

# MODELLING OF THE EMMA ns-FFAG RING USING GPT

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## Abstract

EMMA (Electron Machine with Many Applications) is a prototype non-scaling Fixed-Field Alternating Gradient (ns-FFAG) accelerator whose construction at Daresbury Laboratory, UK, was completed in the autumn of 2010. The energy recovery linac ALICE [1] will serve as an injector for EMMA, within an energy range of 10 to 20 MeV. The injection line consists of a symmetric 30° dogleg to extract the beam from ALICE, a matching section and a tomography section for transverse emittance measurements. This is followed by a transport section to the injection point of the EMMA ring. The ring is composed of 42 cells, each containing one focusing and one defocusing quadrupole. Commissioning of the EMMA ring started in late 2010.

A number of different injection energy and bunch charge configurations are planned; for some the effects of space-charge may be significant. It is therefore necessary to model the electron beam transport in the injection line and ring using a code capable of both calculating the effect of and compensating for space-charge. Therefore the General Particle Tracer (GPT) code [2] has been used. A range of injection beam parameters have been modelled for comparison with experimental results.

## INTRODUCTION

Commissioning of the EMMA ring (the world's first ns-FFAG) commenced in Autumn of 2010. The energy recovery linac ALICE acts as the injector for EMMA, providing single bunches of electrons at an energy between 10 to 20 MeV with a maximum bunch charge of 32 pC. A schematic of the EMMA ring is shown in Fig. 1, highlighting some of the important diagnostic components of the beamline.

The beam injected from ALICE into EMMA is at quite a low energy there may be significant space-charge emittance growth, depending on the input conditions. Therefore rigorous analysis of the effects of space-charge is necessary. Consequently the particle tracking software used must incorporate the ability to model the effects of space-charge; thus GPT was chosen. This paper is an account of the GPT modelling of the EMMA ring using and including the effect of space-charge, for a range of injection parameters.

## THE EMMA RING

### Modelling in MAD and GPT

MAD [3] (Methodical Accelerator Design) is an analytical program designed to model accelerators using a transfer

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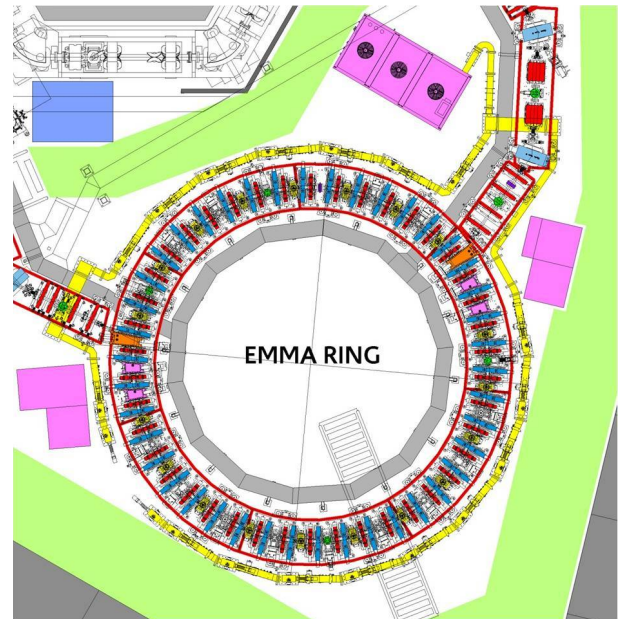


Figure 1: The EMMA ring, with a circumference of 16.57 m

matrix method and, as such, ignores space-charge. The initial modelling of the EMMA ring was conducted using MAD, with the beta functions obtained in both transverse planes in the first magnetic cell downstream of the EMMA injection septum. A schematic of such a magnetic cell can be seen in Fig. 2.

One revolution of the ring was then modelled in GPT for comparison with MAD, transporting a beam of 15 MeV (as well as a nominal bunch charge of 0 pC) with the same initial transverse beam parameters as was used for the MAD modelling. In addition the bunch was initiated with a uniform energy and finite length for ease of computation. The 42 magnetic cells of the ring were modelled in GPT using the repeated input of a single field-map of one magnetic cell. This magnetic field was then held constant, allowing for optimisation of the initial beam parameters to achieve a closed orbit solution. The beam is expected to complete several revolutions of the machine. The constraints, therefore, are required to be extremely specific across one full turn in order to minimise an increase in beam-size across many turns.

Fig. 3 shows  $\beta_{x,y}$  for the first magnetic cell of EMMA, as produced by GPT (without space-charge). The twiss pa-

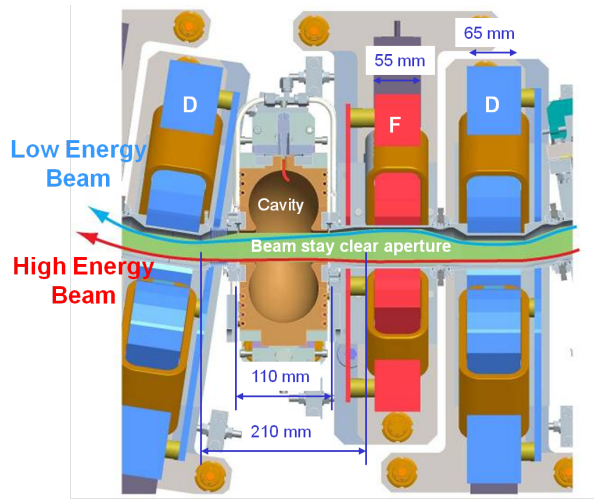


Figure 2: A schematic of a few magnetic cells, which comprise the EMMA ring.

Parameters are matched in GPT such that they become identical at the beginning and end of one complete turn of the machine, as in [4]. Once this matching is complete the beam may be propagated across multiple turns, similar to that of an experimental run.

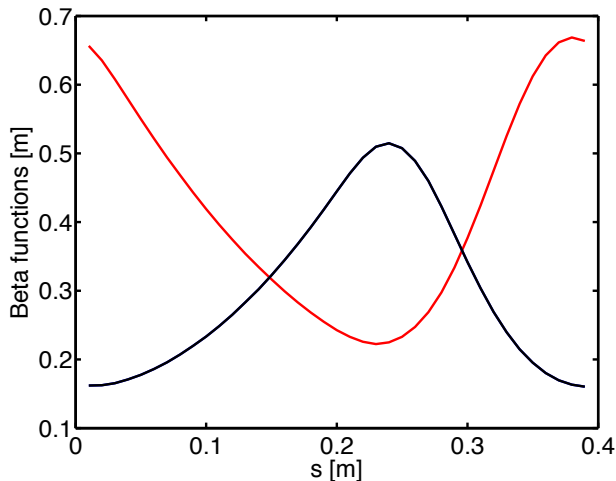


Figure 3: Beta functions,  $\beta_{x,y}$ , for the first magnetic cell of the EMMA ring, as modelled by GPT.

Fig. 4 shows the beta functions and normalised emittances for ten turns of the ring. Both values are recorded at the end of each cell and should remain constant if correctly matched, demonstrating no beam blowup. This is true for the normalised emittance, whereas the beta functions oscillate over a small range. This is due to a combination of low statistics - the betas are calculated by an average across all particles - and an insufficient beam-matching accuracy. These are acknowledged concessions in order to mitigate lengthy simulation times and excessive CPU usage.

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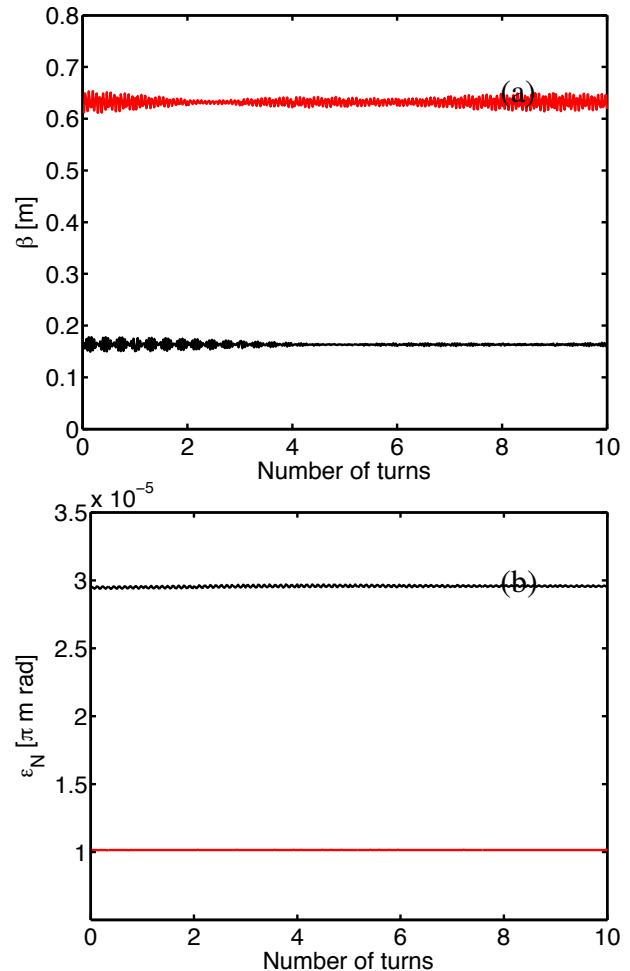


Figure 4: a)  $\beta_{x,y}$ , and b)  $\epsilon_N$  for 100 turns of the ring in both the  $x$ -plane (black) and  $y$ -plane (red). The beam parameters are outputted at the end of each revolution.

### Space-charge in GPT

The optimisation of beam parameters initially performed in GPT was for a bunch charge of 0 pC with the full 3D space-charge function enabled. Within a drift-length, space-charge is locally equivalent to a quadrupole field, defocusing in both the  $x$ - and  $y$ -planes. Thus space-charge directly affects the beam-size such that the transverse beam emittance evolves according to

$$\epsilon_x = \epsilon_{x0} \sqrt{1 + \beta_{x0}^2 \left[ \left\langle \frac{1}{F_x^2} \right\rangle - \left\langle \frac{1}{F_x} \right\rangle^2 \right]}, \quad (1)$$

(for a Gaussian beam only) where  $\epsilon_{x0}$  and  $\beta_{x0}$  are the initial emittance and beta function respectively, and  $F_x$  is the focal length of the bunch. A similar expression describes the vertical plane [5]. As the EMMA ring consists of drift lengths and quadrupole doublets this approximation may not hold for the entire ring; another reason why space-charge calculations are crucial to an accurate model.

The results of Fig. 4 were then recalculated for a dif-

ferent bunch charge at the extreme end of the range that may be injected into EMMA from ALICE: 80 pC. The initial beam parameters of the model (without space-charge) have not been re-optimised for each bunch charge in order to properly demonstrate its effect.

The results displayed in Fig. 5 are again for the first 10 turns of the ring. It can be seen that increasing the bunch charge corresponds with an emittance growth in both transverse planes, however the effects are starkly different. The  $y$ -plane shows results dictated by the relation in Eq. 1, such that the introduction of a non-zero bunch charge increases emittance. The emittance then plateaus after approx. three turns.

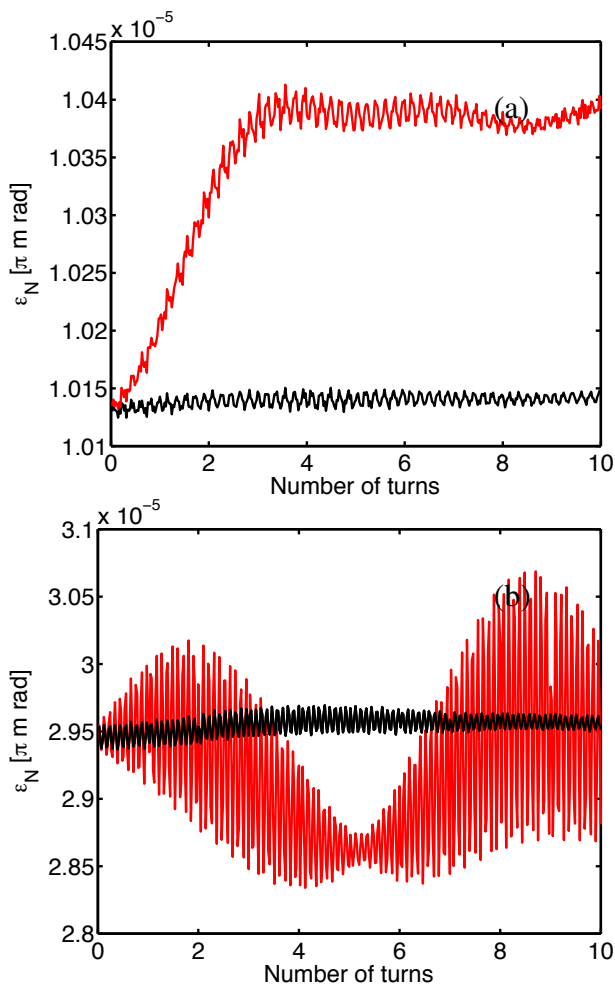


Figure 5: Normalised emittance in both the a)  $y$ -plane, and b)  $x$ -plane for 10 turns of the machine, with two space-charge regimes; 0pC (black) and 80pC (red).

Fig. 5 also shows both an increase and decrease in the  $x$ -emittance after 10 turns. This blow up then oscillates over further turns, growing seemingly larger. This may be due to a slight mismatch in the initial beam parameters exacerbated when the bunch charge is increased. This would require a rematch with a higher level of accuracy. Another cause for this behaviour may be the difference in the mag-

netic effects between the planes; the beam is set off axis in the  $x$ -plane so as to bend the beam without the need for dipoles. The  $x$ -plane therefore sees a different magnetic field to that of the  $y$ -plane.

In this analysis the beam is given a Gaussian one sigma bunch length of  $\sim 4$  ps, however a longer bunch length would reduce space-charge effects as the total charge of the bunch would be extended over a larger physical volume. Similarly the beam substantiated in the GPT model has an energy of 10 MeV - in the middle of the injection energy range - where the effects of space-charge will be more profound than at higher energies.

### FUTURE WORK

The next step is to repeat the analysis over the full space-charge range. If the impact of space-charge on beam size is as slight as demonstrated in this paper then minimal-to-no adjustments will be made to the quadrupole strengths. It would then be prudent to look at these effects at up to 1000 turns to see whether the emittance growth remains plateaued in the  $y$ -plane and continues to oscillate in the  $x$ .

Installation of the EMMA ring and injection/extraction line is now complete. Comprehensive data has already been taken with an overview of the results shown in [7]. Further work also includes time of flight and tune analysis. Data from the entire machine will then be compared to further GPT simulations.

### CONCLUSIONS

The modelling of the EMMA ring in GPT has been completed, with the effect of space-charge on beam parameters at 15 MeV demonstrated to be small, yet apparent. Further work, such as rematching the initial parameters in GPT to attempt to eliminate the effect of space-charge and the evolution of the beam over many more turns, has to be verified.

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