# A 1 GeV CW FFAG HIGH INTENSITY PROTON DRIVER* 

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#### Abstract

The drive for high beam power, high duty cycle, and reliable beams at reasonable cost has focused world attention on fixed-field accelerators, notably Fixed-Field Alternating Gradient accelerators (FFAGs). High-intensity GeV proton drivers are of particular interest, as these encounter duty cycle and space-charge limits in the synchrotron and machine size concerns in the weaker-focusing cyclotron. Recently, the concept of isochronous orbits has been explored and developed for non-scaling FFAGs using powerful new methodologies in FFAG accelerator design. These new breeds of FFAGs have been identified by international collaborations for serious study thanks to their potential applications including Accelerator Driven Subcritical Reactors (ADS) and Accelerator Transmutation of Waste. The extreme reliability requirements for ADS mandate CW operation capability and the FFAGs strong focusing, particularly in the vertical, will serve to mitigate the effect of space charge (as compared with the weak-focusing cyclotron). This paper reports on these new advances in FFAG accelerator technology and presents a stable, 0.251 GeV isochronous FFAG for an accelerator driven subcritical reactor.


## FFAG DESIGN

For a machine to be isochronous, orbital path lengths must be proportional to velocity. However, the orbital path length of particles with a given momentum tracks with the B field and at relativistic energies, momentum is an increasingly nonlinear function of velocity. Therefore, the integrated $B$ field must be proportional to the relativistic velocity and therefore a nonlinear function of radius. A nonlinear field expansion combined with an appropriate edge angle can not only constrain the orbit at each momentum to be proportional to velocity, it can simultaneously control the tune through edge and weak focusing.

In comparison, a cyclotron relies on a predominately dipole field, and is therefore limited in adapting path length to velocity as the energy becomes relativistic. A nonscaling FFAG, however, with nonlinear gradients and simple linear edge contours can maintain isochronous orbits well into strongly relativistic energy regimes well beyond the reach of cyclotrons (and in a compact format).

To achieve these new advances, particularly the application of strongly nonlinear fields, powerful new methodologies in accelerator design and simulation were pioneered using control theory and optimizers in advanced design

[^0]scripts [1] with final description and simulation in the accelerator simulation code, COSY INFINITY [2].

Starting machine parameters are first generated using Mathematica ${ }^{\circledR}$ design scripts and then imported into COSY INFINITY which has a full complement of sophisticated simulation tools to fully and accurately describe the FFAGs complex electromagnetic fields - including realistic edge-field effects and high-order dynamics [3, 4].

The physical design parameters are interpreted and expanded in COSY into a midplane field description (in polar coordinates) which corresponds to realistic components with a further capability of exporting an accurate 3D out-of-plane field data set. This magnetic field lattice data in either 2D or 3D can then be utilized in a number of advanced accelerator codes for studies - such as the space charge ones which are the topic of this work.

FFAG rings are completely periodic and a triplet cell structure containing a vertically defocusing D magnet positioned between two F magnets was chosen as the base lattice unit. A minimum 0.3-0.5 m length has been imposed between magnets to prevent end-field overlap and cross talk between magnets. The long straight is 2 m to accommodate injection, extraction and the acceleration cavities.

A four-cell ring periodicity was found to be a strong initial starting point as shown in Fig. 1 and isochronous over an energy range from $0.25-1 \mathrm{GeV}$. The ring parameters are given in Table 1. Note that a complete CW accelerator system would likely entail an H - injector. (Use of H - in the lower energy ring permits CW injection into the higher-energy ring through charge-changing or stripping methods.)


Figure 1: Magnetic mid-plane field of one cell of the FFAG design.

Table 1: Ring Parameters for the 250 MeV to 1 GeV FFAG Design

| Parameter | $\mathbf{2 5 0} \mathbf{~ M e V}$ | $\mathbf{5 8 5} \mathbf{M e V}$ | $\mathbf{1 0 0 0} \mathbf{~ M e V}$ |
| :--- | :---: | :---: | :---: |
| Avg. Radius | 3.419 m | 4.307 m | 5.030 m |
| $\nu_{x} / \nu_{y}$ (cell) | $0.380 / 0.237$ | $0.400 / 0.149$ | $0.383 / 0.242$ |
| Field F/D | $1.62 /-0.14 \mathrm{~T}$ | $2.06 /-0.31 \mathrm{~T}$ | $2.35 /-0.42 \mathrm{~T}$ |
| Magnet Size F/D | $1.17 / 0.38 \mathrm{~m}$ | $1.59 / 0.79 \mathrm{~m}$ | $1.94 / 1.14 \mathrm{~m}$ |

## SPACE CHARGE

Due to the small gap and relatively large width of the magnets in this design, both the direct and image space charge tune shifts must be considered. The direct space charge tune shift can be estimated assuming an un-bunched round beam and the indirect contribution is calculated for a round beam between parallel conducting walls as:

$$
\begin{align*}
& \Delta Q_{x}=- \frac{r_{0} R}{\beta}\left(\frac{N}{\gamma^{2} L}\left[\frac{1}{2 \epsilon_{n}}-\frac{\pi^{2}\left\langle\beta_{x}\right\rangle}{24 \beta \gamma h^{2}}\right]-\right. \\
&\left.\frac{N_{\text {tot }} \pi\left\langle\beta_{x}\right\rangle}{24 \beta \gamma R}\left(\frac{1}{2 h^{2}}+\frac{\beta^{2}}{g^{2}}\right)\right)  \tag{1}\\
& \Delta Q_{y}=-\frac{r_{0} R}{\beta}\left(\frac{N}{\gamma^{2} L}\left[\frac{1}{2 \epsilon_{n}}+\frac{\pi^{2}\left\langle\beta_{x}\right\rangle}{24 \beta \gamma h^{2}}\right]+\right. \\
&\left.\frac{N_{t o t} \pi\left\langle\beta_{x}\right\rangle}{24 \beta \gamma R}\left(\frac{1}{2 h^{2}}+\frac{\beta^{2}}{g^{2}}\right)\right) . \tag{2}
\end{align*}
$$

Where $L$ is the machine length ( $L=2 \pi R$ ), $\epsilon_{n}$ is the normalised emittance, $h$ is the beam pipe half height, $g$ is the magnet half-gap (both $g$ and $h$ are taken to be 0.025 m ) and all other symbols have their usual meanings. The direct and indirect tune shift contributions for this design at 300 MeV and 1 GeV are given in Table 2.

Table 2: Contributions to Estimated Space Charge Tune Shift Assuming a 10 mA CW Beam with a $10 \pi \mathrm{~mm} \mathrm{mrad}$ Emittance

|  |  | Direct | Indirect |
| :--- | :---: | :---: | :---: |
| $\Delta \nu_{x}$ | 300 MeV | $1.85 \times 10^{-3}$ | $+1.604 \times 10^{-4}$ |
| $\Delta \nu_{z}$ | 300 MeV | $1.85 \times 10^{-3}$ | $-1.891 \times 10^{-4}$ |
| $\Delta \nu_{x}$ | 1 GeV | $-2.686 \times 10^{-4}$ | $+6.496 \times 10^{-5}$ |
| $\Delta \nu_{z}$ | 1 GeV | $-2.686 \times 10^{-4}$ | $-7.662 \times 10^{-5}$ |

## Simulation Setup

The OPAL framework [5] was chosen for space charge studies as it provides accurate tracking and comprehensive benchmarking against existing high intensity accelerators such as the PSI 590 MeV cyclotron. Presently, OPAL is the only freely available code which is capable of comprehensively modelling FFAGs with detailed space charge, although the authors are aware of developments to other
codes such as COSY-INFINITY and ZGOUBI which may, in time, provide a comparison.

A mid-plane field map generated using COSYINFINITY is imported into OPAL which interpolates to find out-of-plane field values. Closed orbits were established by minimisation of the phase space area using turn-by-turn single particle tracking. The basic closed orbit positions and time-of-flight with energy are shown in Figs. 2 and 3. As OPAL is a time-step based code, the PROBE element (as described in the user manual [5] and recently updated) has been utilised to provide accurate turn-by-turn data at a physical position in the ring, without assuming isochronous operation of the machine.


Figure 2: Closed orbit positions as a function of proton kinetic energy.


Figure 3: Time of flight per turn as a function of proton kinetic energy.

Matching in the horizontal and vertical planes was achieved using single particle tracking for 100 turns with a 1 mm offset in each plane respectively. As the injection scheme is not yet finalised, matching was achieved at the centre of the long straight section where the twiss parameter $\alpha_{x, z}=0$, allowing the remaining twiss parameters to be calculated from the ratio of the phase space ellipse axes, in the horizontal $(x)$ plane,

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$$
\begin{equation*}
\beta_{x}=\frac{x_{\max }}{x p_{\max }}, \gamma_{x}=\frac{1}{\beta_{x}} \tag{3}
\end{equation*}
$$

and similarly in the vertical $(z)$ direction. Matching was performed at two different energies for comparison, at 300 MeV and 500 MeV . The resulting twiss parameters are given in Table 3.

Table 3: Twiss parameters used to generate simulation input.

|  | $\mathbf{3 0 0} \mathbf{M e V}$ | $\mathbf{5 0 0} \mathbf{M e V}$ |
| :---: | :---: | :---: |
| $\beta_{x}$ | 3.1130 | 4.1903 |
| $\beta_{z}$ | 3.6717 | 6.7633 |
| $\gamma_{x}$ | 0.3212 | 0.2386 |
| $\gamma_{z}$ | 0.2724 | 0.1479 |

Matched beams were tracked using OPAL for 50 turns at a fixed energy of 300 MeV . A parabolic beam distribution of 1000 macro particles with an unnormalized emittance of $10 \pi \mathrm{~mm}$ mrad was used in the horizontal and vertical planes with a uniform longitudinal bunch filling $1 \%$ of the ring circumference.

## Simulation Results

The evolution of the RMS emittance over 50 turns at varying beam currents is shown in Fig. 4. The increase in emittance in the longitudinal direction reflects the lack of RF in this particular simulation. This will be eliminated from the simulation with the inclusion of RF in the next iteration of isochronous design. Although the $10 \pi \mathrm{~mm} \mathrm{mrad}$ emittance is quite small, with a 10 mA beam current there is no appreciable emittance growth in either transverse direction. The small variability in the transverse emittance is due to a number of factors including coupling between transverse and longitudinal planes. This coupling will be investigated further as work progresses.

## FUTURE WORK

Ongoing work will further optimise the machine design towards isochronicity and study space charge effects in more detail including acceleration, longitudinal effects and multiple bunch interactions.

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Figure 4: Evolution of (normalised) RMS emittance over 50 turns starting with a $10 \pi \mathrm{~mm}$ mrad (unnormalized) parabolic distribution for varying beam currents, where $x$ is horizontal (top), $y$ is longitudinal (middle) and $z$ is vertical (lower).
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