LIMITATION IN MITIGATING COLLECTIVE EFFECTS IN THE BETA-BEAM DECAY RING BY THE USE OF OCTUPOLES*

C. Hansen†, CERN, Geneva, Switzerland
E. Benedetto, NTU-A, Greece and CERN, Switzerland
A. Chancé, CEA, IRFU, SACM, F-91191 Gif-Sur-Yvette, France

Abstract

The beta-beam concept relies on production, by beta decay of radioactive ions, of an electron ν and $\bar{\nu}$ beam towards a detector. After production and acceleration in a rapid cycling synchrotron, the CERN PS and the CERN SPS, the radioactive isotopes are injected into a long racetrack-shaped ring, called the decay ring (DR), where they orbit until they decay or are lost. The required intensities to store in the DR to reach the aimed neutrino fluxes are very high. Among the collective effects, the head tail effect, caused by transversal resonance impedance, is one of the main issues: the beam was shown to be unstable. Octupoles were included in the DR lattice to increase the stability limit with an amplitude detuning. The impact was however not big enough. We here report on the limitation on improving the beta-beam performance with amplitude detuning in the DR and SPS.

INTRODUCTION

One of the proposed next generation neutrino oscillation facilities is the Beta Beams [1]. Beta decaying ions are stored at $\gamma = 100$ in a racetrack shaped storage ring, the "Decay Ring" (DR). One of the straight sections points to an oscillation experiment and the decaying ions create a highly pure (anti) electron ν beam with an opening angle of $1/\gamma$. The aimed annual (anti) neutrino flux of (2.9e18) 1.1e18 from (β^-) β^+ decaying (^{6}He) ^{18}Ne ions gives good sensitivities if the Duty Factor (DF) is smaller than 1% in the DR [2]. One of the accelerating machines in the Beta Beam complex is the SPS. Both SPS and DR will host 20 ion bunches and since DR has same circumference as SPS the limitation of DF $<$ 1% requires the bunches to be less than 2 m. The challenges of how to produce enough ions, group them into 20 bunches, accelerate them through the Beta Beam complex at CERN and merge them into small enough bunches in the DR are described elsewhere [3, 4, 5]. With 20 bunches, the number ^{18}Ne (^{6}He) per bunch has to be $3.4 \cdot 10^{12}$ ($4.5 \cdot 10^{12}$) in the DR and $2.9 \cdot 10^{11}$ ($6.5 \cdot 10^{11}$) in SPS to reach the nominal (anti) neutrino fluxes. These required bunch intensities are shown as red dashed lines in fig. 1. To be able to host the nominal number of ions in as small bunches as needed the DR has requirements of seemingly insurmountable low transverse broadband impedance. This was shown in [6] where also the methods of measuring the bunch ion intensity limits, N_b^{th}, with HEADTAIL [7] simulations and calculating it with the "Coasting Beam Equation" (CB Eq.) [8] were described in detail. This report focuses on the question if it is possible to damp the instabilities causing these strict limitations on N_b^{th}. Damping with amplitude detuning was studied in the SPS where experience already exist [9]. The technical challenges of providing similar octupole strengths to the DR will be discussed and then the possibility of mitigate the collective effects in the DR will be explored.

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† Corresponding author: Christian.Hansen@cern.ch

Parameters	Description	SPS Inj ^{6}He	DR ^{18}Ne
Z	Charge Num.	2	10
A	Mass Num.	6	18
V_{RF} [MV]	RF Voltage	0.53	26.75
E_{rest} [MeV]	Rest Energy	5606	16767
L_b [m]	Bunch Length	4.07	1.97
δ_{max} [10^{-3}]	Mom. Spread	0.9	2.5
$N_{b_{inj}}^{nom}$ [10^{11}]	Nom. Injected	6.5	2.7
$N_{b_{sat}}^{nom}$ [10^{11}]	Nom. Saturated	-	34
E_{tot} [GeV] $= \gamma \cdot E_{rest}$		52.3	1676.7
$\varepsilon_l^{2\sigma}$ [eVs] $= \frac{\pi\beta}{2c} E_{tot} L_b \delta_{max}$		1.00	43.20
Q_s [10^{-3}] $= \sqrt{\frac{hZeV\lvert\eta\cos\phi_s\rvert}{2\pi\beta^2 E_{tot}}}$		2.5	8.1

Table 1: Values are shown for ^{6}He in SPS injection and for ^{18}Ne in the DR. For SPS some parameters are given by a non-linear matching routine. For the DR all values above the line are input parameters.

SPS INJECTION SCANS

When 20 ion bunches are injected from PS into SPS they are too long for SPS' 200 MHz RF system (h = 4620). Therefore a 40 MHz (h = 924) will be added to SPS [3]. Bunches from PS need to be long since direct space charge (dsc) effects are worse for low energy ($\Delta Q_{dsc} \propto \gamma^{-2}$). E.g. ^{6}He bunches enter with bunch length, L_b, of more than 4 m. This means that the bunch is in a nonlinear region of the RF bucket (although h = 924). A nonlinear matching routine, based on LOBO[11] was implemented into HEADTAIL. The parameters for the resulting matched bunch and voltage are shown in table 1. The constraint for this match was to keep the longitudinal emittance not bigger than 1 eVs, since that is what was used for the DR injection simulation. Differences with the results from the CB eq. are due to this nonlinear matching.

Parameters	Description	SPS Inj	DR
N_B	Number Bunches	20	20
h	Harmonic Number	924	924
C [m]	Circumference	6911.6	6911.6
ℓ_{eff}	Eff. Straight Sec.	-	39%
ρ [m]	Magnetic Radius	740.0	155.6
γ_{tr}	Gamma Transition	23.00	18.6
γ	Relativistic Gamma	9.3	100.0
Q_x	Horizontal Tune	26.13	22.23
Q_y	Vertical Tune	26.18	19.16
$\langle\beta\rangle_x$ [m]	Av. x-βtron Func.	54.55	124.70
$\langle\beta\rangle_y$ [m]	Av. y-βtron Func.	54.59	160.40
$\langle D\rangle_x$ [m]	Av. Dispersion	1.83	-0.60
$\xi_{x,y}$	x,y Chromaticity	0.0	0.0
$Q_\perp$	Quality Factor	1.0	1.0
f_r [GHz]	Resonance Freq.	1.0	1.0
$R_\perp[\frac{M\Omega}{m}]$	Shunt Impedance	20.0	1.0
$\eta = \gamma_{tr}^{-2} - \gamma^{-2}$	Phase Slip Factor	-9.6e-3	2.8e-3
$T_{rev}[\mu s] = \frac{C}{\beta c}$	Revolution Time	23.19	23.06
R [m] $= C/2\pi$	Machine Radius	1100	1100

Table 2: Input parameters (some from [3] and some from updated DR design [10]) above the first line. Assumed transversal impedance parameters between the lines. Calculated parameters below the last line.

The transversal shunt impedance of SPS is about $R_\perp^{SPS}$ = 20 MΩ/m and there is not much flexibility to lower this value, so there cannot be big impact on N_b^{th} by the change of $R_\perp$, as can be seen in fig. 1**(a)**. We see that the allowed number ^{6}He ions per bunch in SPS injection is about one order of magnitude lower than required. Results shown in fig. 1 are all based on probing for instabilities with long rise time, $(1/\tau)^{th}$ = 0.2 s^{-1}.

Attempts to damp instabilities, and thereby relax N_b^{th}, have been made. Instabilities can be damped by avoiding resonances, i.e. making particles in the bunch oscillate with different frequencies. Tune spread in the bunch can be introduced by e.g. octupole magnets since they can introduce a tune dependence on the oscillation amplitude. The achieved tune spreads follow $\Delta Q_{x,y} = \frac{\partial Q_{x,x}}{\partial \varepsilon_{x,y}} a_x + \frac{\partial Q_{x,y}}{\partial \varepsilon_{y,y}} a_y$ where the single particle "action" $a_\alpha = \frac{\alpha^2 + \alpha'^2 \beta_\alpha}{\beta_\alpha}$, $\alpha = x, y$, and $\frac{\partial Q_x}{\partial \varepsilon_x}$, $\frac{\partial Q_y}{\partial \varepsilon_x}$, $\frac{\partial Q_x}{\partial \varepsilon_y}$ and $\frac{\partial Q_y}{\partial \varepsilon_y}$ are the "amplitude detuning coefficients". By changing one of the amplitude detuning coefficients, $\frac{\partial Q_y}{\partial \varepsilon_y}$, we explored the dependence on N_b^{th} the octupole magnets could have. The other coefficients are then fixed to $\partial Q_x/\partial\varepsilon_x$ = 424.9 m^{-1}, $\partial Q_{x,y}/\partial\varepsilon_{y,x}$ = −878.0 m^{-1}. These are estimations from SPS measurements [9] from where $\partial Q_y/\partial\varepsilon_y \approx$ 1155.0 m^{-1} is also concluded. A slight relaxation in N_b^{th} with amplitude detuning could be accomplished at the SPS injection, but not enough to reach required bunch intensity (compare fig. 1**(a)** and **(c)**). This is due to transversal emittance growth caused by the octupoles. This can be seen in fig. 1**(b)** where a scan over $\partial Q_y/\partial\varepsilon_y \in$ [0,2000] m^{-1} was performed. We see a growth of allowed number ions per bunch from $\partial Q_y/\partial\varepsilon_y$ = 0 to 800 m^{-1} due to growth in resonance damping when the octupole strength is increased. However, increasing $\partial Q_y/\partial\varepsilon_y$ further has no effect since then the emittance growth limit was reached which was here loosely put to 200 %.

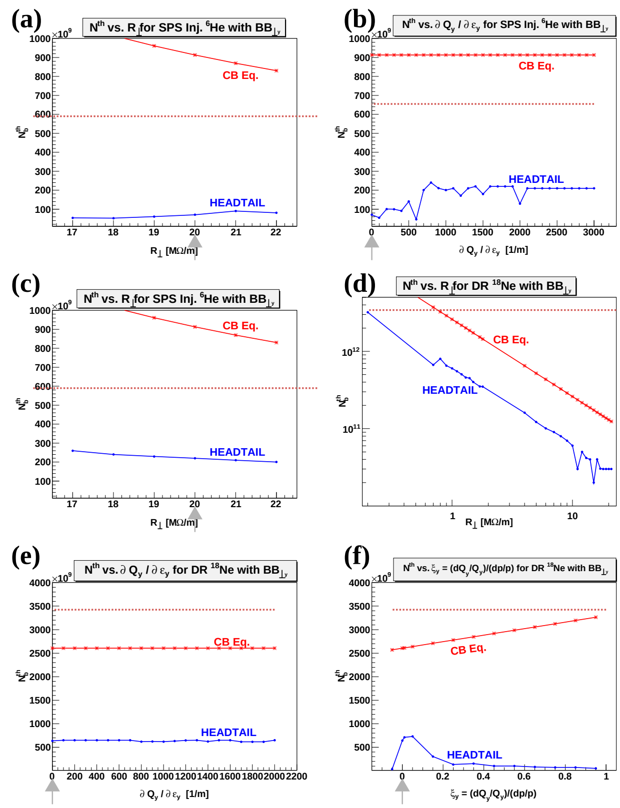

Figure 1: N_b^{th} as a function of **(a, c, d)** the transversal shunt impedance, **(b, e)** one amplitude detuning coefficient and **(f)** the chromaticity for **(a, b, c)** ^{6}He in the SPS injection and **(d, e, f)** ^{18}Ne in the DR, according to the CB eq. [8] and HEADTAIL [7] as described in [6]. The damping of the instability damping in **(b, e)** is due to a limit on the emittance blow up. The difference between **(a)** and **(b)** is amplitude detuning with $\partial Q_y/\partial\varepsilon_y$ = 1155 m^{-1}. The red dashed lines indicate required bunch intensities.

OCTUPOLES IN THE DR

The tune derivative $\partial Q_u/\partial\varepsilon_v$ is proportional to the integrate $\oint K_3 \beta_u \beta_v ds$ where $u, v = x, y$ and where K_3 is the normalized octupole strength. Three octupole families are then necessary to obtain any value of the tune derivatives, but only two will be used to match the value of the derivatives $\partial Q_x/\partial\varepsilon_x$ and $\partial Q_y/\partial\varepsilon_y$ ($\partial Q_x/\partial\varepsilon_y$ is then free). Two octupole locations were studied: in the arcs or in the long straight section. The arcs are made of regular FODO lattices. The phase advance per period is $\pi/2$ rad in both planes. The advantage of the arcs is to have phase advances for which the octupoles should compensate each other. The long straight section is made of FODO lattices as the arcs but the phase advances are not optimum. The phase advances per period are respectively 0.34π rad and 0.27π rad in both planes. However, the amplitude detuning varies as the square of the betatron functions at the octupole location. In the arcs β_x = 55 m and β_y =9.7 m at the center of the quadrupoles, whereas, in the long straight section, they are respectively 273 m and 124 m in the focusing quadrupoles and 97 m and 330 m in the defocusing quadrupoles. The betatron functions in the long straight section are then 5-6 times larger than in the arcs. Therefore, the needed octupoles should be weaker if they are located in the long straight section. If the octupoles are in the arcs, the required integrated strength for each octupole is 0.064 m^{-3} and 0.177 m^{-3}. If they are located in the long straight section, the needed integrated strengths are 0.006 m^{-3} and 0.011 m^{-3}, i.e. one order of magnitude below.

After evaluating the needed strengths of the octupoles, we have calculated the dynamic aperture at the center of the injection region for 10000 turns with SixTrack [12] for the lattice presented in[10]. The betatron functions are $\beta_x = 22$ m and $\beta_y = 7.3$ m. The used rms emittances are $\epsilon_x = 0.15\pi$ mm.mrad and $\epsilon_y = 0.08\pi$ mm.mrad. The dynamic aperture is shown in fig. 2. In both cases, the dynamic aperture is more than twice smaller but stays larger than 10 sigmas. Nevertheless, a compromise could be done by wisely choosing the derivatives of the tune.

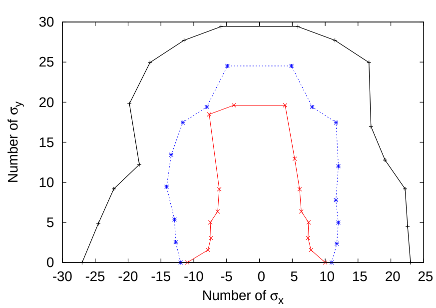

Figure 2: Comparison of the dynamic apertures at the center of the injection region for 10,000 turns in the reference case (black, plus sign), in the case of octupoles in the arc (red, cross sign) and in the case of octupoles in the long straight section (blue, star sign). The rms beam sizes are $\sigma_x = 1.8$ mm and $\sigma_y = 0.8$ mm. The amplitude detuning is $\partial Q_x/\partial\varepsilon_x = 425$ m^{-1} and $\partial Q_y/\partial\varepsilon_y = 1155$ m^{-1}.

DECAY RING SCANS

Assuming the DR's transversal shunt impedance is half of RHIC's ($R_\perp^{RHIC}$ = 2 MΩ/m [14]) since the DR will be a modern machine, i.e. $R_\perp^{DR}$ = 1 MΩ/m, HEADTAIL simulations give that N_b^{th} = 0.7·10^{11} for ^{18}Ne according to fig. 1**(d)**. Attempts to damp the instabilities that limit N_b^{th} were made with same octupole strengths as was used for SPS. Amplitude detuning does damp instabilities in the DR, however for every instability damping, there is an unacceptable transversal emittance growth of the beam. This is shown in fig. 1**(e)** where a scan over $\partial Q_y/\partial\varepsilon_y \in$ [0,2000] m^{-1} was performed but no relaxing in N_b^{th} could be claimed due to a parallel check in emittance growth. Even if as much as double transversal emittance growth was allowed the damping due to amplitude detuning had no impact.

With sextupoles it is possible to introduce a tune dependence on the momentum offset. The achieved tune spreads follow $\Delta Q_{x,y} = \xi_{x,y} Q_{x,y} \frac{\Delta p}{p}$ where $\Delta p/p$ is the momentum spread and ξ is the "chromaticity". By changing ξ we investigated if N_b^{th} could be relaxed by increasing the sextupole magnet strength. It is known that the "rigid bunch mode" (n = 0) is stable for negative (positive) ξ below (above) transition, and unstable otherwise. So since $\eta > 0$ for the DR and since n = 0 is the most crucial mode (most likely to cause beam loss) we scanned $\xi \in [0, 1]$ ($\xi \approx 1$ is a normal scale [13]). Fig. 1**(f)** shows that increasing ξ would not relax N_b^{th}. The contrast with CB eq. is due to different mode instabilities that are represented in the simple CB eq. Fig. 1**(f)** also confirms more instabilities for $\xi < 0$.

CONCLUSIONS AND OUTLOOK

Studies of damping of collective effects for the Beta Beam DR and SPS have been performed with $R_\perp^{DR} \approx$ 1 MΩ/m and $R_\perp^{SPS}$ = 20 MΩ/m. None or very little instability damping was shown trying amplitude detuning and chromaticity. The DR study indicates that there will be large challenges due to requirements of seemingly insurmountable low transverse broadband impedance. The bunch intensity limits in the SPS indicates that new solutions are necessary. New results from the T2K experiment show however indications [15] that would relax the bunch length requirements in the DR and open new opportunities for the Beta Beams.

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