

SURFACE WAVES FOR TESTING OF BEAM INSTRUMENTATION

F. Stulle*, J. Bergoz, Bergoz Instrumentation, Saint Genis-Pouilly, France

Abstract

The fundamental TM wave can be guided as a surface wave along a single dielectric coated wire. Such a setup is known as a Goubau line. Close to the wire this TM wave resembles closely the radial electric and azimuthal magnetic fields of a charged particle beam moving in an accelerator. Hence, it can be used to test beam instrumentation in the workshop. We introduce the principle, discuss benefits, and compare measurements of a beam instrumentation device performed with a Goubau line to measurements performed with a standard bench testing setup.

INTRODUCTION

A novel technique for bench testing of beam instrumentation has been proposed and successfully applied to BPMs some time ago by J. Musson et al. [1, 2]. The idea is to excite an electromagnetic surface wave which is guided by a single wire surrounded by air without the need for an outer conductor. This wave consists of a fundamental TM mode closely resembling the electromagnetic fields around a beam of charged particles. Such a setup is known as Goubau line and has found previous applications in signal transmission [3].

Recently, we have setup a Goubau line at Bergoz Instrumentation using cones provided by J. Musson, JLab. First measurements have been presented at BIW12 [4]. In this paper we focus on a description of the theoretical basis.

ELECTROMAGNETIC SURFACE WAVES

A first evaluation of electromagnetic surface waves traveling along wires of finite conductivity has been performed more than a century ago by A. Sommerfeld [5].

Starting from Faraday's law of induction and Ampère's law,

$$-\frac{\partial \vec{B}}{\partial t} = \nabla \times \vec{E} \quad \text{and} \quad \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B},$$

he made only two assumptions. The first is azimuthal symmetry $\frac{\partial}{\partial \phi} = 0$, which means the evaluation is valid only for straight and round wires. The second is the limitation to TM modes, i.e. $E_\phi = 0$, which is acceptable since one expects a general solution that contains the solution for a perfect wire, i.e. a TEM mode with radial electric and azimuthal magnetic field, as a special case. In [6] Hondros has shown that indeed this assumption is justified since only the fundamental TM mode can travel with low losses along a single wire while higher order TM and all TE modes are strongly damped.

*stulle@bergoz.com

What remains are differential equations for the radial electric, the azimuthal magnetic and the longitudinal electric field components:

$$\begin{aligned} \mu \sigma E_r + \mu \epsilon \frac{\partial E_r}{\partial t} &= -\frac{\partial B_\phi}{\partial z} \\ \frac{\partial B_\phi}{\partial t} &= \frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} \\ \mu \sigma E_z + \mu \epsilon \frac{\partial E_z}{\partial t} &= \frac{1}{r} \frac{\partial r B_\phi}{\partial r} \end{aligned} \quad (1)$$

with $\vec{J} = \sigma \vec{E}$, i.e. Ohm's law for linear, isotropic media.

Note that the longitudinal component is not assumed to vanish on the wire surface since this would only be justified on the surface of a perfect wire. As Sommerfeld has shown imperfections allow certain phenomena which are not present in perfect cases.

Introducing a harmonic wave of propagation constant h traveling in z -direction, i.e

$$\begin{aligned} E_r &= E_{r0} e^{i(\omega t - h z)} \\ B_\phi &= B_{\phi 0} e^{i(\omega t - h z)} \\ E_z &= E_{z0} e^{i(\omega t - h z)}, \end{aligned}$$

into Eqns. (1) leads to a 0th-order Bessel differential equation

$$E_{z0} = r^2 \frac{\partial^2 E_{z0}}{\partial r^2} + r \frac{\partial E_{z0}}{\partial r} + r^2 \gamma^2 E_{z0} \quad (2)$$

and the relations $E_{r0} = \frac{h}{i\gamma^2} \frac{\partial E_{z0}}{\partial r}$, $B_{\phi 0} = \frac{k^2}{i\omega\gamma^2} \frac{\partial E_{z0}}{\partial r}$ with $\gamma^2 = k^2 - h^2$ and $k^2 = \omega^2 \mu \epsilon - i\omega \mu \sigma$. k is the propagation constant of a free wave in a medium with permeability μ , permittivity ϵ and conductivity σ .

The solution of Eqn. (2) can be any linear combination Z_0 of the 0th-order Bessel functions with amplitude factor A and argument γr , i.e. $A Z_0(\gamma r)$:

$$\begin{aligned} E_r &= \frac{iAh}{\gamma} Z_1(\gamma r) e^{i(\omega t - h z)} \\ B_\phi &= \frac{iAk^2}{\omega\gamma} Z_1(\gamma r) e^{i(\omega t - h z)} \\ E_z &= A Z_0(\gamma r) e^{i(\omega t - h z)} \end{aligned} \quad (3)$$

since $\frac{\partial Z_0(\gamma r)}{\partial r} = -\gamma Z_1(\gamma r)$.

Eqns. (3) are valid inside the wire, outside the wire and even in a surface layer which might surround the wire. Sommerfeld limited his study to wires with finite conductivity. Harms [7] and Goubau [3] generalized his results to wires with dielectric surface layers and surface corrugations. Since the boundary conditions differ also the solutions consist of different linear combinations of Bessel functions. h , on the other hand, is the same everywhere.

For bench testing of beam instrumentation we are only interested in the field configuration outside the wire. First, these fields must vanish for $r \rightarrow +\infty$. Second, the wire will be surrounded by air and $k \approx \omega\sqrt{\mu_0\epsilon_0}$. The phase velocity of such a wave is speed of light. Any guided wave must have a lower phase velocity and its propagation constant h is larger than k . Hence, γ and the argument γr of the Bessel functions Z_0 and Z_1 are imaginary.

Assuming γ to be a positive imaginary number the Hankel functions $Z_0 = H_0^{(1)}$ and $Z_1 = H_1^{(1)}$, i.e. the Bessel functions of the third kind, show the proper behavior¹. For small arguments $H_0^{(1)}(\gamma r) \propto -\log(\gamma r/2)$ and $H_1^{(1)}(\gamma r) \propto 1/(\gamma r)$. That means, near the wire E_r and B_ϕ are proportional to $1/r$ while E_z decays slower. For large arguments $H_0^{(1)}(\gamma r) \propto e^{-\gamma r}$ and $H_1^{(1)}(\gamma r) \propto e^{-\gamma r}$. That means, further away from the wire E_r , B_ϕ and E_z decay exponentially. Hence, the power is confined to a limited distance and the wire acts as a wave guide.

Up to which distance $1/r$ remains a valid approximation for E_r and B_ϕ depends on γ , i.e. wire and surface properties. Of course, for a perfect wire $1/r$ will be fulfilled exactly up to infinite distance. For all imperfect cases γ needs to be calculated from the fact that on the wire surface, i.e. at $r = r_w$, E_z and $H_\phi = B_\phi/\mu$ must be continuous:

$$\left(\frac{E_z}{H_\phi}\right)_{\text{outside}} = \left(\frac{E_z}{H_\phi}\right)_{\text{surface}}. \quad (4)$$

The left side is given by Eqns. (3) with $Z_0 = H_0^{(1)}$ and $Z_1 = H_1^{(1)}$. The right side is given by Eqns. (3) with $Z_0 = J_0 + bY_0$ and $Z_1 = J_1 + bY_1$. b is defined by boundary conditions. For example, assuming an uncoated wire of finite conductivity would lead to $b = 0$, whereas for a coated wire with infinite conductivity $b = -J_0(\gamma_s r_c)/Y_0(\gamma_s r_c)$ ($\gamma_s^2 = k_{\text{surface}}^2 - h^2$ and $r_c = \text{radius of conductor}$) [3].

In general Eqn. (4) cannot be solved analytically. Further discussions and approximate solutions for different cases were given in [3, 5, 6, 7]. Today numerical solutions are most convenient for all practical applications. Using standard enamel coated copper wires the imaginary component of h is found to be very small. That means, the wave is truly propagating and just weakly damped.

The last variable to be evaluated is the amplitude A . It is connected to the power of the incoming signal and can be calculated by equating the time-averaged input power \bar{P}_{in} to the time-averaged power traveling along the Goubau line

$$\bar{P}_{\text{in}} = \bar{P}_{\text{G}} = \frac{1}{2} \int_S \text{Re}(\vec{E} \times \vec{H}^*) \cdot d\vec{S}. \quad (5)$$

We are only interested in cases which are not too strongly distorted from the perfect case. In these cases by far the largest power fraction is traveling in the fields outside the wire and we can perform the integration in Eqn. (5)

¹One could also assume γ to be negative imaginary and use $H_n^{(2)}$ for the solution. Equivalent solutions can be found if the modified Bessel functions K_n are used.

between r_w and $+\infty$:

$$\begin{aligned} \bar{P}_{\text{G}} &\approx \frac{1}{2} \int_{r_w}^{+\infty} \int_0^{2\pi} E_r H_\phi^* d\phi dr \\ &= \pi \frac{AA^* k^2 h}{\omega \mu_0 \gamma \gamma^*} r_w^2 \left(\frac{2}{\gamma r_w} H_0^{(1)}(\gamma r_w) H_1^{(1)}(\gamma r_w) \right. \\ &\quad \left. - [H_0^{(1)}(\gamma r_w)]^2 - [H_1^{(1)}(\gamma r_w)]^2 \right). \end{aligned} \quad (6)$$

Another way to approximate A is to equal on the wire surface the B_ϕ given by Eqns. (3) to the B_ϕ on the surface of a perfect wire with the same radius:

$$\begin{aligned} \frac{iAk^2}{\omega\gamma} H_1^{(1)}(\gamma r_w) &\approx \frac{I\mu}{2\pi r_w} \\ \Rightarrow A &\approx \frac{I}{2\pi r_w} \frac{\omega\mu\gamma}{ik^2} \frac{1}{H_1^{(1)}(\gamma r_w)}. \end{aligned} \quad (7)$$

In practice the current I can be calculated using Eqns. (6) and (7). The input power \bar{P}_{in} depends on the way the surface wave is excited and can be extracted, for example, from reflection measurements.

Finally, in good approximation the fields outside the wire can be written as:

$$\begin{aligned} E_r &\approx \frac{I e^{i(\omega t - hz)}}{2\pi r_w} \frac{h}{\omega\epsilon} \frac{H_1^{(1)}(\gamma r)}{H_1^{(1)}(\gamma r_w)} \\ B_\phi &\approx \frac{I e^{i(\omega t - hz)}}{2\pi r_w} \mu \frac{H_1^{(1)}(\gamma r)}{H_1^{(1)}(\gamma r_w)} \\ E_z &\approx \frac{I e^{i(\omega t - hz - \pi/2)}}{2\pi r_w} \frac{\gamma}{\omega\epsilon} \frac{H_0^{(1)}(\gamma r)}{H_1^{(1)}(\gamma r_w)}. \end{aligned} \quad (8)$$

The similarity of E_r and B_ϕ to the fields around a charged particle beam is obvious. The Hankel functions lead only to a modified radial evolution in the aforementioned way.

Fig. (1) shows examples of the fields versus distance for 1 GHz and 10 GHz signals. For bench testing of beam instrumentation the field distribution has to maintain its $1/r$ proportionality over the full aperture of the device under test. Wire and surface properties have to be adjusted to achieve this up to the highest frequency of interest.

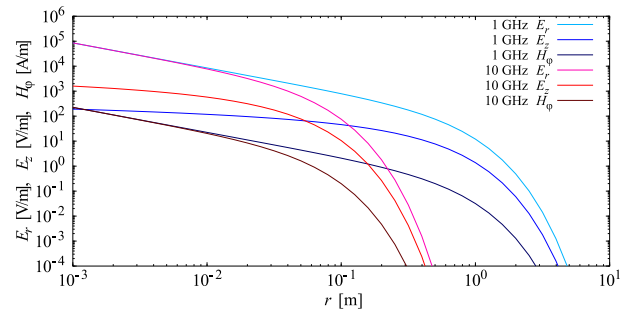


Figure 1: Radial evolution of field strengths for 1 GHz and 10 GHz assuming wire diameter = 200 μm , enamel coating thickness = 20 μm and average current = 1 A.

GOUBAU LINE

The surface wave needs to be properly excited. A simple possibility is to use a tapered cone which matches the mode in a coax cable, i.e. a quasi-TEM mode, to the fundamental TM mode on the wire. Such a cone can also be used to capture the TM mode and re-feed it into a coax cable. To reduce reflections the cones and the conductors inside have to follow a proper impedance profile. These cones and the wire form the basic setup of a Goubau line (Fig. 2).

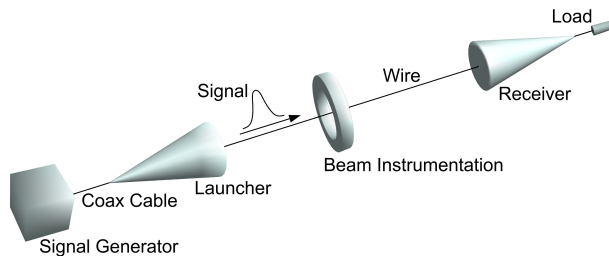


Figure 2: Sketch of a Goubau line for bench testing.

MEASUREMENTS AND DISCUSSIONS

We have set up a Goubau line using aluminium cones provided by J. Musson from JLab². The wire has a diameter of 0.9 mm and is enamel coated. To reduce reflections the shape of the conductors inside the cones was calculated to achieve an impedance evolution of the Klopfenstein type [8]. They have been assembled from brass tubes of 1 - 7 mm diameter.

The response of a current transformer (CT) has been measured on our Goubau line and in our standard setup, a so called spider which is a structure terminated by 50 Ω (Fig. 3). The CT response on the Goubau line has been normalized to the 50 Ω case using Eqns. (6) and (7).

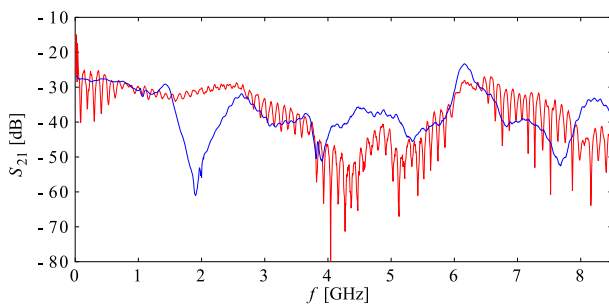


Figure 3: CT response in a spider (blue) and CT response in a Goubau line normalized to a 50 Ω environment (red).

Around 1 GHz the match between the two measurements is very good. And also around 3 GHz the match is quite good. That means, the notch around 2 GHz in the spider measurements is an artifact of the measurement setup. The CT itself works well. It is well-known that these artifacts

²The cones were originally manufactured by S. Rubin from Rubytron Inc. who provided them to JLab for Goubau line tests.

appear at high frequencies leading to false interpretations of the performance of the device under test. Improving this situation is the main motivation for studying the Goubau line.

These measurements and additional reflection measurements of the Goubau line allowed us to identify standing waves between the cones as the main perturbation of the CT response measurements [4]. They are the cause for the equidistant resonances seen in Fig. 3 and are a result of too strong reflections by the cones, i.e. imperfect geometries.

SUMMARY

In view of an application to bench testing of beam instrumentation we have reviewed the theory of electromagnetic surface waves traveling along straight, round wires. These are non-radiating waves which are bound to the wire due to imperfections, e.g. finite conductivity, surface coatings or corrugations. They consist of a weakly damped fundamental TM mode. Near the wire this mode resembles closely the fields around a beam of charged particles. Thus a Goubau line can indeed be used as a tool for bench testing of beam instrumentation devices.

A first Goubau line has been assembled and tested with very promising results. We could identify signal reflections as a single cause for the strongest of the observed perturbations. To reduce these reflections we will improve the cones and their inner conductors. This will increase power transfer to the TM mode and more importantly reduce standing waves between the cones.

ACKNOWLEDGMENT

We would like to thank J. Musson from JLab for sharing his experience with us and for providing the cones.

Bergoz Instrumentation is associated partner of the oPAC network.

REFERENCES

- [1] J. Musson et al., "Application of Goubau Surface Wave Transmission Line for improved Bench Testing of Diagnostic Beamline elements", Proc. of PAC09, 2009.
- [2] M. Spata et al., "Evaluation and Correction of the non-linear distortion of CEBAF Beam Position Monitors", Proc. of IPAC11, 2011.
- [3] G. Goubau, "Surface Waves and their Application to Transmission Lines", J. Appl. Phys. 21, 1950.
- [4] F. Stulle, J. Bergoz, "The Goubau line — Surface Waves for Bench Testing of Beam Instrumentation at high Frequencies", in Proc. of BIW12, 2012.
- [5] A. Sommerfeld, "Ueber die Fortpflanzung elektrodynamischer Wellen längs eines Drahtes", Ann. Phys. 303, 1899.
- [6] D. Hondros, "Über elektromagnetische Drahtwellen", Ann. Phys. 335, 1909.
- [7] F. Harms, "Elektromagnetische Wellen an einem Draht mit isolierender zylindrischer Hülle", Ann. Phys. 328, 1907.
- [8] R.W. Klopfenstein, "A Transmission Line Taper of improved Design", Proc. of the IRE 44, 1956.