complex-valued eigenmode solver can be applied to properly calculate the eigenmodes. At the Computational Electromagnetics Laboratory (TEMF) a robust parallel eigenmode solver based on complex-valued finite element analysis is available. In this paper, the realvalued and complex-valued eigenmode solver have been applied to the PETRA 7-cell cavity to determine the resonance frequency, the quality factor and the corresponding field distribution of eigenmodes.

INTRODUCTION

As shown in earlier studies [2], the higher order modes (HOMs) can cause instabilities of the particle beam in the accelerating cavities. For this reason, during the design phase of the accelerating cavities, a challenging and difficult task is to determine the eigenmodes inside the accelerating cavities with the help of proper computer simulations. So far, the most efficient commercially available eigenmode solvers are based on real-valued analysis, which is sufficient to describe the entire electromagnetic field in the lossless acceleration structure. But in reality, the PETRA 7-cell cavity requires the input coupler to transfer energy from the sources to the beam. Therefore, a complex-valued eigenmode solver can be applied to calculate the eigenmodes efficiently [3]. In this paper, we used the real-valued and complex-valued eigenmode solver to calculate the eigenmodes in the PETRA 7-cell cavity.

THEORETICAL BACKGROUND

Generally, Maxwell's equations are the mathematical foundation of the eigenmode analysis for accelerating structures. To determine the eigenmodes with high precision, the

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continuous Maxwell's formulation has been transformed to a suitable matrix equation with the help of the Finite Element Method (FEM) [4]. For the FEM discretization the tetrahedral grids and higher order curvilinear elements (see Fig. 1) have been applied to satisfy the demand for high-precision modeling of the curved geometry [5].



Figure 1: Curvilinear tetrahedral element [5].

Real-valued Eigenmode Solver

The real-valued eigenmode solver is applying the idealized boundary conditions (the perfect electric conductive material (PEC) or the perfect magnetic conductive material (PMC)) and utilizes the basis functions up to third order on curved tetrahedral elements. The determination of the eigenmodes for the accelerating structures by the real-valued eigenmode solver is performed with CST MICROWAVE STUDIO [6].

Complex-valued Eigenmode Solver

In the real installation, energy transfer can occur with the help of dedicated couplers of the accelerating cavity. To represent this fundamental behavior in the computational model, a complex-valued eigenmode solver applies the port boundary condition as one of the lossy boundary conditions, which can be applied to formulate the necessary energy exchange in the port plane [4].

For implementation of the complex-valued eigenmode solver, the geometric modeling of the accelerating structure with the tetrahedral meshing is performed with CST MICROWAVE STUDIO [6], then the essential information will be delivered to the complex-valued eigenmode solver by means of ASCII or binary file transfer. The eigenmode solver is generally computationally demanding due to the precision modeling of elliptical cavities with curved tetrahedral meshes as well as the complex-valued calculation process. To achieve a good performance on simulation time, a distributed memory architecture using MPI parallelization strategy has been utilized for the implementation [5].

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COMPUTATION OF EIGENMODES WITH REAL-VALUED EIGENMODE SOLVER

Firstly, the real-valued eigenmode solver has been applied to the unperturbed PETRA 7-cell cavity (see Fig. 2) to determine the characteristic values (resonance frequency and shunt impedance R/Q) for all modes in the 1st monopole passband. The electric boundary condition (PEC) is applied on the entire surface. In addition, the magnetic boundary condition (PMC) is used to realize the symmetry planes to reduce the computational costs.



Figure 2: The PETRA 7-cell cavity (500 MHz) with beam tubes.



Figure 3: Shunt impedance (R/Q) versus frequency for the monopole modes in the 1st monopole passband for different discretizations. The squares denote the values obtained from the MAFIA calculations. The MAFIA calculations are performed on meshes with 94.231 and 698.412 'hexahedrons' in cylindrical coordinate system indicated by black and magenta squares. The results obtained with the PMC symmetry planes from the real-valued FEM (CST MWS) calculations using basis functions up to the second order on second order curved tetrahedral elements are marked by cyan data points, while the red points mark the results obtained with the PMC symmetry planes from CST MWS calculations using basis functions up to the third order on fourth order curved tetrahedral elements. The CST MWS calculations are performed on meshes with 307.200 tetrahedrons.

A graphical representation of the shunt impedance (R/Q)as a figure of merit for the calculated monopole modes using the electromagnetic field solver MAFIA based on the realvalued Finite Integration Technique (FIT) [7] as well as on the real-valued FEM eigenmode solver is given in Fig. 3. According to Fig. 3, the colored data points obtained with various tetrahedral meshes indicate a robust calculation of frequency and shunt impedance for the eigenmodes in the 1st monopole passband. The eigenmode solver on the basis of real-valued finite element analysis is sufficient to describe the

entire electromagnetic field in the lossless PETRA cavity. In addition, with the increase of the hexahedral mesh cells, the difference between the eigenfrequencies from the MAFIA and CST MWS calculations becomes smaller.

The simulation results summarized in Fig. 3 have shown a remarkable competence of the real-valued eigenmode solver for the PETRA 7-cell cavity. Starting from the design values, the green curve in Fig. 4 indicates a bad field flatness of the longitudinal electric field component E_z in the PETRA 7-cell cavity for the TM₀₁₀, π mode. According to the fundamental operating principle of the PETRA accelerating cavity, a homogeneous field distribution is desirable.



Figure 4: Evaluation of the longitudinal electric field component Ez along the cavity axis. Green curve: unperturbed 7-cell cavity. Black curve: the penetration depth of the plunger is set to -20 mm, while the length of the pump is set to 120 mm. Red curve: the penetration depth of the plunger is set to -50 mm, while the length of the pump is set to 120 mm. Blue curve: the penetration depth of the plunger is set to -80 mm, while the length of the pump is set to 120 mm. All calculations are performed with 0.5 million curvilinear tetrahedrons.

COMPUTATION OF EIGENMODES WITH COMPLEX-VALUED EIGENMODE SOLVER



Figure 5: PETRA 7-cell cavity (500 MHz) equipped with beam tubes, plungers, pumps as well as the input coupler.

In reality, the PETRA acceleration cavity is equipped with two vacuum pumps, two plungers as well as an input coupler (see Fig. 5). The input coupler is installed onto the central cell of the PETRA cavity and required as an additional extension to transfer the energy from the sources to the particle beam. Therefore, the complex-valued eigenmode solver, which utilizes basis function up to the second order on curved tetrahedral elements, has been applied to the PE-TRA cavity to determine the characteristic values (resonance frequency, quality factor and shunt impedance) for all modes in the 1st monopole passband. A port boundary condition is used to define the boundary condition for the input coupler.

Firstly, by means of adjusting the penetration depth of the plungers, the field flatness of the PETRA cavity has been adjusted, as is shown in Fig. 4. In the future, the field flatness will be adjusted to a homogeneous level.

In Fig. 6, the colored markers indicate the values of frequency, external quality factor for all eigenmodes in the 1st monopole passband for a specific geometric setup where the penetration depth of the plungers is set to -80 mm, while the length of the pumps is set to 120 mm. According to Fig. 6, the quality factor of the accelerating mode TM_{010} , π is about 2.3·10³, while the quality factors of three monopole modes can reach to about 10⁸. The reason for those large values is that the electric fields of the three monopole modes in the central cell of the PETRA cavity are so weak that the energy transfer between the cavity and input coupler is restricted. Furthermore, a graphical representation of the shunt impedance (R/Q) for all modes in the 1st monopole passband are given in Fig. 7. The calculated results in Fig. 7 have shown a robust calculation of the shunt impedance.



Figure 6: Quality factors for the monopole modes in the 1st monopole passband for various mesh resolutions. The calculations are performed on meshes with 117.012, 347.427 and 1.145.408 tetrahedrons indicated by red squares, blue circles and black points.

CONCLUSION

In this paper, the eigenmodes in the PETRA 7-cell cavity have been preliminary calculated by using the real-valued and complex-valued eigenmode solver. Firstly, the fundamental monopole modes in the unperturbed 7-cell cavity has been computed with different tetrahedral meshes by using the real-valued eigenmode solver. The obtained results are in good agreement with the values of the eigenfrequency and shunt impedance. Due to the bad field flatness of the



Figure 7: Shunt impedances (R/Q) for the monopole modes in the 1st monopole passband for various mesh resolutions. The calculations are performed on meshes with 117.012, 347.427 and 1.145.408 tetrahedrons indicated by red squares, blue circles and black points.

longitudinal electric field components in the unperturbed PETRA 7-cell cavity, the penetration depth of the equipped plungers has to be set properly. Finally, a robust parallel eigenmode solver on the basis of complex-valued finite element analysis, has been successfully applied to calculate the eigenmodes inside the PETRA 7-cell cavity.

In the future, the eigenfrequency, the quality factor as well as the shunt impedance of the fundamental monopole modes and higher order modes in the PETRA 7-cell cavity will be calculated by using the complex-valued eigenmode solver, thereby the eigenmodes, which impair the stability of particle beam, can be discovered.

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