INVESTIGATION OF INTRABEAM SCATTERING IN THE HEAVY ION STORAGE RING TSR

S. Artikova, M. Grieser, J. Ullrich,
Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany

Abstract

Intrabeam scattering (IBS) is a multiple scattering effect between stored beam particles. It leads to diffusion in all three spatial dimensions and thus, causes an expansion of the whole ion beam. IBS plays an important role in the equilibrium diameter of a low- velocity, electron-cooled ion beam. IBS effects for coasting and bunched $^{12}C^{6+}$ ion beams at an energy of 73.3 MeV were studied using the TSR heavy ion cooler storage ring. Experimental results of the IBS rates are presented.

INTRODUCTION

Especially in dense and low-energy ion beams intrabeam scattering is one of the dominant effects which might give an important contribution to the beam properties. IBS is a result of Coulomb collisions between the charged particles. It causes an exchange of energy between the transverse and longitudinal degree of freedom. In an earlier work by Piwinski [1] the IBS theory was developed for weak focusing machines where the lattice functions are constant. Later, the theory was extended by Martini [2] for strong focusing machines taking into account the variation of the lattice functions along the storage ring. Based on these IBS theories [1, 2] the growth times defined by:

$$\frac{1}{\tau_{x,y}} = \frac{1}{\sigma_{x,y}}\frac{d\sigma_{x,y}}{dt} \qquad \frac{1}{\tau_p} = \frac{1}{\sigma_p}\frac{d\sigma_p}{dt}, \tag{1}$$

of the assumed Gaussian transverse beam width $\sigma_{x,y}$ and momentum spread $\sigma_{\frac{\Delta p}{p}}$ can be described by a set of three coupled differential equations:

$$\frac{1}{\sigma_i}\frac{d\sigma_i}{dt} = \begin{cases} c_i \frac{Z^4}{A^2}\frac{N}{\beta^3}\frac{1}{\epsilon_x\epsilon_y\Delta p/p\cdot C} & \text{coasting beam} \\ c_i \frac{Z^4}{A^2}\frac{N}{\beta^3}\frac{1}{\epsilon_x\epsilon_y\Delta p/p l_{eff} h} & \text{bunched beam,} \end{cases} \tag{2}$$

where N is the number of ions with charge state Z, mass A, velocity β, number of bunches h, elective bunch length l_{eff} and C is the circumference of the storage ring. c_i (i=x,y,$\frac{\Delta p}{p}$) are lattice dependent functions, which weakly depend on the beam energy. For a coasting ion beam the growth rates are inversely proportional to the horizontal ϵ_x and the vertical ϵ_y emittances and the momentum spread $\Delta p/p$. In the case of bunched ion beams the growth rates also scale with the inverse effective bunch length l_{eff}.

A SIMPLE APPROACH

The IBS equations given in [1, 2] are rather complex. To simplify the IBS equations (Eq. 2), it is assumed that in the IBS process the horizontal and vertical emittances, the square of the momentum spread and the square of the bunch length are proportional to each other, so that: $\epsilon_x\epsilon_y\frac{\Delta p}{p} \propto \sigma_i^5$ for a coasting beam and $\epsilon_x\epsilon_y\frac{\Delta p}{p}l_{eff} \propto \sigma_i^6$ for bunched beams. These assumptions will decouple and simplify the system of differential equations (Eq. 2):

$$\frac{1}{\sigma_i}\frac{d\sigma_i}{dt} = \frac{D_i}{\sigma_i^\gamma}, \tag{3}$$

where γ=5 for coasting beams and γ=6 for bunched ion beam. The heating term D_i:

$$D_i \propto c_i \frac{Z^4}{A^2}\frac{1}{\beta^3}N, \tag{4}$$

leads to a growth of the emittance. The solution of Eq. 3 is given by:

$$\sigma_i(t) = (\gamma\cdot D_i\cdot t + \sigma_o^\gamma)^{\frac{1}{\gamma}}, \tag{5}$$

where σ_0 is the initial beam width at equilibrium between IBS and beam cooling. Eq. 5 is valid to explain IBS effects starting from the equilibrium state.

IBS EXPERIMENTS WITH COASTING AND BUNCHED ION BEAM

IBS experiments were conducted with $^{12}C^{6+}$ coasting and bunched ion beams at an injection energy of E=73.3 MeV in order to test the validity of Eq. 5. Electron cooling is applied to achieve an equilibrium between IBS and cooling. To investigate IBS the cooler was switched off and the development of the beam profile was recorded with a beam profile monitor. One second before switching off cooling the beam profile monitor starts recording the beam profile. At the same intensity the profile development measurements were conducted for total data taking times of 2 s and 10 s, each with 20 individual intervals. The 2 s measurement aimed to record the rapid increase of the beam size right after switching off the electron cooler. The development of the beam profile was also measured for 10 s to observe the further changes after the rapid expansion of the profile due to IBS. Because the lifetime of the ion beam is in the order of hours it is possible to continue the experiment with the same ion beam without further injections. In order to get better statistics the measurements were repeated several times at the same intensity. In the current range of 5μA to 105μA the development of the horizontal beam profile was determined as a function of time. In Fig. 1 the development of the beam width at $I = 50\mu$A for $^{12}C^{6+}$

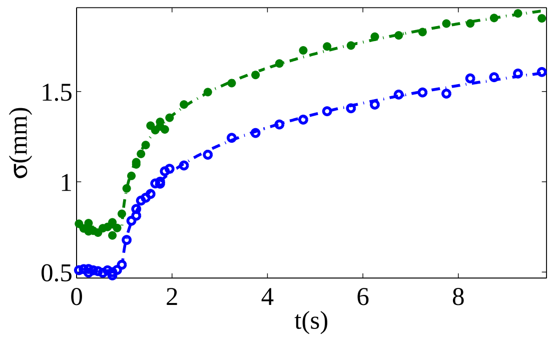

Figure 1: Measured horizontal beam width as a function of time for $^{12}C^{6+}$ coasting (blue) and bunched (green) ion beams.

ions is shown. The blue circles are the data of a coasting beam and the green dots represents the data of a bunched ion beam. The dashed lines through the data points are fits using Eq. 5, where σ_0, D_i and γ are the fit parameters. In

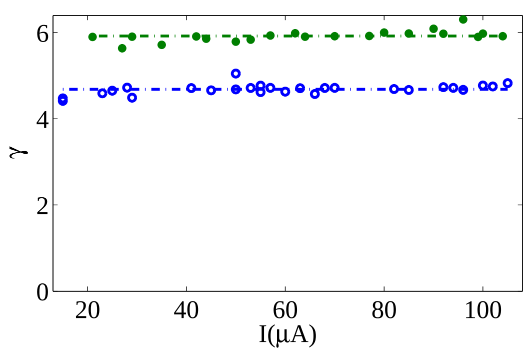

Figure 2: Measured γ parameter for $^{12}C^{6+}$ coasting (blue) and bunched (green) ion beams as a function of the beam intensity.

the same current range of 5 μA to 105 μA the γ parameter was determined for coasting and for bunched ion beams. The results are shown in Fig. 2, where the blue circles and green dots are values of the γ parameter for a coasting and for a bunched beam, respectively. For bunching a resonator voltage of 195.44 V was used. For a coasting beam $\bar{\gamma} = 4.68$ was found, which is close to $\gamma = 5$ as predicted in the simple IBS approximation. For a bunched ion beam $\bar{\gamma} = 5.9$ which closely resembles the value of γ=6. In Table 1 the average γ parameter for bunched ion beams for different resonator voltages are shown. The experimental γ parameter slightly depends on the resonator voltage. The maximum deviation to the value γ=6 was found at the lowest resonator voltage of 46.5 V. In the same intensity range the measured heating D_i terms are shown in Fig. 3 as a function of intensity for coasting and bunched ion beams. The resonator voltage was varied in the range 46.5-232.5 V. As shown in Fig. 3, the heating terms D_i increase linearly with the ion intensity I: $D_i \propto I$, where the slope of the straight lines fitted to data has the maximum value at the maximum resonator voltage due to the increased ion density. The heating term D_b of a bunched ion beam can be

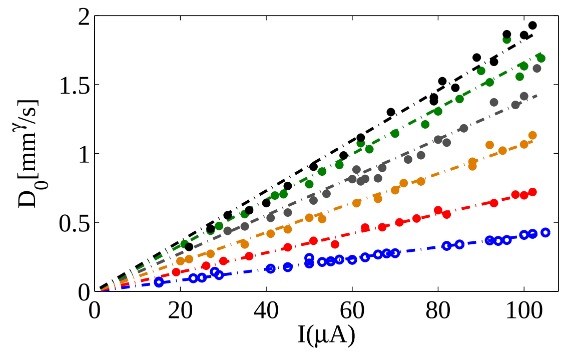

Figure 3: Measured heating terms D_i of coasting (blue) and bunched $^{12}C^{6+}$ ion beams as a function of intensity. At resonator voltages of 46.5 V (red), 93.0 V (yellow), 139.5 V (grey), 186.5 V (green) and 232.5 V (black) the heating term of bunched ion beams were measured.

calculated from the heating term D_c of a coasting beam. By re-writing the Eqs. 2 and 3 as:

$$c_i \cdot \frac{Z^4}{A^2}\frac{N}{\beta^3}\frac{1}{\epsilon_x \epsilon_y \Delta p/p \cdot C} = \frac{D_c}{\sigma^{\gamma_c}} \tag{6}$$

$$c_i \cdot \frac{Z^4}{A^2}\frac{N}{\beta^3}\frac{1}{\epsilon_x \epsilon_y \Delta p/p \cdot h \cdot l_{eff}} = \frac{D_b}{\sigma^{\gamma_b}}. \tag{7}$$

By dividing Eq. 6 with Eq. 7 the following expression is obtained:

$$\frac{h \cdot l_{eff}}{C} = \frac{D_c}{D_b}\sigma^{(\gamma_b - \gamma_c)}, \tag{8}$$

where the γ_c parameter belongs to the coasting beam and γ_b parameter belongs to the bunched ion beam. The ratio on the left side in Eq. 8 is the bunching factor B, defined by:

$$B = \frac{h \cdot l_{eff}}{C}. \tag{9}$$

In Fig. 3 it is pointed out that $D_c \propto I$ and $D_b \propto I$, therefore the bunching B factor, compare Eq. 8, should scale as:

$$B \propto \sigma^{(\gamma_b - \gamma_c)}. \tag{10}$$

In the equilibrium between electron cooling and IBS the bunch length [3] and therewith the bunching factor of an electron cooled ion beam is well known. The measured equilibrium beam width σ_0 for bunched $^{12}C^{6+}$ ion and coasting ion beams are shown in Fig. 4 as a function of intensity. The blue circles are data of coasting beam. Moreover, the equilibrium beam width increases with the intensity and resonator voltage due to the increased ion density. As it is indicated in Fig. 4, the equilibrium beam width $\sigma = \sigma_0$ of a bunched ion beam scales as $\sigma_0 \propto I^\alpha$, resulting in:

$$B \propto I^{\alpha(\gamma_b - \gamma_c)}. \tag{11}$$

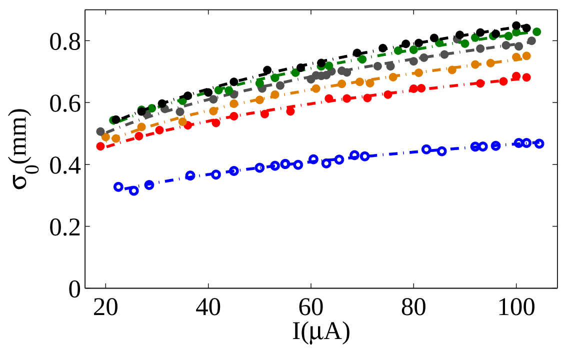

Figure 4: Measured horizontal equilibrium beam width as a function of intensity for $^{12}C^{6+}$ coasting (blue) and bunched ion beams. At resonator voltages of 46.5 V (red), 93.0 V (yellow), 139.5 V (grey), 186.5 V (green) and 232.5 V (black) the equilibrium beam widths of the bunched ion beams were measured.

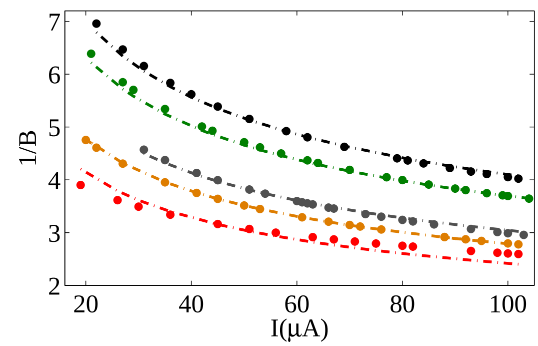

Figure 5: Inverse bunching factor for different resonator voltages as a function of the beam intensity. At resonator voltages of 46.5 V (red), 93.0 V (yellow), 139.5 V (grey), 186.5 V (green) and 232.5 V (black) the inverse bunching factor of bunched ion beams were measured.

Because the bunching factor B for cooled ion beams is proportional to the effective bunch length l_{eff} and $l_{eff} \propto I^{1/3}$[3]:

$$\frac{1}{3} = \alpha(\gamma_b - \gamma_c). \tag{12}$$

The experimental parameters α, γ_b for different resonator voltages are summarized in Table 1 for the horizontal degree of freedom. In Table 1 the experimental values of the $\alpha \cdot (\gamma_b - \gamma_c)$ term are calculated, where $\gamma_c = 4.68$. The theoretical value for the $\alpha \cdot (\gamma_b - \gamma_c)$ term is $\frac{1}{3}$, compare Eq. 12. As it is shown in Table 1 the experimental $\alpha \cdot (\gamma_b - \gamma_c)$ product is very close to the theoretical value of $\frac{1}{3}$, if a resonator voltage higher than 93 V is chosen. From the measured IBS data: D_c, D_b, γ_c, γ_b and α it is possible to calculate with Eq. 8 the bunching factor of bunched ion beams. In Fig. 5 the inverse bunching factor, derived from the experimental IBS data, for different resonator voltages and ion intensities are shown as colored dots. The bunching factor calculated from the half bunch length w [3] of an electron cooled ion beam are fitted to experimental data, shown as dashed lines. In the fit it was assumed that $l_{eff} = a \cdot w$, where a is the fit parameter. By fitting the experimental data $a = 2.45$ was found, which predicts a slightly larger effective bunch length than the maximum theoretical possible bunch length of $2 \cdot w$.

Table 1: Measured α, γ_b Parameters at Various Resonator Voltages

U[V]	α	γ_b	$\alpha \cdot (\gamma_b - \gamma_c)$
46.5	0.24	5.68	0.24
93.0	0.28	5.83	0.30
139.5	0.28	5.90	0.32
186.0	0.28	5.90	0.32
232.5	0.30	5.84	0.33

CONCLUSION

The purpose of the work was obtaining a simple, decoupled equations from IBS theories [1, 2] to explain intrabeam scattering between stored beam particles. The expansion of the beam width starting from equilibrium state of bunched and coasting $^{12}C^{6+}$ ion beams were measured with beam profile monitor at different intensities. The approximated equations for IBS are turned out to fit the experimental data quite well. The results of experimental measure of the parameters for bunched and coasting ion beams corresponds with the theoretical values. It was found for the equilibrium state between electron cooling and intrabeam scattering that the equilibrium beam width for bunched ion beams scales as $\sigma_0 \propto I^{1/3}$, which is in agreement with the measurements if a resonator voltage larger than 93 V. From the experimental IBS data of bunched ion beams it was possible to calculate the bunching factors as a function of intensity and resonator voltage. Equation 5 is applicable to describe IBS for different energies if IBS starts from the equilibrium state of the cooled ion beam.

REFERENCES

[1] A. Piwinski Pros. 9th Int. Conference on High Energy Acc., Stanford p.405(1974).

[2] M. Martini, CERN PS/84-9(AA) (1984).

[3] M. Grieser, R. Bastert, K. Blaum, H. Buhr, R. von Hahn, M. B. Mendes, R. Repnow, A. Wolf. Acceleration, deceleration and bunching of stored and cooled ion beams at the TSR, Heidelberg HIAT 09 Proceedings, Venice, June 2009.