

ELECTRON BUNCH SLICE EMITTANCE MEASUREMENT WITH THE SPACE CHARGE EFFECTS*

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Abstract

Since slice transverse emittance of the electron beam is critical to a high-gain short-wavelength FEL, its characterization is very important. For space charge dominated electron beam, conventional emittance measurement techniques, such as solenoid scanning and quadruple scanning, without considering space charge forces lead to large errors of emittance evaluation. This essay introduces a modified solenoid-scan method of slice emittance measurement for space charge dominated beam, and simulations show that the new method brings the emittance evaluations much closer to actual values.

MOTIVATION

The high-gain short-wavelength free-electron laser (FEL) requires a high-brightness, low-emittance electron beam generated by a photocathode RF gun. In SASE FEL a self-amplification process mainly occurs in a short length of an electron bunch, called slippage distance, typically far shorter than the bunch length [1]. Accordingly, the radiation of FEL depends on the transverse emittance and charge density of a time slice instead of those of the entire bunch.

The emittance of each slice is usually not equal to the projected emittance of the entire bunch [2]. Hence a 3-cell transverse RF deflecting cavity (TCAV), which provides strong vertical deflecting force depending linearly on the longitudinal position within the bunch, is used to convert the longitudinal bunch distribution to a vertical streak performed on the beam profile monitor downstream for time-resolved measurement [3].

In contrast to the strong focusing quadrupole magnet, an emittance compensating solenoid immediately following the RF gun, can provide much weaker focus and be used for solenoid-scan emittance measurement of the electron beam generated by RF gun at low range energy (~MeVs) [4]. Because the electron beam is not primarily emittance dominated, the space charge forces, which can strongly contribute to the beam envelope evolution along the scanning solenoid and the drift space towards the beam profile screen, lead to much larger emittance evaluation than the actual value.

To take the space charge into the transfer matrices of solenoid and drift space, beam sizes on the measurement screen can be fitted for each solenoid current value, to calculate the beam emittance and Twiss parameters. PARMELA simulations of solenoid scan algorithm with considering the linear space charge forces applied to round beams is presented. Moreover, the algorithm with

considering the non-linear space charge effects of Gaussian beams are also be discussed.

SOLENOID SCAN METHOD

Three gradients method is one of the most commonly used methods for measuring beam emittance. An electron beam with beam matrix σ_0 at start point 0 is considered,

$$\sigma_0 = \begin{bmatrix} \langle x_0^2 \rangle & \langle x_0 x_0' \rangle \\ \langle x_0 x_0' \rangle & \langle x_0'^2 \rangle \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{bmatrix} \quad (1)$$

transported through an focusing element and a drift space, and then striking a screen with the beam matrix σ_p . If M is the transfer matrix precisely between the point 0 and the screen, σ_p and σ_0 are related by the expression of $\sigma_p = M\sigma_0M^T$, and the term $\sigma_p(1,1)$ can be written as

$$\sigma_p(1,1) = M_{11}^2\sigma_0(1,1) + M_{11}M_{12}\sigma_0(1,2) + M_{22}^2\sigma_0(2,2) \quad (2)$$

By changing the focussing strength, the elements of σ_0 can be fitted from the measurements of the corresponding various profiles on screen. Note that change of the strength of the focusing element results in variation of the corresponding transfer matrix.

For the coupling of horizontal and vertical motions in solenoid fields, the transfer matrix M and beam matrix σ should be four-dimensional in phase space (x,x',y,y') and the number of unknown elements in Eq (2) turns to be six [5]. The beam rms emittance is given by $(\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2)^{1/2}$, where the average is taken over the beam distribution of all particles in rotation coordinates without transverse motion coupling.

Due to the beam rotation in magnetic solenoid fields, the measured emittance is actually not in the horizontal direction. The rotation angle can be calculated by the solenoid field strength and bunch energy.

SPACE CHARGE FORCES

Envelope Equation

With the assumption of linear space charge forces, the envelope equation can be written as follows [6],

$$\sigma_x'' + k_x^2(z)\sigma_x - \frac{\varepsilon_n^2}{\gamma^2\sigma_x^3} - \frac{I}{\gamma^3I_A(\sigma_x + \sigma_y)} = 0 \quad (3)$$

where I is peak current and $I_A=17A$ is Alfven current.

At low energy, the space charge term in Eq (3) can strongly contribute to the beam envelope evolution to

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cause large emittance evaluation errors, thus it is standard to use the pepper-pot or single-slit emittance measurement instead of the three gradients method [7]. But high time resolution in slice emittance measurement requires a small beam profile on screen, so the pepper-pot or single-slit method is not suitable.

Taking the space charge into the transfer matrices of solenoid and drift space can help to describe the beam evolution more accurately. And these matrices can be used in solenoid-scan emittance measurement to reduce the evaluation errors.

Linear Space Charge Forces

The solenoid fields are fragmented into N pieces along the axis with the assumption that the paraxial solenoid field $B(z)$ is piecewise constant. Moreover, we make up two assumptions on the electron beam: 1. Beam envelope is slowly changed seen as a constant in each piece of the solenoid. 2. Electron beam is ideal KV distribution after laser shaping and spatial filtering. By these assumptions the space charge forces are linearly correlated with particle transverse position and written as:

$$\begin{cases} F_{SC,x} = -\frac{eI}{\pi\epsilon_0\beta c\sigma_x(\sigma_x + \sigma_y)\gamma^2}x = a_x x \\ F_{SC,y} = -\frac{eI}{\pi\epsilon_0\beta c\sigma_y(\sigma_x + \sigma_y)\gamma^2}y = a_y y \end{cases} \quad (4)$$

where the coefficients a_x and a_y depend on beam current, energy and specially the size of beam profile.

By solving the motion function of particles with the above linear space charge forces, it can be deduced the transfer matrix of a short piece of solenoid:

$$\begin{bmatrix} \text{ch}K_x z & \frac{K_x \text{sh}K_x z + K \sin Kz}{K^2 + K_x^2} & 0 & \frac{K(\cos Kz - \text{ch}K_x z)}{K^2 + K_x^2} \\ K_x \text{sh}K_x z & \frac{K_x^2 \text{ch}K_x z + K^2 \cos Kz}{K^2 + K_x^2} & 0 & \frac{K(K \sin Kz + K_x \text{sh}K_x z)}{K^2 + K_x^2} \\ 0 & \frac{K(\text{ch}K_y z - \cos Kz)}{K^2 + K_y^2} & \text{ch}K_y z & \frac{K_y \text{sh}K_y z + K \sin Kz}{K^2 + K_y^2} \\ 0 & \frac{K(K_y \text{sh}K_y z + K \sin Kz)}{K^2 + K_y^2} & K_y \text{sh}K_y z & \frac{K_y^2 \text{ch}K_y z + K^2 \cos Kz}{K^2 + K_y^2} \end{bmatrix} \quad (5)$$

where the solenoid focusing factor $K=eB_z/mv_z$, and the space charge factors $K_x=a_x/\gamma\beta^2 c^2 m_e$, $K_y=a_y/\gamma\beta^2 c^2 m_e$.

For the i -th piece of solenoid, if σ_i is given, σ_{i+1} can be calculated by using a standard transfer matrix, where σ_i and σ_{i+1} refer to the start and end beam sizes in this piece. Then the transfer matrix M_i with space charge factors is determined by an average of beam size $(\sigma_i + \sigma_{i+1})/2$, so as to be used to calculate a new σ_{i+1} . After several iterations, the end beam size σ_{i+1} of this piece can be calculated accurately and used as a start beam size in next piece.

Through these calculations, the evolution of beam sizes in solenoid can be deduced to fix on the transfer matrix of each piece of solenoid with space charge factors. Note that the initial beam size σ_1 in the first piece is fitted by a

06 Beam Instrumentation and Feedback

T03 Beam Diagnostics and Instrumentation

conventional solenoid-scan method without considering the space charge. Accordingly, the transfer matrix of the whole solenoid is multiplied by those of all pieces in reverse order $M_{\text{sol}}=M_N M_{N-1} \cdots M_1$.

As in the case with a solenoid matrix, the transfer matrix of a drift space can also be deduced only if the solenoid focusing factor K in Eq (5) is zero.

PARMELA Simulations

Simulations by the PIC code PARMELA have been applied to round beams to verify the solenoid-scan method with linear space charge forces. The simulation beamline is given in Figure 1. The initial bunch distribution is both longitudinal and transverse Gaussian with rms bunch radius of 0.6 mm, thermal emittance of 0.72 mm-mrad at the cathode, and accelerated to 3.5 MeV in RF gun with bunch compression to 200 fs (rms), then transported through an solenoid and a drift space towards the end where the assumed screen is located.

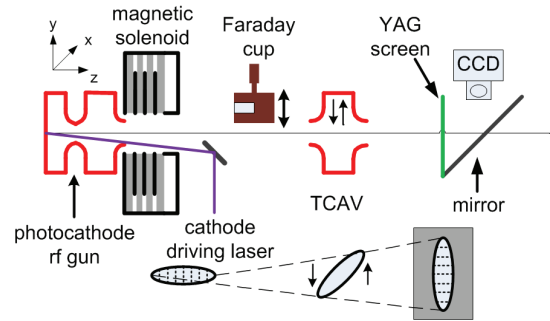


Figure 1: Schematic of simulation beamline.

For the Gaussian bunch, we use 2.355 times the rms bunch length to compute the peak current which is assumed to be constant [8]. The solver using the transfer matrix with space charge, which is used to fit the various profiles on the screen as a function of solenoid currents, is compared with that without space charge at bunch charge of 30 pC, as shown in Figure 2. The solver which includes the space charge fits better beam sizes.

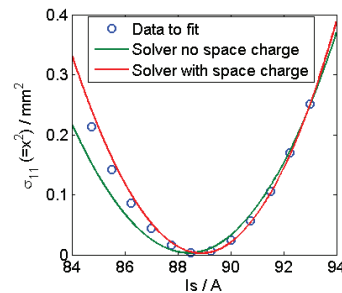


Figure 2: Simulated beam sizes on profile screen.

The simulation results of the projected emittance evaluations by solenoid scan are shown in Figure 3. As the bunch charge increases, the emittance measurement errors get obviously large. Taking the linear space charge forces into solenoid-scan method helps to reduce the evaluations from 1.82 mm-mrad to 1.51 mm-mrad which is closer to the actual value at bunch charge of 30 pC.

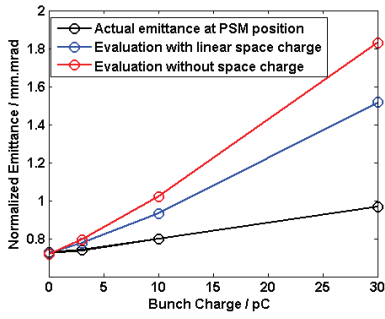


Figure 3: Simulated projected emittance evaluations as a function of bunch charge. The PSM position refers to phase space mapping position as the start point of matrix.

The simulation results of slice emittance evaluations at bunch charge of 10 pC are shown in Figure 4. Emittance evaluations are closer to the actual value at the core slices, but turn so bad at both ends owing to the extreme changes of beam current, that the non-linear space charge effects cannot be ignored in Gaussian bunch.

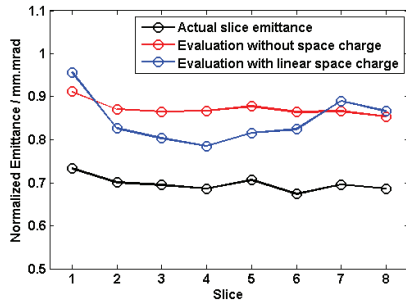


Figure 4: Simulated slice emittance evaluations with slice length of 0.1 ps. Bunch head is on the left side.

Non-Linear Space Charge Forces

Gaussian distribution, which is closer to the actual distribution of electron bunch generated by photocathode RF gun than KV distribution, has a potential function of space charge in series form to bring out non-linear space charge effects. We can use Lie algebraic method to deduce a high order transfer matrix with non-linear space charge in some transport elements.

In classical mechanics, particle motion can be described as Hamilton's equations with the Hamiltonian,

$$H = \left[m_0^2 c^4 + c^2 (p_x - qA_x)^2 + c^2 (p_y - qA_y)^2 + c^2 (p_z - qA_z)^2 \right]^{1/2} + q(\psi_e + \psi_s) \quad (6)$$

where ψ_s denotes the space charge potential function with a series form. According to the Hamilton's equations and definition of Lie operator [9], derivatives of phase-space variables ξ can be expressed by a Lie operator acting on ξ associated by the Hamiltonian,

$$\dot{\xi} = \sum_i \left(\frac{\partial \xi}{\partial q_i} \dot{q}_i + \frac{\partial \xi}{\partial p_i} \dot{p}_i \right) = \sum_i \left(\frac{\partial \xi}{\partial q_i} \frac{\partial H}{\partial p_i} - \frac{\partial \xi}{\partial p_i} \frac{\partial H}{\partial q_i} \right) = [\xi, H] = -H : \xi \quad (7)$$

$$d^n \xi / dt^n = -H :^n \xi$$

where the symbol $:H$ is the Lie operator associated by the corresponding Hamiltonian.

Substituting the Lie operators for derivatives, the coordinates mapping from one point to another in the phase space, which is a symplectic map, can be written in the series form of Lie operators called Lie transformation in Eq (8). Based on the factorization theorem in Lie algebra, this map can be written in a factored product form and truncated at any stage to achieve a high order approximation of the original symplectic map in Eq (9).

$$\xi(t) = \sum_{m=0}^{\infty} (1/m!) (t-t_0)^m (d/dt)^m \xi(t)|_{t_0} = \sum_{m=0}^{\infty} (1/m!) (t-t_0)^m : -H :^m \xi_0 \quad (8)$$

$$= \exp[-(t-t_0) : H :] \xi_0$$

$$M = \exp\left(-\int_{t_0}^t (H_2 + H_3 + \dots) dt\right) = \dots \exp(:f_3 :) \exp(:f_2 :) \quad (9)$$

$$= \exp(:f_2 :) + :f_3 : \exp(:f_2 :) + \left(:f_4 : + \frac{1}{2} :f_3 :^2\right) \exp(:f_2 :) + \dots$$

Lie algebraic method can be used to deduce the high order transfer matrix to describe the beam evolution more accurately to help to reduce the emittance measurement errors of solenoid-scan method, but some details in solenoid-scan measurement need to be dealt with.

For a single particle, transverse motion is coupled with longitudinal motion by non-linear space charge forces. We may assume that particles have the same Δt and Δp in a short slice of a bunch, but their values are difficult to determine. Another problem is that the bunch shape varies from a Gaussian distribution after passing the TCAV.

SUMMARY

With the linear space charge effects considered, a modified solenoid-scan method for emittance measurement is developed. This method is applied for the low energy electron beam which is not primarily emittance dominated, and has been demonstrated by the PARMELA simulations. In spite of the emittance evaluation errors down to some extent, it's not satisfactory yet. For a Gaussian beam generated by photocathode RF gun, taking the non-linear space charge forces into the transfer matrix may help to describe the beam evolution more accurately.

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