

PRODUCTION OF COHERENT OPTICAL ČERENKOV RADIATION IN SILICA AEROGEL *

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Abstract

As a demonstration of the apposite properties of silica aerogel as an electron beam diagnostic we intend to use it to produce coherent optical Čerenkov radiation (COCR). In this paper we propose an experiment and provide details of the challenges to be overcome in producing COCR.

INTRODUCTION

Electromagnetic radiation production has been a focus of accelerator physics since synchrotron radiation was first seen in 1947 [1]; it is used both as a product for scientific users, such as synchrotrons and free-electron lasers, as well as a beam diagnostic tool for accelerator physicists, i.e. transition radiation. In both cases, over the past few decades there has been increasing interest in coherent radiation production from electrons. During this time, because of electron beam scattering in the materials required to produce it, coherent Čerenkov radiation has been limited to *mm* wavelengths, usually in non intercepting geometries [2, 3]. Presently, production of optical Čerenkov radiation requires the beam to pass through the material and thus a low density material is required to increase the mean free path, a role generally filled by gas cells [2]. We propose a different media for COCR production: silica aerogel. Herein we will describe the apposite characteristics of aerogel and introduce an experiment at the Next Linear Collider Test Facility (NLCTA) [4] to demonstrate the production of COCR.

AEROGEL

Silica aerogel is a solid material comprised of tenuous amorphous structures of SiO_2 surrounding pockets of air or vacuum [5]. It has excellent vacuum properties [6] and the index of refraction is nearly constant over the optical regime [7]. The index of refraction of aerogel at optical frequencies is a simple function of the density of the aerogel [5, 6, 7]: $n = 1 + 0.21\rho$ (g/cm^3). The range of densities available from various manufacturers means a range of indexes are available.

Although Čerenkov radiation based techniques have been used to make velocity measurements of protons and other heavy particles, they have been less useful for electrons because of their higher scattering in the dense materials used to produce the radiation [8]. The very low density of aerogel ameliorates the scattering problem.

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To produce COCR we will be using an intercepting scheme where the electron beam passes through a block of aerogel. For aerogel assumed to be pure silica, the mean free path (l) for elastic scattering of a 47 MeV electron is $l = 770\mu m/\rho$, with ρ in mg/cm^3 , calculated using the static field approximation [9], while the radiation length (X_0) is many meters [10] – no bremsstrahlung is expected. Because the elastic scattering range is so short the aerogel slab must be made as short as possible to minimize growth of both electron beam emittance and the thickness of the Čerenkov ring.

To model the electron beam emittance growth caused by scattering in the aerogel, we used GEANT4 [11]. The electron beam was focused into the center of a 1 *mm* thick aerogel block of density $26 mg/cm^3$ ($l = 30 \mu m$, $X_0 = 10 m$). The rms emittance growth of 90% of the electrons as a function of focal radius, σ^* , is shown in Fig. 1 for a 1 *mm* thick aerogel sample. To minimize the emittance growth the beam should be focused as much as possible, which we will see is also a requirement to maximize the coherent signal.

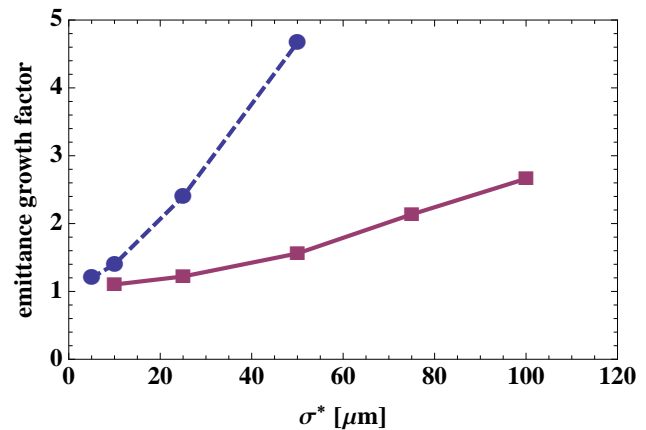


Figure 1: rms emittance growth factor of 90% of the particles for a 47 MeV electron beam with initial emittance of 2 *mm mrad* (dashed line, circles) and 8 *mm mrad* (solid line, squares) after being focused at the center of a 1 *mm* thick aerogel block. The beam waist size is given by σ^* .

ČERENKOV RADIATION

Čerenkov radiation occurs when a charged particle passes through a material with high enough velocity such that the region of the material that is polarized is trailing behind the particle itself. The result is a net dipole moment between the polarized material and the charged particle which can radiate [12]. Accordingly, it is readily shown that the particle must be superluminal in the mate-

rial (called the Čerenkov condition, $\beta n \geq 1$) and in this case, the angle between the emitted photons and the direction of electron motion, called the Čerenkov angle, is $\cos \theta_C = 1/(\beta n)$, where β is the velocity of the electron normalized to the speed of light. θ_C is the half angle of the characteristic Čerenkov cone created in a medium with isotropic index of refraction.

The angle-spectral density of the photons radiated by a single electron during linear motion is, from Tamm [13],

$$\left. \frac{d^2 N_\gamma}{d(\cos \theta) dk} \right|_{e^-} = \frac{\alpha n k L^2}{2\pi} \left(\frac{\sin(X(k, \theta))}{X(k, \theta)} \right)^2 \sin^2 \theta, \quad (1)$$

where N_γ is the number of photons, α is the fine structure constant, θ is the angle between the electron direction of motion and the emitted photon, $\omega = kc$ is the frequency of the radiation *in vacuo*, L is the length of Čerenkov emission and $X(k, \theta) = \frac{kL}{2\beta}(1 - \beta n \cos \theta)$. Because the thickness of the Čerenkov ring, $(\sin(X)/X)^2$, decreases with increasing wavenumber, Eq. 1 shows that the photon number spectral distribution is flat for all wavelengths with the same index of refraction. The finite thickness of the Čerenkov ring is a diffraction effect caused by the finite length of the radiating material; when $kL \gg 1$ the ring thickness becomes a δ -function and we recover the Čerenkov angle given above.

It should be noted that the Čerenkov radiation distribution from a single electron is itself a coherent interaction between photons emitted at varying positions along the path of electron motion. In light of particle scattering, the length L in Eq. 1 is somewhat poorly defined. In the limit of no scattering L is the length of the medium (L_m), in the limit of strong scattering it is the mean free path. For cases in between, it is the distance the particle travels before changing angle enough to disturb the coherence. To estimate the effect of scattering, we compare the growth of the *rms* electron angle relative to the initial direction of motion, θ_{rms} , as a function of distance within the aerogel and the *rms* diffraction angle, θ_D . When $\theta_D \geq \theta_{rms}$ the length of the whole medium (L_m) can be used, otherwise the length to equality of the two terms is used. Fig. 2 shows how the length which results in equality between these terms changes with aerogel density for the beam given by Table 1.

COHERENT RADIATION

The coherence of radiation from an assembly of emitters is an interference effect between the waves emitted which depends on the method of emission, the wavelength of the light and the distribution of the source emitters [14]. When the emitters are spread out over a region large compared to the wavelength of interest the waves arrive at a distant detector with a large spread in phase and the collected radiation is incoherent. In order to generate coherence the emitters must be compressed into a region small enough such that there can be a known phase relationship between photons.

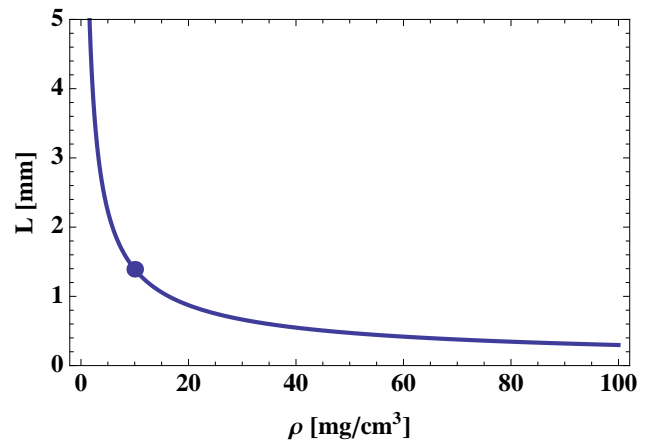


Figure 2: Length of the aerogel sample at which there is equality between the *rms* diffraction angle and the Čerenkov ring broadening due to scattering. The dot indicates the $\rho = 10 \text{ mg/cm}^3$ point.

To examine what this means for Čerenkov radiation we start with the relation between the angle-spectral density of a beam of electrons and a single electron [2],

$$\left. \frac{dN_\gamma}{d(\cos \theta) dk} \right|_{e^-} = N(1 + N f_L f_T \chi) \left. \frac{dN_\gamma}{d(\cos \theta) dk} \right|_{e^-}, \quad (2)$$

where N is the number of electrons in the bunch, χ is the divergence factor of the electrons (taken to be equal to 1 here) and f_L and f_T are the square magnitude of the longitudinal and transverse Fourier transforms of the beam distribution, respectively, assuming those dimensions are separable, i.e. $\rho(x, y, z) = g(x, y)h(z)$. It can be seen that coherent optical radiation requires not only longitudinal bunching at an optical frequency, which is straightforward using IFEL modulation in an undulator [15], but also a transverse beam size that is quite small. For example, 800 nm radiation requires a Gaussian beam size of $\sigma_r = 2 \mu\text{m}$ for $f_T = 0.1$, meaning that producing COCR is quite challenging. Particularly so because the emission angle of Čerenkov radiation can be quite large – unlike transition radiation the emission angle can be much larger than the inverse of the Lorentz factor [16]. Further, COCR spatially overlaps with the incoherent Čerenkov radiation meaning that the presence of COCR must be confirmed with either a total energy measurement or a measurement of the spectrum.

There are, however, techniques for increasing the coherence of the optical radiation without resorting to extreme focusing by using the properties of the Fourier transform to increase the allowed frequency content of the radiation. Naturally occurring charge structures on the micron scale in the beam can increase the allowed bandwidth of the coherent process or masks can be used to create sharp edges in the distribution. Charge distribution fluctuation is a result of the non equilibrium nature of electron beams in linear accelerators and is expected to lead to shot-to-shot fluctuations in the coherent signal. Recent experiments measur-

ing the far field distribution of coherent optical transition radiation indicate that this effect will be large. While the masking process can be used to produce a regular structure within the charge on the scale necessary to enhance coherence.

Table 1: Beam parameters used in simulations.

Parameter	Symbol	Value
charge (pC)	Q	50
energy (MeV)	E	47
rms (x,y) emittance (mm-mrad)	$\epsilon_{x,y}$	8
radius at focus (μm)	σ_r	25
rms beam length (μm)	σ_z	150
bunching wavelength (nm)	λ_r	795
bunching factor	b_1	0.18

EXPERIMENT AT NLCTA

As an example of how a mask can be used to enhance coherence we have calculated the total photon production per unit length by a partially microbunched electron beam at the m^{th} harmonic of the microbunching ($k_r = 2\pi/\lambda_r$), using typical parameters at NLCTA (see Table 1), passing through a circular aperture in a Tungsten plate of radius r_0 :

$$N'_{\gamma,m} = \frac{\alpha}{\sqrt{\pi}\sigma_z} \frac{\sin^2 \theta_C}{\cos \theta_C} N^2 b_m^2 \left(1 - e^{-\frac{r_0^2}{2\sigma_r^2}}\right)^2 \times \left(\frac{r_0}{\sigma_r}\right)^4 e^{-\frac{r_0^2}{\sigma_r^2}} \left[\frac{J_1(mk_r r_0 \tan \theta_C)}{mk_r r_0 \tan \theta_C}\right]^2, \quad (3)$$

where $J_1(x)$ is the Bessel function of the first kind. Fig. 3 shows how the transverse coherence term depends on the ratio r_0/σ_r .

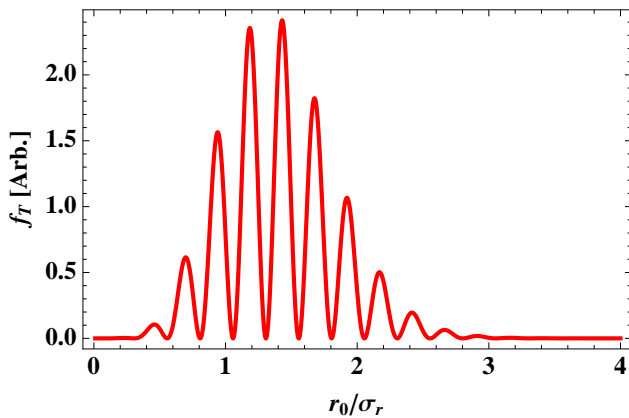


Figure 3: The transverse coherence function for a Gaussian beam of width σ_r passing through a hole of radius r_0 . The aerogel density is 10 mg/cm^3 . The maximum occurs for $r_0/\sigma_r = 1.43$.

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The bandwidth for the fundamental bunching mode ($m = 1$) is 1.1 meV. This is a key difference between the incoherent and coherent radiation spectra. While, the incoherent radiation is wide bandwidth, limited only by the wavelengths that meet the Čerenkov condition ($\sim eV$), the incoherent radiation has a much narrower bandwidth which will show up as bright peaks in the spectrum of the collected radiation. We take the full length of the material as the emission length because the coherence effect causes each slice of aerogel block, dz , to emit $N'_{\gamma,m}$ photons and in the limit of infinitesimally thin slices $N_{\gamma,m} = \int N'_{\gamma,m} dz$. When $L_m \sim \frac{2\epsilon_n \sqrt{N_l}}{\gamma \sigma^* \theta_{rms}}$ (N_l is the number of mean free paths in the sample), the aerogel is short enough such that the beam size is not affected by the scattering and $L = L_m$. We estimate that there will be 1.7 pJ of 790 nm COCR from a 1 mm thick block of 10 mg/cm^3 aerogel.

SUMMARY

The two largest hurdles to producing coherent optical Čerenkov radiation are scattering in the material and the large angle of Čerenkov radiation. Both of these problems can be ameliorated by using a low density, low index of refraction material such as silica aerogel. To maintain emittance and maximize coherent radiation production the beam must be strongly focused into the aerogel block, while the length of the block is limited by diffraction effects and scattering. An example experiment has been proposed for the NLCTA accelerator.

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