TRANSVERSE IMPEDANCE CALCULATION FOR SIMPLIFIED MODEL OF FERRITE KICKER MAGNET WITH BETA < 1

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Abstract

In high intensity rings, kicker magnet is usually considered as a main source to the total impedance. Transverse coupling impedance of a simplified kicker model has been derived analytically in the ultrarelativistic limit. We extend the result to the general case of $v < c$, and present the analytical formulae of both horizontal and vertical transverse impedances. Numerical results are given for the CSNS extraction kicker magnets.

INTRODUCTION

The transverse coupling impedance in high intensity circular accelerator can lead to collective instabilities and must be carefully estimated. An important transverse impedance contribution is from the ferrite kicker magnets. Thus a lot of experimental and theoretical works have been carried out to investigate the impedance of the ferrite kickers [1-4]. Most of the studies consider the ultrarelativistic beams, moving with the speed of light c. However, for many low energy proton or heavy-ion rings, the velocities of the beam may significantly differ from c. Thus, possible performance limitations may arise from the non relativistic effect. In this paper, the horizontal and the vertical transverse coupling impedance of a window frame ferrite kicker magnet with $\beta < 1$ have been derived. The method is similar to the one developed in Ref. [2] to compute the impedance of the same structure in the ultra-relativistic limit. The analytical expressions of the impedance are given. Numerical results for the extraction kicker in the rapid cycling synchrotron (RCS) of China Spallation Neutron Source (CSNS) are presented.

THE IMPEDANCE MODEL FOR FERRITE KICKERS

The window frame kicker consists of a rectangular ferrite block and two metallic sheets, i.e. the busbar. A simplified model as shown in Fig. 1 has been used in order to estimate the impedance analytically.

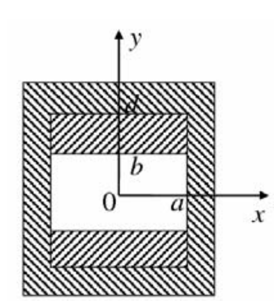

Figure 1: Cross section for the simplified kicker model.

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The beam pipe is rectangular with two opposing sides consist of ferrite ($|x| < a$ and $b < |y| < d$) and the others consist of perfect conductors ($|x| > a$ or $|y| > d$). There is vacuum inside the beam pipe. The length of the kicker in axial direction is assumed to be infinite for analytical calculations. The impedance is computed with the technique of field matching method.

HORIZONTAL COUPLING IMPEDANCE

The Beam Source

A beam travels at the center of the beam pipe with horizontal dipole moment P. The axial component of the current density of the beam is expressed as

$$j_z = \frac{P}{\pi r^2}\delta(r-h)\cos\theta\exp\big(j\omega(t-z/v)\big), \qquad (1)$$

where ω is the angular frequency, $v = \beta c$ is the speed of the beam, and h is the beam offset which is small compared to the aperture of the beam pipe. Therefore, all quantities have the same dependence of the exponential term exp ($j\omega z/v$), which will be omitted below.

In order to satisfy the boundary conditions at $|x| = a$, we add image current densities at $(x, y) = (2ma, 0)$, $m = 0, \pm1, \pm2$ … Thus, the beam source field is the sum of the direct beam field and the field from the image current,

$$\begin{aligned}
&E_x^{(s)} = Z_0/\beta H_y^{(s)} = -\frac{Pkk_r}{\pi\varepsilon_0\omega h} I_1(k_r h) \\
&\sum_{m=-\infty}^{\infty}\left(K_1^{'}(k_r r)\frac{(x-2ma)}{r^2} + \frac{K_1(k_r r)}{k_r r}\frac{y^2}{r^2}\right), \\
&E_y^{(s)} = -Z_0/\beta H_x^{(s)} \\
&= \frac{Pkk_r}{\pi\varepsilon_0\omega h} I_1(k_r h)\sum_{m=-\infty}^{\infty}\left(K_2(k_r r)\frac{(x-2ma)y}{r^2}\right), \\
&E_z^{(s)} = \frac{jPk_r^2}{\pi\varepsilon_0\omega h} I_1(k_r h)\sum_{m=-\infty}^{\infty}\left(K_1(k_r r)\frac{x-2ma}{r}\right), \\
&H_z^{(s)} = 0,
\end{aligned} \qquad (2)$$

where $k = k_0/\beta = \omega/\beta c$, $k_r = \omega/\beta\gamma c$, $r = \sqrt{(x-2ma)^2 + y^2}$, ε_0 is the vacuum permittivity, I_1, K_1, and K_2 denote modified Bessel functions, $Z_0 = 377\ \Omega$ is the characteristic impedance in vacuum.

Field Matching

The electromagnetic field in the vacuum region is the sum of the beam source field and the waveguide modes, while the field in the ferrite region can be expressed with only the waveguide modes.

The waveguide modes are determined by the following wave equations,

$$\nabla^2\vec{E}+\omega^2\mu\varepsilon\vec{E}=0,\quad \nabla^2\vec{H}+\omega^2\mu\varepsilon\vec{H}=0, \tag{3}$$

where μ and ε are permittivity and permeability of the medium.

Field matching at boundary $y = b$ gives

$$\begin{aligned}
&E_{zn}^{(s)}\Big|_{y=b}+A_n\cos x_1=C_n\sin x_2,\\
&E_{xn}^{(s)}\Big|_{y=b}+\frac{j}{k_r^2}(\omega\mu_0 k_{2n}B_n+kk_{1n}A_n)\cos x_1\\
&=j\frac{\omega\mu k_{3n}D_n-kk_{1n}C_n}{k^2-k_0^2\mu_r\varepsilon_r}\sin x_2,\\
&E_{yn}^{(s)}\Big|_{y=b}+\frac{j}{k_r^2}(\omega\mu_0 k_{1n}B_n-kk_{2n}A_n)\sin x_1\\
&=j\varepsilon_r\frac{\omega\mu k_{1n}D_n+kk_{3n}C_n}{k^2-k_0^2\mu_r\varepsilon_r}\cos x_2,\\
&B_n\sin x_1=D_n\cos x_2,
\end{aligned} \tag{4}$$

where μ_r and ε_r are relative permittivity and permeability of ferrite, $k_{1n} = n\pi/a$, $n = 0, 1, 2, \ldots$, $k_{1n}{}^2+k_{2n}{}^2 = -k_r{}^2$, $k_{1n}{}^2+k_{3n}{}^2 = k_0{}^2\mu_r\varepsilon_r-k_r{}^2$ are wave numbers, $x_1 = k_{2n}b$, $x_2 = k_{3n}(b-d)$, $E_{xn}{}^{(s)}$, $E_{yn}{}^{(s)}$ and $E_{zn}{}^{(s)}$ are Fourier expansion factors of the beam source fields, A_n, B_n, C_n and D_n are unknown coefficients to be determined. The symmetry of the structure in the transverse plane has been applied in deriving the equations. For simplicity, the conductivity of the ferrite has been assumed to be zero.

Impedance

The transverse impedance is the integration of the horizontal electromagnetic field

$$Z_{\perp,x}(\omega)=\frac{j}{P}\int_{-\infty}^{\infty}dz\left[E_x-v\mu H_y\right]e^{jkz}. \tag{5}$$

The horizontal impedance per unit length is derived as

$$Z_{\perp,x}/L=-\sum_n k_{1n}N_{xn}\exp(jk_{2n}b)\frac{M_{1n}\cos x_1+M_{2n}\sin x_1}{kk_{2n}(M_{3n}-M_{4n})}, \tag{6}$$

where

$$N_{xn}=\frac{Z_0 I_1(k_r h)}{a\pi\beta^2\gamma h\exp(jk_{2n}b)}\int_{-a}^{a}dx\sum_{m=-\infty}^{\infty}K_2(k_r r)\frac{(x-2ma)b}{(x-2ma)^2+b^2}\sin(k_{1n}x),$$

$$\begin{aligned}
M_{1n}&=jk_{2n}^2k_0^2(1-\varepsilon_r\mu_r)\\
&\quad+k_{2n}k_r^2(jk_{2n}-k_{3n}\varepsilon_r\cot x_2)\\
M_{2n}&=-k_{1n}^2k_0^2(1-\varepsilon_r\mu_r)\\
&\quad+k_{2n}k_r^2\mu_r(-k_{2n}\varepsilon_r+jk_{3n}\tan x_2)\\
M_{3n}&=-k_{2n}k_{3n}(\varepsilon_r\cos^2 x_1\cot x_2+\mu_r\sin^2 x_1\tan x_2)\\
M_{4n}&=(k_0^2(-1+\varepsilon_r\mu_r)+k_{2n}^2(1+\varepsilon_r\mu_r))\cos x_1\sin x_1
\end{aligned}$$

In the ultra-relativistic limit, the horizontal impedance turns to

$$\frac{Z_{\perp,x}}{L}=\sum_n kk_{1n}N_{xn}\frac{jk_{1n}^2(1-\varepsilon_r\mu_r)}{k_{2n}(M_{3n}-M_{4n})}, \tag{7}$$

which agrees with Eq. (7) in Ref. [2].

VERTICAL COUPLING IMPEDANCE

In the vertical impedance calculation, the current density of the source beam can be expressed by

$$j_z=\frac{P}{\pi r^2}\delta(r-h)\sin\theta\exp\big(j\omega(t-z/v)\big), \tag{8}$$

The vertical impedance is defined as

$$Z_{\perp,y}(\omega)=\frac{j}{P}\int_{-\infty}^{\infty}dz\left[E_y+v\mu H_x\right]e^{jkz}. \tag{9}$$

Using the similar method developed in the calculation of the horizontal impedance, we derive the vertical coupling impedance per unit length is

$$Z_{\perp,y}/L=\sum_n k_{2n}N_{yn}\exp(jk_{2n}b)\frac{M_{1n}\sin x_1-M_{2n}\cos x_1}{kk_{2n}(M_{5n}-M_{4n})}, \tag{10}$$

with

$$M_{5n}=k_{2n}k_{3n}(\varepsilon_r\sin^2 x_1\cot x_2+\mu_r\cos^2 x_1\tan x_2),$$

$$N_{yn}=-\frac{jZ_0k_{2n}bI_1(k_r h)}{a\pi\beta\gamma h\exp(jk_{2n}b)}\int_{-a}^{a}dx\sum_{m=-\infty}^{\infty}(-1)^m\frac{K_1\left(k_r\sqrt{(x-2ma)^2+b^2}\right)}{k_r\sqrt{(x-2ma)^2+b^2}}\cos(k_{1n}x).$$

In the ultra-relativistic limit, the impedance becomes

$$\frac{Z_{\perp,y}}{L}=\sum_n kk_{2n}N_{yn}\frac{jk_{1n}^2(1-\varepsilon_r\mu_r)}{k_{2n}(M_{5n}+M_{4n})}, \tag{11}$$

which agrees with Eq. (14) in Ref. [2].

APPLICATIONS

We apply the formulae to calculate the transverse impedance of the extraction kicker of CSNS/RCS. The RCS is a high intensity proton ring, which works at injection energy of 80 MeV and extraction energy of 1.6 GeV [5]. In this energy region, the proton has relativistic γ

from 1.1 to 2.7. The extraction kickers are important contributions to the total transverse coupling impedance for the RCS.

One typical kicker of the RCS has the parameters of a = 103 mm, b = 67 mm, d = 122 mm, and l = 0.27m [6]. The kicker magnet core is made from Ni-Zn type ferrite blocks, whose performance is very close to that of CMD5005. The relative permeability of the CM5005 ferrite [7] is used in the numerical calculations. The relative permittivity is assumed to be $\varepsilon_r = 12$.

The horizontal and the vertical impedances are computed at both injection and extraction energies. The numerical results are presented and compared with the relativistic limit in Fig. 2 and Fig. 3. The curves with triangles correspond to injection energy, the curves with diamonds correspond to extraction energy, and the curves with stars correspond to the ultra-relativistic case. As the bunch is tens of meters in length, we only interest in the low frequency impedance in the range of 0 ~ 100 MHz.

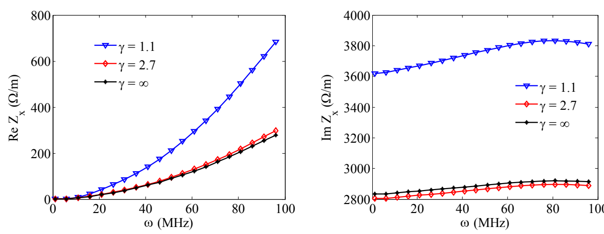

Figure 2: Horizontal coupling impedance of a typical kicker of CSNS/RCS. Left and right plots show the real and the imaginary parts, respectively.

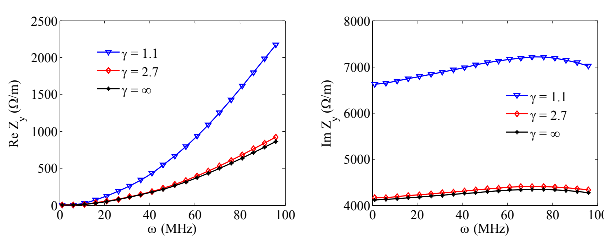

Figure 3: Vertical coupling impedance of a typical kicker of CSNS/RCS. Left and right plots show the real and the imaginary parts, respectively.

We can see from the figures that the vertical impedance is about two times of the horizontal one. The distinction between the horizontal and the vertical impedance mainly comes from the different boundary conditions at the orientation of the beam displacement.

The results also illustrate that the difference between the impedance at injection and extraction energy is significant, and the non ultra-relativistic results converge to the relativistic limit as the energy of the beam increases.

SUMMARY

The transverse impedance of a simplified model of window frame kicker magnet has been extended to the general case of $v < c$. Both the horizontal and vertical transverse coupling impedance are derived by using the field matching method. The analytical expressions of the impedances are presented. The results agree with the ultra-relativistic value in the limit of $\gamma\to\infty$.

Using the formulae, we calculated the horizontal and vertical transverse impedance of a typical kicker of CSNS RCS at different energies. The result is compared with the relativistic limit, which demonstrates that the non-relativistic effect is not negligible in evaluating the impedance of low energy proton beam, especially for the small synchrotrons.

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