# LHC OPTICS DETERMINATION WITH PROTON TRACKS MEASURED IN THE ROMAN POT DETECTORS OF THE TOTEM EXPERIMENT 

F. Nemes, Eötvös University, Budapest, Hungary<br>H. Niewiadomski, H. Burkhardt, CERN, Geneva, Switzerland<br>on behalf of the TOTEM collaboration

## Abstract

The TOTEM experiment at the LHC is equipped with near beam movable devices - called Roman Pots (RP) which detect protons scattered at the interaction point (IP5) $\dot{2}$ arriving to the detectors through the magnet lattice of the LHC. Proton kinematics at IP5 is reconstructed from positions and angles measured by the RP detectors, on the basis of the transport matrix between IP5 and the RP locations. The precision of optics determination is therefore of the key importance for the experiment. TOTEM developed a novel method of machine optics determination making use of angle-position distributions of elastically scattered protons observed in the RP detectors together with the data retrieved from several machine databases. The method has been successfully applied to the data samples registered in 2010 and 2011. The studies show that the transport matrix could be estimated with a precision better than $1 \%$.

## THE ROMAN POTS OF THE TOTEM EXPERIMENT

Proton-proton elastic scattering was measured by the TOTEM experiment at the CERN Large Hadron Collider at $\sqrt{s}=7 \mathrm{TeV}$ in dedicated runs [1,2]. To detect leading protons scattered at angles as small as $1 \mu \mathrm{rad}$, silicon sensors are placed in movable beam-pipe insertions, so-called Roman Pots (RP), located symmetrically on either side of the LHC intersection point IP5 at distances up to 220 m from it. Each RP station is composed of two units separated by a distance of about 5 m . A unit consists of 3 RPs, two approaching the outgoing beam vertically and one horizontally, allowing for a partial overlap between horizontal and vertical detectors and the alignment precision of $10 \mu \mathrm{~m}$.

## PROTON TRANSPORT FROM IP5 TO THE ROMAN POTS

Scattered protons are detected in the Roman Pots after having moved through a segment of the LHC lattice containing 29 magnets per beam. The trajectory of protons with transverse positions ${ }^{1}\left(x^{*}, y^{*}\right)$ and angles $\left(\Theta_{x}^{*}, \Theta_{y}^{*}\right)$ at IP5 are described with a linear formula

$$
\begin{equation*}
\vec{d}=T \cdot \overrightarrow{d^{*}}, \tag{1}
\end{equation*}
$$

where $\vec{d}=\left\{x, \Theta_{x}, y, \Theta_{y}, \Delta p / p\right\}^{T}$ with nominal beam momentum $p$ and momentum loss $\Delta p$. The transport matrix $T$

[^0]ISBN 978-3-95450-115-1
is defined by the optical functions

$$
T=\left(\begin{array}{ccccc}
v_{x} & L_{x} & m_{13} & m_{14} & D_{x}  \tag{2}\\
v_{x}^{\prime} & L_{x}^{\prime} & m_{23} & m_{24} & D_{x}^{\prime} \\
m_{31} & m_{32} & v_{y} & L_{y} & D_{y} \\
m_{41} & m_{42} & v_{y}^{\prime} & L_{y}^{\prime} & D_{y}^{\prime} \\
0 & 0 & 0 & 0 & 1
\end{array}\right) .
$$

The magnification $v_{x, y}=\sqrt{\beta_{x, y} / \beta^{*}} \cos \Delta \phi_{x, y}$ and the effective length $L_{x, y}=\sqrt{\beta_{x, y} \beta^{*}} \sin \Delta \phi_{x, y}$ are functions of the betatron amplitude $\beta_{x, y}$ and the relative phase advance $\Delta \phi_{x, y}=\int_{\mathrm{IP}}^{\mathrm{RP}} \beta(s)_{x, y}^{-1} d s$ and are particularly important for the proton kinematics reconstruction. The coupling coefficients $m_{i, j}$ are close to 0 and the vertex contributions are canceled due to the anti-symmetry of the scattering angles. Therefore, the kinematics of elastically scattered protons at IP5 can be reconstructed from Equation (1) as:

$$
\begin{equation*}
\Theta_{y}^{*} \approx \frac{y_{\mathrm{RP}}}{L_{y, \mathrm{RP}}} \quad \Theta_{x}^{*} \approx \frac{1}{\frac{d L_{x, \mathrm{RP}}}{d s}}\left(\Theta_{x, \mathrm{RP}}-\frac{d v_{x, \mathrm{RP}}}{d s} x^{*}\right), \tag{3}
\end{equation*}
$$

where "RP" defines the measurement location. As the values of the reconstructed angles are directly inversely proportional to the optical functions, the accuracy of optics defines the systematic errors of the final physics results.

The proton transport matrix $T(s ; \mathcal{M})$ over a distance of $s$ is defined by the machine settings $\mathcal{M}$. It is calculated with the MAD-X [3] code for each group of runs with identical optics based on several data sources. The magnet currents are retrieved from TIMBER [4] and are converted to strengths with LSA [5], which implements the conversion curves measured by FIDEL [6]. The WISE database [7] contains the measured imperfections (field harmonics, magnet displacements and rotations).
However, the lattice is subject to additional $\Delta \mathcal{M}$ imperfections, not measured well enough so far, which alter the transport matrix by $\Delta T$ :

$$
T(s ; \mathcal{M}) \rightarrow T(s ; \mathcal{M}+\Delta \mathcal{M})=T(s ; \mathcal{M})+\Delta T
$$

The $5-10 \%$ precision of $\Delta \beta / \beta$ beating measurement does not allow to estimate $\Delta T$ with the accuracy required by the TOTEM physics program. However, the magnitude of $|\Delta T|$ can be evaluated from the tolerances of the LHC imperfections of which the most important are:

- Strength conversion error $I \rightarrow B, \sigma(B) / B \approx 10^{-3}$
- Beam momentum offset $\sigma(p) / p \approx 10^{-3}$.

Their impact on optical functions is presented in Table 1. It is clearly visible that the imperfections of the inner triplet
(the MQXA and MQXB magnets) are of high influence on the transport matrix while the optics is less sensitive to the quadrupoles MQY and MQML.

Other imperfections are of lower significance:

- Magnet rotations $\sigma(\phi) \approx 1 \mathrm{mrad}$
- Beam harmonics $\sigma(B) / B \approx 10^{-4}$
- Power converter errors $\sigma(I) / I \approx 10^{-4}$
- Magnet positions $\Delta x, \Delta y \approx 100 \mu \mathrm{~m}$.

Generally, as can be seen in Table 1, for large $\beta^{*}$ optics the magnitude of $\Delta T$ is sufficiently small from the viewpoint of data analysis and therefore $\Delta T$ does not need to be precisely estimated. However, the low $\beta^{*}$ optics sensitivity to the machine imperfections is significant and cannot be neglected. Fortunately, in this case $\Delta T$ can be determined precisely enough from the proton tracks in the Roman Pots.

Table 1: Sensitivity of the vertical effective length $L_{y}$ to magnet strengths and beam momentum perturbed by $1 \%$ for low- and large- $\beta^{*}$ optics.

|  | $\delta \mathbf{L}_{\mathbf{y}} / \mathbf{L}_{\mathbf{y}}[\mathbf{\%}]$ |  |
| :---: | :---: | :---: |
| Perturbed element | $\boldsymbol{\beta}^{*}=\mathbf{3 . 5} \mathbf{~ m}$ | $\boldsymbol{\beta}^{*}=\mathbf{9 0} \mathbf{~ m}$ |
| MQXA.1R5 | 0.98 | 0.14 |
| MQXB.A2R5 | -2.24 | -0.23 |
| MQXB.B2R5 | -2.42 | -0.25 |
| MQXA.3R5 | 1.45 | 0.20 |
| MQY.4R5.B1 | -0.10 | -0.01 |
| MQML.5R5.B1 | 0.05 | 0.04 |
| $\Delta \mathrm{p} / \mathrm{p}$ | -2.19 | 0.01 |

## CONSTRAINTS FROM PROTON TRACKS IN THE ROMAN POTS

The elements of the transport matrix are functions of the betatron amplitudes $\beta_{x, y}$ and the phase advances $\phi_{x, y}$. Therefore they are mutually related. Moreover, the elastic scattering ensures that the scattering angles in both arms are identical:

$$
\begin{equation*}
\Theta_{x, b_{1}}^{*}=\Theta_{x, b_{2}}^{*}, \Theta_{y, b_{1}}^{*}=\Theta_{y, b_{2}}^{*} \tag{4}
\end{equation*}
$$

which allows to compute ratios between the effective lengths of the two beams. From Equation (1) we get:

$$
\begin{gather*}
R_{1} \equiv \frac{\Theta_{x, b_{1}, \mathrm{RP}}}{\Theta_{x, b_{2}, \mathrm{RP}}} \approx \frac{\frac{d L_{x, b_{1}, \mathrm{RP}}}{d s} \Theta_{x, b_{1}}^{*}}{\frac{d L_{x, b_{2}, \mathrm{RP}}}{d s} \Theta_{x, b_{2}}^{*}}=\frac{\frac{d L_{x, b_{1}, \mathrm{RP}}}{d s}}{\frac{d L_{x, b_{2}, \mathrm{RP}}}{d s}},  \tag{5}\\
R_{2} \equiv \frac{y_{b_{1}, \mathrm{RP}}}{y_{b_{2}, \mathrm{RP}}} \approx \frac{L_{y, b_{1}, \mathrm{RP}}}{L_{y, b_{2}, \mathrm{RP}}} \tag{6}
\end{gather*}
$$

where $b_{1}$ and $b_{2}$ indicate beam 1 and beam 2. The ratios $R_{1}$ and $R_{2}$ can be estimated with a $0.5 \%$ precision.

Furthermore, the distributions of proton angles and positions detected in Roman Pots define ratios of certain elements of the transport matrix $T$. First of all, $d L_{y} / d s$ and
$L_{y}$ are related by

$$
\begin{equation*}
R_{3} \equiv \frac{\Theta_{y, b_{1}}}{y_{b_{1}}} \approx \frac{\frac{d L_{y, b_{1}}}{d s}}{L_{y, b_{1}}}, R_{4} \equiv \frac{\Theta_{y, b_{2}}}{y_{b_{2}}} \approx \frac{\frac{d L_{y, b_{2}}}{d s}}{L_{y, b_{2}}} \tag{7}
\end{equation*}
$$

with a $0.5 \%$ precision.
Similarly, we exploit the horizontal distributions to quantify the relation between $d L_{x} / d s$ and $L_{x}$. Contrary to the previous case, $L_{x}$ is close to 0 and instead of defining the ratio we rather estimate the position $s$ (with the precision of about 1 m ) along the beam where $L_{x}$ equals to 0 by solving

$$
\begin{equation*}
\frac{L_{x}(s)}{d L_{x}\left(s_{1}\right) / d s}=\frac{L_{x}\left(s_{1}\right)}{d L_{x}\left(s_{1}\right) / d s}+\left(s-s_{1}\right)=0 \tag{8}
\end{equation*}
$$

where $s_{1}$ is the beginning of the Roman Pot station. The ratio $\frac{d L_{x}\left(s_{1}\right)}{d s} / L_{x}\left(s_{1}\right)$ is defined by the proton distributions

$$
\begin{equation*}
R_{5} \equiv \frac{x_{b_{1}}}{\Theta_{x, b_{1}}} \approx \frac{L_{x, b_{1}}}{\frac{d L_{x, b_{1}}}{d s}}, R_{6} \equiv \frac{x_{b_{2}}}{\Theta_{x, b_{2}}} \approx \frac{L_{x, b_{2}}}{\frac{d L_{x, b_{2}}}{d s}} \tag{9}
\end{equation*}
$$

Finally, tracks determine as well the coupling components of $T$. Due to $L_{x} \approx 0$ at the Roman Pot locations, the further four constraints can be defined

$$
\begin{equation*}
R_{7} \equiv \frac{x_{b_{1}, \text { near pots }}}{y_{b_{1}, \text { near pots }}} \approx \frac{m_{14, b_{1}, \text { near pots }}}{L_{y, b_{1}, \text { near pots }}}, \tag{10}
\end{equation*}
$$

$R_{8}$ is defined with the far pots, and $R_{9,10}$ respectively for beam 2. These four constraints can be estimated with a 3\% accuracy.

## OPTICS MATCHING

On the basis of the constraints $R_{1} \ldots R_{10}, \Delta T$ can be determined with the $\chi^{2}$ minimization procedure. The relevant lattice imperfections were selected forming a 26 dimensional optimization phase space, which includes the magnet strengths, rotations and beam momenta. Due to the high dimensionality of the phase space and approximately linear structure of the problem there is no unique solution. Therefore, the optimization is subject to additional constraints defined by the machine tolerances. Finally, the $\chi^{2}$ is composed of the part defined by the values measured with the Roman Pots (discussed in the previous section) and such reflecting the LHC tolerances:

$$
\begin{equation*}
\chi^{2}=\chi_{\text {Measured }}^{2}+\chi_{\text {Design }}^{2} \tag{11}
\end{equation*}
$$

where the design part

$$
\begin{aligned}
\chi_{\text {Design }}^{2}= & \sum_{i=1}^{12}\left(\frac{k_{i}-k_{i, \mathrm{MADX}}}{\sigma\left(k_{i}\right)}\right)^{2}+\sum_{i=1}^{12}\left(\frac{\phi_{i}-\phi_{i, \mathrm{MADX}}}{\sigma\left(\phi_{i}\right)}\right)^{2}+ \\
& \sum_{i=1}^{2}\left(\frac{p_{i}-p_{i, \mathrm{MADX}}}{\sigma\left(p_{i}\right)}\right)^{2}
\end{aligned}
$$

defines the nominal machine as an attractor in the phase space, and the measured part

$$
\begin{equation*}
\chi_{\text {Measured }}^{2}=\sum_{i=1}^{10}\left(\frac{R_{i}-R_{i, \mathrm{MADX}}}{\sigma\left(R_{i}\right)}\right)^{2} \tag{12}
\end{equation*}
$$

ISBN 978-3-95450-115-1
contains the track based constraints $R_{1} \ldots R_{10}$ together with their errors. The subscript "MADX" defines the parameter optimized with the MAD-X software.
Table 2 presents the results of the optimization procedure for $\beta^{*}=3.5 \mathrm{~m}$. The obtained value of the effective length $L_{y}$ of beam 1 is close to the nominal one, while beam 2 shows a significant change. The same pattern applies to the values of $d L_{x} / d s$.

Table 2: Selected optical functions of both LHC beams obtained with the matching procedure compared to their nominal values for $\beta^{*}=3.5 \mathrm{~m}$.

|  | $\mathbf{L}_{\mathbf{y}, \mathbf{b}_{\mathbf{1}}}[\mathrm{m}]$ | $\mathbf{d L}_{\mathbf{x}, \mathbf{b}_{\mathbf{1}}} / \mathbf{d s}$ | $\mathbf{L}_{\mathbf{y}, \mathbf{b}_{\mathbf{2}}}[\mathbf{m}]$ | $\mathbf{d L}_{\mathbf{x}, \mathbf{b}_{\mathbf{2}}} / \mathbf{d s}$ |
| :--- | :---: | :---: | :---: | :---: |
| Nominal | 22.4 | $-3.21 \cdot 10^{-1}$ | 18.4 | $-3.29 \cdot 10^{-1}$ |
| Matched | 22.6 | $-3.12 \cdot 10^{-1}$ | 20.7 | $-3.15 \cdot 10^{-1}$ |

## MONTE-CARLO VALIDATION

The procedure has been extensively verified with Monte Carlo studies. The nominal machine settings were perturbed in order to simulate the LHC imperfections and the simulated proton tracks were used afterwards to calculate the optimization constraints $R_{1} \ldots R_{10}$. The study included the impact of

- magnet strengths
- beam momenta
- displacements, rotations
- kickers, harmonics
- elastic scattering $\Theta$-distributions

The results obtained for the $\beta^{*}=3.5 \mathrm{~m}$ study are summarized in Figures 1 and 2 and their statistical description is given in Table 3. The distributions of optical functions' errors indicate that the optical functions can be reconstructed with a precision of $0.2 \%$, which confirms the validity of the proposed approach.

Table 3: Monte-Carlo validation results of Roman Pot track based optics estimation. The machine imperfections induce large spread of optical functions. The matching procedure estimates the optics with errors lower than $2.1 \%$.

|  | Machine with imperfections |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | before |  | after matching |  |
| $\begin{array}{\|c\|} \hline \text { Optical function } \\ \text { relative error } \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \text { Mean } \\ {[\%]} \\ \hline \end{array}$ | $\begin{gathered} \mathbf{R M S} \\ {[\%]} \end{gathered}$ | $\begin{gathered} \hline \text { Mean } \\ {[\%]} \end{gathered}$ | $\begin{gathered} \hline \text { RMS } \\ {[\%]} \end{gathered}$ |
| $\frac{\delta L_{y, b_{1}}}{L_{y} b_{1}}$ | 0.77 | 3.0 | $5.7 \cdot 10^{-3}$ | $9.9 \cdot 10^{-2}$ |
| $\frac{\delta d L_{x, b_{1}} / d s}{d L_{x, b_{1}} / d s}$ | 1.0 | 1.1 | $-1.2 \cdot 10^{-1}$ | 2. $2.1 \cdot 10^{-1}$ |
| $\frac{\delta L_{y, b_{2}}}{L}$ | 2.0 | 3.8 | $1.5 \cdot 10^{-1}$ | $9.5 \cdot 10^{-2}$ |
| $\begin{gathered} \overline{L_{y, b_{2}}} \\ \delta \frac{\delta d L_{x, b_{2}} / d s}{} \end{gathered}$ | -1.14 | 1.2 | -7.6 $10^{-2}$ | $2.1 \cdot 10^{-1}$ |
| $\overline{d L_{x, b_{2} / d s}}$ | -1.14 | 1.2 | $-7.6 \cdot 10$ | $2.1 \cdot 10$ |



Figure 1: Relative error distribution of $L_{y}$ for beam 1 before and after matching.


Figure 2: Relative error distribution of $d L_{x} / d s$ for beam 1 before and after matching.

## CONCLUSIONS AND OUTLOOK

TOTEM proposed a novel approach to optics estimation. First of all, the method allows to asses the optical functions' errors from machine tolerances. Secondly, it allows to determine the real optics solely from the Roman Pot proton tracks. The method has been validated with the Monte Carlo studies both for large- and low- $\beta^{*}$ optics. With its application TOTEM has published elastic scattering distributions obtained with different running conditions. It is foreseen to extend the proposed approach to model the transport of protons with large momentum loss.

## REFERENCES

[1] G. Antchev et al., [TOTEM Collaboration], Europhys. Lett. 95 (2011) 41001 [arXiv:1110.1385 [hep-ex]].
[2] G. Antchev et al., Europhys. Lett. 96 (2011) 21002 [arXiv:1110.1395 [hep-ex]].
[3] Title MAD-X : An Upgrade from MAD8, CERN-AB-2003-024-ABP.
[4] The LHC Logging Service, CERN-AB-Note-2006-046.
[5] The LSA Database to Drive the Accelerator Settings, CERN-ATS-2009-100.
[6] FIDEL - The Field Description for the LHC, LHC-C-ES0012 ver.2.0.
[7] WISE: A Simulation of the LHC Optics Including Magnet Geometrical Data, Proc. of EPAC08, Genoa, Italy.


[^0]:    ${ }^{1}$ The $*$ superscript indicates the LHC Interaction Point 5

